DOCUMENT RESUME

ED 406 126

SE 052 746

| | |
|---|---|
| TITLE | Mathematics 26: Teacher Resource Manual. Integrated Occupational Program. Interim 1991. |
| INSTITUTION | Alberta Dept. of Education, Edmonton. Curriculum Branch. |
| REPORT NO | ISBN-1-55006-212-3 |
| PUB DATE | 91 |
| NOTE | 331p.; For Science 26 Teacher Resource Manual, see SE 052 747. |
| PUB TYPE | Guides - Classroom Use - Teaching Guides (For Teacher) (052) -- Tests/Evaluation Instruments (160) |
| EDRS PRICE | MF01/PC14 Plus Postage. |
| DESCRIPTORS | Calculators; Computer Uses in Education; Curriculum Design; Curriculum Guides; *Daily Living Skills; Educational Strategies; Estimation (Mathematics); Foreign Countries; Geometry; High Schools; Instructional Materials; Integrated Activities; Integrated Curriculum; Learning Activities; Lesson Plans; Manipulative Materials; *Mathematical Applications; Mathematics Education; *Mathematics Instruction; Mathematics Skills; Measurement; Problem Solving; *Resource Materials; *Secondary School Mathematics; Student Projects; Teaching Guides; Teaching Methods; Thinking Skills |
| IDENTIFIERS | Alberta |

ABSTRACT

The Integrated Occupation Mathematics 26 Program is an optional program designed to allow students in Alberta, Canada to meet the credit requirements of the Certificate of Achievement and facilitate transfer to the General High School Diploma Program. This Teacher Resource Manual is provided to assist classroom teachers to implement the Mathematics 26 Program. Teachers can use this manual as a practical planning and instructional tool in translating the intentions of the mathematics program into classroom practice. The manual provides: (1) information about the goals of the mathematics program, (2) four themes that address the development of prescribed concepts, skills, and attitudes, (3) suggestions for planning, relating the mathematical instruction to essential life skills, and utilizing community resources throughout the mathematics program, (4) strategies for developing student computational skills, solving problems, and using various technologies, (5) suggestions for planning experiential learning activities, and (6) strategies for assessing and evaluating student progress. The document is divided into four sections. The Introduction describes the manual's purpose and modes of utilization, and gives a scope and sequence for Math 26 and how it relates to other programs. The second section describes the sub-themes of earning money, budgeting and banking, skills for the consumer, and math in the workplace that are seen throughout the manual. The next section presents the program emphases of problem solving, uses of technology, computational facility and estimation, situational and concrete approaches, and evaluation. The last section contains three appendices listing resources, consumer affairs offices, and regional offices of education. (MDH)

ED 406 126

# INTEGRATED OCCUPATIONAL PROGRAM

## Teacher Resource Manual

# MATHEMATICS 26

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Alberta
EDUCATION

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# Teacher Resource Manual

## Mathematics 26

INTERIM – 1991

**NOTE:** This publication is a support document. The advice and direction offered is suggested only. Consult the Program of Studies/Curriculum Guide to identify the prescriptive contents of the Mathematics 26 Program.

ALBERTA EDUCATION CATALOGUING IN PUBLICATION DATA

Alberta. Curriculum Branch.
Mathematics 26 : teacher resource manual.

ISBN 1-55006-212-3

1. Mathematics -- Study and teaching (Secondary).
I. Title. II. Integrated Occupational Program.

QA14.A333t 1991 372.7

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# ACKNOWLEDGEMENTS

Alberta Education acknowledges with appreciation the contributions of the following individuals and school jurisdictions to the development and validation of this document.

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# INTRODUCTION

## PURPOSE

This *Teacher Resource Manual* (TRM) has been developed to assist classroom teachers to implement the Integrated Occupational Mathematics 26 Program. Teachers are encouraged to use this manual as a practical planning and instructional tool in translating the intentions of the mathematics program, as outlined in the *Mathematics 26 Program of Studies/Curriculum Guide*, into classroom practice. Although shaded statements or segments within this manual indicate prescriptive content of the Program of Studies, all other advice and direction provided are suggested only and are not legally binding.

This TRM provides:

- further information about the goals and objectives of the mathematics program
- four themes that address the development of prescribed concepts, skills and attitudes
- suggestions for planning and implementing the program
  - instructional strategies
  - sequenced activities
  - a correlation of strategies and activities to learning resources
- suggestions for relating mathematical instruction to essential life skills and other applications across the curriculum
- suggestions for utilizing community resources throughout the mathematics program
- strategies for developing student competence in performing computations, in solving problems, and in using various technologies (e.g., the calculator and computer)
- situational and concrete approaches that give meaning to abstract concepts and skills in mathematics
- strategies for assessing and evaluating student progress.

The binder format was chosen for this manual to enable teachers to add strategies, samples of student work and other activities that have proven effective through experience. During cooperative planning sessions, pages may be easily removed and shared with other Integrated Occupational Program teachers. This exchange will facilitate awareness of content in other subject areas, thereby increasing opportunities for students to relate mathematical competencies to applications in everyday life, the occupational courses and other academic disciplines.

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## HOW TO USE THE TEACHER RESOURCE MANUAL

This manual has been organized to provide ready access to both thematic units and suggestions for addressing program emphases and methodology within the Mathematics 26 program.

### THEMES

It is recommended that themes provide the vehicle for instruction in Mathematics 26. Four thematic units of instruction that assure coverage of the prescribed concepts, skills and attitudes have been provided in this manual. The themes can be accessed by using the first tab along the side of the manual.

The chart below indicates the nature of investigations within themes developed for Mathematics 26, and illustrates their relationship to investigations outlined in themes developed for Mathematics 16.

| | MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|---|
| Theme A<br>EARNING MONEY | • FINDING EMPLOYMENT<br>• COMPUTING INCOME | • PAYING TAXES<br>• BUYING AND SELLING |
| Theme B<br>BUDGETING AND BANKING | • USING BANK ACCOUNTS<br>• PREPARING BUDGETS | • USING CREDIT<br>• SAVING AND INVESTING |
| Theme C<br>SKILLS FOR THE CONSUMER | • BUYING GOODS AND SERVICES<br>• OPERATING A VEHICLE | • ESTABLISHING A PLACE OF YOUR OWN<br>• BUYING A VEHICLE |
| Theme D<br>MATH IN THE WORKPLACE | • ESTIMATING AND MEASURING<br>• USING SCALE DRAWINGS AND MODELS | • DESIGNING AND BUILDING<br>• MONITORING INVENTORY AND CASH FLOW |

Teachers are encouraged to examine each theme and its corresponding activities and learning resources well in advance of instruction. The activities and suggestions provided within each theme are numerous. Advance planning should include a synthesis of effective strategies from the teacher's repertoire of personal experience together with suggestions in this manual considered most appropriate to student needs.

### PROGRAM EMPHASES AND METHODOLOGY

Suggestions for addressing program emphases and methodology within the Mathematics 26 program are provided in the five sections that follow the themes. These sections of the manual can also be accessed by using tabs, and are identified as:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Situational and Concrete Approaches
- Assessment/Evaluation.

Each of these sections clarifies the intent of a particular component of instruction, and suggests teaching practices that will assist students to acquire an understanding of prescribed concepts, skills and attitudes. Teachers are encouraged to reference the ideas provided throughout these sections of the manual as they plan learning activities within each theme of the mathematics program.

# PROGRAM PLANNING

## GENERAL COURSE PLANNING

Course planning and organization must take into consideration the sequential and developmental nature of learning in mathematics, and address students' abilities, needs, interests and preferred ways of approaching tasks. Prescribed concepts, skills and attitudes should be appropriately clustered and applied to progressively difficult and/or age-appropriate situations. The four themes developed in this manual are intended to be descriptive rather than prescriptive. Teachers are encouraged to plan for instruction using activities and suggestions provided throughout each theme, relevant sections of the *Program of Studies/Curriculum Guide*, other locally developed themes and materials, or a combination of approaches.

Through cooperative conferencing, teachers may find that students are required to use certain mathematics-related competencies in other courses before they are learned in mathematics. Joint planning and negotiation with teachers of other courses will assist in establishing an integrated program that places consistent expectations upon the student. (For example, students may benefit from the study of the theme "Math in the Workplace" early in the program as this theme addresses skills frequently demanded of students in their occupational courses.)

Course planning should also focus attention on effective strategies for solving problems, using technology (e.g., the calculator and computer) and performing computation/estimation in practical situations. The strategies and activities suggested throughout both the *Program of Studies/Curriculum Guide* and this manual are numerous, but by no means exhaustive. **Be prepared to add, delete, and modify activities in adapting instructional materials and methodology to the particular circumstances of the classroom and student.**

## TIME ALLOCATIONS

Mathematics 26 is a three-credit course. Instructional time for this course is to be allocated at the discretion of the school or jurisdiction administration to reflect the needs of individual students.

The chart below suggests time distribution among themes developed in the *Teacher Resource Manual*. These suggestions are intended to provide guidance for the teacher.

| THEME | ELECTIVE COMPONENT | SUGGESTED TIME ALLOCATION |
|---|---|---|
| EARNING MONEY | REMEDIATION AND/OR ENRICHMENT | $\frac{1}{4}$ |
| BUDGETING AND BANKING | | $\frac{1}{4}$ |
| SKILLS FOR THE CONSUMER | | $\frac{1}{4}$ |
| MATH IN THE WORKPLACE | | $\frac{1}{4}$ |

## COMMUNITY PARTNERSHIPS

The mathematics program should enable students to recognize the relevance of computational competence and problem-solving skills in daily life experiences within the home, community and work environments. Within this context, students will be expected to demonstrate competencies that will enable them to:

- apply mathematical concepts and skills to practical situations
- set goals, solve problems and make informed decisions
- prepare for a chosen occupation or career.

Community partnerships (i.e., community-based learning experiences) will foster an appreciation of mathematics for its usefulness and relevance. Community involvement will assist students to transfer specific mathematical concepts and skills to more generalized situations in everyday life and the world of work. Guest speakers, field trips, job shadowing and mentorship are a few examples of meaningful community associations.

Suggestions for relevant community partnerships in the mathematics program include:

- inviting guest speakers from local government, business and industry to discuss topics related to those studied in thematic investigations
- visiting local business, industry, and recreational facilities for first-hand observation and real life experience in areas related to the themes studied
- walking through the community in search of applications made of the concepts and skills being studied (e.g., advertising billboards, information/direction signs, tools/units of measure, presence of geometric form)
- visiting local TV stations or newspaper plants to gather information related to mathematics and the media
- investigating career and employment opportunities in areas that require specific mathematical competencies (e.g., job shadowing, mock employment interviews).

Community agencies and groups that may provide meaningful contributions to the mathematics program include:

- businesses involved in retail sales and the promotion of consumer products/services (e.g., supermarkets, drug stores, department stores, hardware stores)
- financial institutions (e.g., banks, credit unions, loan companies)
- government agencies (e.g, Consumer and Corporate Affairs, Labour and Employment Standards, Career Development and Employment, Revenue Canada, Unemployment Insurance Commission, Workers' Compensation Board)
- businesses and industries offering potential career opportunities (e.g., construction, decorating and repair service, retail sales and marketing, food production and service).

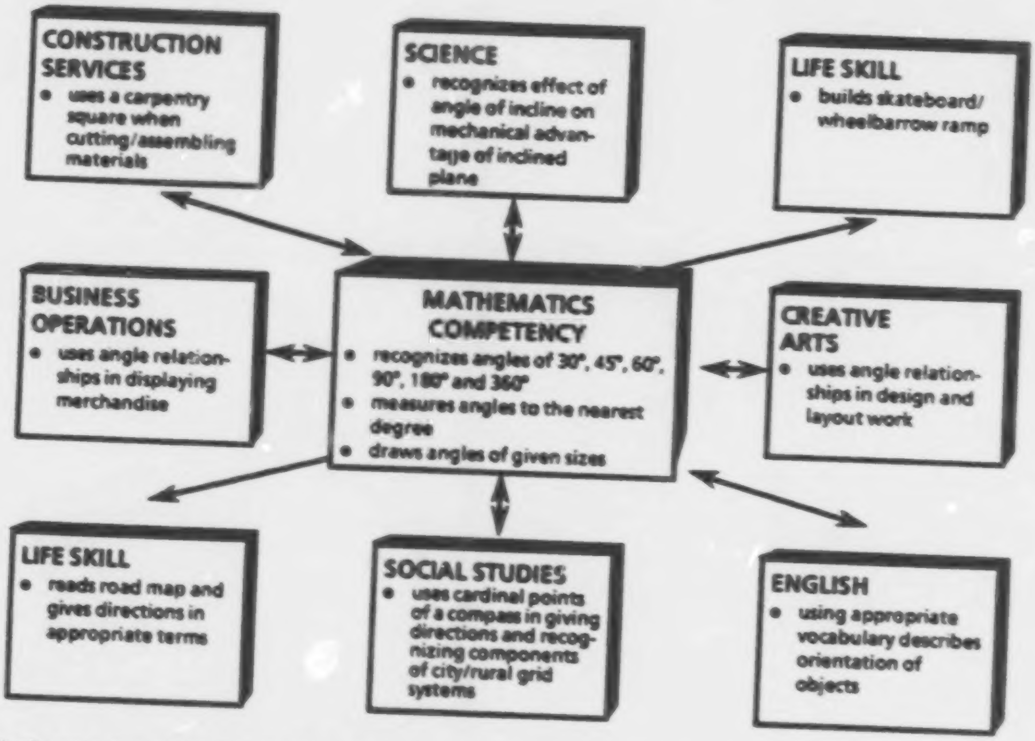
Additional suggestions for inviting members of the community into the classroom, or having students involved in the community by way of meaningful activities linked to mathematics are provided throughout each theme developed in this manual. These suggestions may be effective in furthering the objectives of each theme, as well as in reducing classroom barriers to "real life" experience.

## CURRICULAR INTEGRATION

Course content should be presented within the context of its application in daily living, the occupational program and other academic disciplines. Curricular integration will become a motivating factor as students recognize the relevance and utility of course content. Student ability to transfer concepts, skills and attitudes to a variety of situations in everyday life will improve as a result of relevant learning experiences that link mathematics with practical applications.

Teachers need to be familiar with the mathematical competencies required of students in everyday life, the occupational program and other academic disciplines. Cooperative planning and conferencing with other teachers is central to understanding differing contexts in which basic mathematical skills are used, and will assist teachers in providing practical learning experiences that encourage transfer of knowledge and skill in student learning.

### EXAMPLES OF CURRICULAR INTEGRATION BASED ON THE MATHEMATICAL COMPETENCY OF ANGLE MEASUREMENT



**CONSTRUCTION SERVICES**
- uses a carpentry square when cutting/assembling materials

**SCIENCE**
- recognizes effect of angle of incline on mechanical advantage of inclined plane

**LIFE SKILL**
- builds skateboard/wheelbarrow ramp

**BUSINESS OPERATIONS**
- uses angle relationships in displaying merchandise

**MATHEMATICS COMPETENCY**
- recognizes angles of 30°, 45°, 60°, 90°, 180° and 360°
- measures angles to the nearest degree
- draws angles of given sizes

**CREATIVE ARTS**
- uses angle relationships in design and layout work

**LIFE SKILL**
- reads road map and gives directions in appropriate terms

**SOCIAL STUDIES**
- uses cardinal points of a compass in giving directions and recognizing components of city/rural grid systems

**ENGLISH**
- using appropriate vocabulary describes orientation of objects

Angle concepts and skills are developed in mathematics and related to their applications in other areas. While in this instance application is shown in all subject areas, some skills may have a more limited base for application.

The *Program of Studies/Curriculum Guide* provides suggestions for relating prescribed content to daily life skills and to its applications in other subject areas across the curriculum. Additional suggestions and approaches for linking prescribed content with practical applications are provided throughout this manual.

## DEVELOPING AN INTEGRATED UNIT OF INSTRUCTION

Teachers are encouraged to consider a variety of factors when expanding upon an existing theme or developing a new thematic unit. The guidelines that follow provide a useful structure for developing integrated units of instruction.

1. Identify a possible theme, based on:
    - curriculum objectives
    - student needs/interests/abilities
    - availability of suitable learning resources.

2. Develop a purpose for the theme. Include:
    - appropriate thematic objectives
    - a checklist of concepts, skills and attitudes that lend themselves to the theme. Identify those skills that may need some focussed/direct teaching.

3. Consider suitable resources:
    - books, pamphlets, monographs
    - computer software and other technology
    - resources from the occupational program
    - community contacts
    - newspaper and magazine articles
    - other resources to which students may have access.

4. Design activities:
    - allocate activities to the purposes developed in Step 2
    - develop a checklist of process objectives
    - separate activities into lessons with general and specific objectives
    - sequence the lessons.

5. Develop ongoing strategies to build community partnerships into your theme:
    - field trips
    - guest speakers.

6. Plan for evaluation:
    - student's self-evaluations
    - teacher's ongoing (formative) and summative evaluations
    - peer evaluations.

7. Share the unit:
    - celebrate achievement
    - share thematic units with other teachers
    - expand, update and revise units as they are used
    - develop strategies to evaluate the updated and revised activities with a variety of student groups.

## MAKING UNIT AND DAILY LESSON PLANS

Preliminary plans for a thematic unit can be organized in a variety of ways. One possible format is illustrated below. This format provides a useful overview of intended learning outcomes, methodology, student activities and learning resources, and facilitates more detailed planning prior to each daily lesson.

**THEME:** ______________________ **NUMBER OF CLASS PERIODS:** ____________

| Period | Learning Objectives and General Activity Description | Evaluation Technique | Student Activity | Required Resources (e.g., print materials, software, audio-visual materials, manipulatives) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

The format of daily lesson plans may vary from one class period to the next, and can be adjusted to suit instructional objectives and the interests/needs of students. However, each lesson plan should:

- emphasize student involvement/activity
- give explicit guidelines for developing knowledge and process components
- establish evaluation criteria.

A sample lesson plan that illustrates the integration of these components in a problem-solving activity is provided in a subsequent section of this manual (see "Problem Solving", Resource 1: Sample Lesson Plan for Problem Solving).

# SCOPE AND SEQUENCE FOR MATHEMATICS 16–26

The scope and sequence chart provided on the following pages outlines the mathematical process and skill that is developmentally addressed throughout Mathematics 16 and 26. In recognizing that students differ in the rate at which they acquire mathematical competencies, the chart is intended to assist teachers in assessing present levels of student performance, and in diagnosing individual student strengths and weaknesses. In using the scope and sequence, it should be noted that:

- the chart is not intended to provide a sequenced plan for instruction. **Instruction should be sequenced throughout Mathematics 16 and 26 in a manner that will suit individual student needs and growth patterns**

- learning objectives that provide meaningful **learning contexts** (i.e., problem solving, use of technology, computational facility and estimation) are followed by learning objectives that support the development of process and skill within five **strands of mathematics** (i.e., number systems and operations, ratio, proportion and percent, algebra, geometry and measurement, data interpretation and display) established on the basis of tasks frequently demanded of students in everyday life

- **the skills are developmental** through Mathematics 16 and 26 (i.e., the spiral approach). Mathematics 26 provides opportunities for students to reinforce and extend their understanding of skills developed in Mathematics 16 through more extensive applications, and also prescribes new skills related to each of the five strands of mathematics

- most students will continue to use concrete operational thinking, depending upon personal and tangible experience in order to link ideas. Instruction should generally **begin with an operational understanding of ideas at the Mathematics 16 level, and encourage students to extend their thinking to more abstract levels that are cognitively appropriate** at the Mathematics 26 level. (The *Teacher Resource Manuals* for Mathematics 16 and 26 provide for the development of thinking skills through themes that reflect increasing levels of analysis and critical thought)

- **the skills are interdependent and are not meant to be taught in isolation.** Although some skills may be mastered more effectively through discrete instruction, this approach is not advocated as a primary focus of instruction. The thematic structure permits a more holistic view of instruction through the linking of strategies and skills.

**Prescribed components of the mathematics program outlined in the scope and sequence chart should be appropriately clustered and applied to progressively difficult and/or age-appropriate situations as students advance through senior high school. Teachers are encouraged to organize for instruction in ways that are consistent with the abilities, needs and interests of students, using relevant sections of the *Program of Studies/Curriculum Guide*, thematic units provided in this manual, locally developed themes, or a combination of approaches.**

Teachers may also wish to examine a scope and sequence chart for the IOP Junior High School Mathematics Program (Grades 8 and 9). An understanding of the developmental progression of process and skill occurring in junior high school will facilitate program articulation and continuity.

PROBLEM SOLVING

| MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|
| DEMONSTRATES ABILITY TO USE A VARIETY OF STRATEGIES/SKILLS IN SOLVING PRACTICAL PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY. | |
| • Applies strategies/skills that are useful in understanding a problem.<br>e.g., – reads the problem several times<br>– asks questions and refers to other sources to clarify meaning<br>– identifies key words and their meanings<br>– looks for patterns<br>– identifies given and needed information<br>– identifies extraneous information<br>– internalizes the problem by restating in his or her own words, or by visualizing the problem<br>– draws pictures/diagrams<br>– uses concrete manipulatives<br>– interprets pictures/charts/graphs<br>– relates the problem to other problems previously encountered<br>– considers alternative interpretations of the problem. | • Applies strategies/skills that are useful in understanding a problem.<br>→<br>e.g., – determines if there are hidden assumptions that contain information necessary to the solution of the problem. |
| • Applies strategies/skills that are useful in developing and carrying out a problem-solving plan.<br>e.g., – guesses and checks the result (thus improving the guess)<br>– uses logic or reason<br>– identifies and applies relationships<br>– chooses and sequences the operations needed<br>– sorts and classifies information<br>– applies selected strategies<br>– presents ideas clearly<br>– selects appropriate calculating/measuring devices and methods<br>– acts out or simulates the problem<br>– applies patterns<br>– estimates the answer<br>– documents the process used<br>– works carefully<br>– works in a group situation where ideas are shared<br>– visualizes the problem<br>– makes positive statements (e.g., "I can solve this.")<br>– uses a simpler problem (making an analogy)<br>– identifies factors relevant to the problem<br>– collects and organizes data into diagrams, charts, tables, pictures, graphs or models<br>– experiments through the use of manipulatives<br>– breaks the problem down into smaller parts<br>– formulates an equation<br>– recognizes limits and eliminates possibilities. | • Applies strategies/skills that are useful in developing and carrying out a problem-solving plan.<br>→<br>e.g., – constructs flow charts<br>– works backward through the problem<br>– examines the problem from varying perspectives/points of view. |
| • Applies strategies/skills that are useful in reviewing and applying the results of problem solving.<br>e.g., – states an answer to the problem<br>– restates the problem with an answer<br>– explains the answer in oral/written form<br>– determines if the answer is reasonable<br>– discusses with others the process used<br>– suggests other ways of solving the problem<br>– checks the answer<br>– considers the possibility of other answers/solutions<br>– makes and solves similar problems<br>– generalizes the solution and applies the process used to other situations. | • Applies strategies/skills that are useful in reviewing and applying the results of problem solving.<br>→<br>e.g., – creates other routine/non-routine problems that require the use of a similar process. |

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USE OF TECHNOLOGY

<table>
<tr><th>MATHEMATICS 16</th><th>MATHEMATICS 26</th></tr>
<tr><td colspan="2">DEMONSTRATES ABILITY TO USE A HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.</td></tr>
<tr><td>• Identifies appropriate and inappropriate uses of the calculator.<br>• Clears the display of the calculator and corrects entry errors.<br>• Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent:<br>– identifies and uses basic functions (i.e., +, −, ÷, ×, %, =, decimal, clear)<br>– enters numbers in correct sequence for subtraction and division<br>– determines whole number remainders for division<br>– uses subtotals and memory for adding, subtracting, multiplying and dividing<br>– converts fractions to decimal and percent form<br>– generates sets of multiples for a given number<br>– follows order of operations.<br>• Selects from calculator display the number of decimal places appropriate to the context of a calculation/problem.<br>• Uses estimation and mental arithmetic to check the reasonableness of answers obtained on the calculator.</td><td>• Applies previously developed calculator skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Identifies and uses additional functions on the calculator (i.e., $x^2$, $y^x$, $\sqrt{\ }$).</td></tr>
<tr><td colspan="2">DEMONSTRATES AN UNDERSTANDING OF THE BASIC OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS.</td></tr>
<tr><td>• Identifies major parts of a computer.<br>• Distinguishes between hardware and software.<br>• Explains how computers get their instructions from a program written to accomplish a specific task.</td><td>• Displays knowledge of previously developed concepts/skills related to the basic operation of computers.<br>→</td></tr>
<tr><td>• Gives examples of the applications of computers in performing tasks that require speed, accuracy, repeated operations and the processing of large amounts of data.<br>• Identifies major areas in society where computers are used and the tasks performed by computers in these areas.<br>• Gives examples of tasks that computers cannot accomplish.</td><td>• Explains the applications and limitations of computers in progressively difficult and/or age-appropriate situations.<br>→</td></tr>
</table>

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## LEARNING CONTEXTS (continued)

**COMPUTATIONAL FACILITY AND ESTIMATION**

<table>
<tr><th>MATHEMATICS 16</th><th>MATHEMATICS 26</th></tr>
<tr><td colspan="2">DEMONSTRATES ABILITY TO PERFORM COMPUTATIONS AS APPROPRIATE IN DAILY LIFE AND WORK-RELATED SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.</td></tr>
<tr><td>• Uses mental arithmetic skills that are based upon:<br>– all single-digit operations<br>– sequences of operations<br>– doubling and halving<br>– multiplying and dividing by powers of 10<br>– applications of the commutative, associative and distributive properties<br>– properties of zero and one<br>– strategies appropriate to the situation (e.g., compensation, computing from left to right).</td><td>• Uses previously developed mental arithmetic skills in progressively difficult and/or age-appropriate situations.<br>→</td></tr>
<tr><td>• Uses paper-and-pencil algorithms to perform computations with whole numbers, decimals, fractions, integers and percent within parameters provided in the Scope and Sequence (see "Number Systems and Operations" and "Ratio, Proportion and Percent").</td><td>• Uses paper-and-pencil algorithms to perform computations with whole numbers, decimals, fractions, integers and percent within parameters provided in the Scope and Sequence (see "Number Systems and Operations" and "Ratio, Proportion and Percent").</td></tr>
<tr><td>• Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent within parameters established by the nature of the problem (see "Use of Technology").</td><td>• Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent within parameters established by the nature of the problem (see "Use of Technology").</td></tr>
<tr><td>• Uses estimation skills that are based upon:<br>– stating the largest and smallest reasonable answer to a problem before solving the problem<br>– predicting whether a computation will result in a larger or smaller number<br>– forecasting an order of magnitude for the result of a computation (e.g., 10's, 100's, 1000's)<br>– predicting the magnitude of the result of a computation through use of strategies appropriate to the situation (e.g., front-end estimation, rounding, clustering, compatible numbers).</td><td>• Uses previously developed estimation skills in progressively difficult and/or age-appropriate situations.<br>→</td></tr>
</table>

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## NUMBER SYSTEMS AND OPERATIONS

| | MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|---|
| | **APPLIES WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS.** | |
| WHOLE NUMBERS | • Identifies place value to one million.<br>• Reads numbers to one million in context.<br>• Write numbers in numeric or word form to one million.<br>• Quantifies and orders numbers to one million.<br>• Identifies even and odd numbers.<br>• Rounds numbers to nearest 10, 100, 1000, 10 000 and 100 000.<br>• Interprets/uses numerical terms (i.e., pair, triplet, decade, dozen, century).<br>• Adds and subtracts whole numbers (limit for paper-and-pencil computation: numbers less than 10 000).<br>• Uses mental arithmetic to determine products when multiplying by multiples of ten.<br>• Multiplies whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100).<br>• Applies divisibility rules for 2, 3, 5 and 10.<br>• Divides whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100).<br>• Determines the lowest common multiple for pairs of numbers less than 10.<br>• Expresses numbers up to 50 as the product of prime factors.<br>• Determines the greatest common factor for pairs of numbers less than 50. | • Uses previously developed whole number skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Describes the meaning of terms related to the use of exponents (i.e., exponent, base, power, "squared", "cubed").<br>• Determines the value of a power, using whole number base and exponents of 2 and 3.<br>• Determines the square root of whole numbers, using estimation/square root tables/calculator. |
| | **APPLIES DECIMAL SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS.** | |
| DECIMALS | • Identifies place value to thousandths.<br>• Reads decimals to thousandths in context.<br>• Writes decimals in numeric form to thousandths.<br>• Quantifies and orders numbers to three decimal places in applications.<br>• Rounds decimals to the nearest whole number, tenth and hundredth.<br>• Adds and subtracts decimals (limit for paper-and-pencil computation: numbers to thousandths).<br>• Multiplies decimals (limit for paper-and-pencil computation: numbers that yield products to thousandths, using one- or two-digit multipliers).<br>• Divides decimals (limit for paper-and-pencil computation: numbers having five digits or less by one- or two-digit divisors; dividend may have 0, 1, 2 or 3 decimal places, divisor may be a whole number or decimal).<br>• Uses mental arithmetic to determine products/quotients when multiplying/dividing by multiples of 10. | • Uses previously developed decimal skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Recognizes place value beyond the thousandths place in work-related situations (e.g., reading the micrometer).<br>• Rounds to the nearest thousandth/ten thousandth as required in work-related situations. |

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| | | MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|---|---|
| NUMBER SYSTEMS AND OPERATIONS (cont'd) | FRACTIONS | DEMONSTRATES ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16). | |
| | | • Illustrates a fraction as part of a whole, part of a set, or a point on a number line.<br>• Describes proper/improper fractions and mixed numbers through the use of objects, pictures and diagrams.<br>• Using a number line, illustrates the relationship between whole numbers, decimals and fractions.<br>• Compares and orders fractions in applications.<br>• Identifies and determines equivalent fractions (including proper and improper fractions, mixed numbers).<br>• Identifies and expresses fractions in basic form.<br>• Converts proper/improper/mixed fractions to decimals using a calculator.<br>• Recalls decimal equivalents for one-half, thirds, quarters, fifths and tenths.<br>• Determines common denominators for fractions having denominators of 2, 3, 4, 5, 8, 10 and 16.<br>• Adds/subtracts fractions and mixed numbers having like/unlike denominators that are within the parameters stated above.<br>• Multiplies/divides proper fractions and mixed numbers by whole numbers.<br>• Calculates a fractional part of an amount in applications. | • Applies previously developed fraction skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Multiplies/divides proper fractions and mixed numbers by whole numbers/proper fractions. |
| | INTEGERS | DEMONSTRATES ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS. | |
| | | • Identifies situations that require the use of integers.<br>• Uses vocabulary related to integers (i.e., positive, negative, plus, minus, above, below, gain, loss).<br>• Places integers on the number line.<br>• Compares and orders integers in practical applications.<br>• Adds pairs/groups of one- and two-digit integers. | • Applies previously developed integer skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Adds/subtracts pairs of integers containing three digits or less.<br>• Multiplies/divides pairs of one- and two-digit integers (where multiplier and divisor are positive).<br>• Describes the use of integers in locating points on the coordinate plane. |

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STRANDS OF MATHEMATICS (continued)

<table>
<tr><th></th><th>MATHEMATICS 16</th><th>MATHEMATICS 26</th></tr>
<tr><td rowspan="2">RATIO, PROPORTION AND PERCENT</td><td colspan="2">DEMONSTRATES ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS.</td></tr>
<tr><td>• Compares two quantities in the same unit by writing ratios in the form a/b, a:b and a to b.<br>• Generates equivalent ratios.<br>• Calculates rates/unit rates by writing ratios that involve numbers with different units (e.g., 90 km/2h, 3 items for $1.00).<br>• Identifies proportions as statements about equivalent ratios.<br>• Writes proportions that describe practical problem situations.<br>• Calculates the unknown value in a proportion.<br>• Expresses whole number percents as ratios/decimals.<br>• Expresses ratios as decimals and percents (i.e., $\frac{a}{b} = \frac{?}{100}$ where b = 2, 4, 5, 10, 20, 25 or 50).<br>• Recalls fraction, decimal and percent equivalents for one-half, quarters and tenths.<br>• Expresses one-, two- and three-place decimals as percents.<br>• Determines other fraction, decimal and percent equivalents through the use of the calculator.<br>• Calculates/estimates a percent of a number.</td><td>• Applies previously developed ratio, proportion and percent skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Using a calculator, converts fractional percents to fraction and decimal forms (e.g., 12 1/2% = $\frac{1}{8}$ = 0.125).<br>→<br>• Calculates/estimates what percent one number is of another.<br>• Calculates/estimates any one of the missing elements in practical problems involving application of percent (i.e., determines percentage, rate or base).</td></tr>
<tr><td rowspan="2">ALGEBRA</td><td colspan="2">DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF EVERYDAY AND WORK-RELATED SITUATIONS.</td></tr>
<tr><td>• Distinguishes between the use of variables and constants in practical situations.<br>• Uses variables to write mathematical expressions/equations that describe familiar situations.<br>• Evaluates expressions for given whole number values of the variable.<br>• Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$).<br>• Applies equation-solving skills to practical problem situations.<br>• Interprets formulas as word statements.<br>• Uses formulas and substitution to solve problems in practical situations (e.g., if $P = 2L + 2w$, find P when $L = 10$ and $w = 8$).</td><td>• Applies previously developed algebra skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Evaluates expressions for given decimal values of the variable.<br>• Uses estimation, guess/check, concrete manipulatives and formal strategies to find decimal solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$).<br>• Applies equation-solving strategies/skills to practical situations involving decimals.<br>• Uses substitution and equation-solving strategies to find the missing element in a formula (e.g., if $P = 2L + 2w$, find L when $P = 16$ and $w = 3$).</td></tr>
</table>

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| | | MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|---|---|
| GEOMETRY AND MEASUREMENT | | DEMONSTRATES ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | GEOMETRY | • Identifies line relationships (i.e., horizontal, vertical, parallel, perpendicular and intersecting).<br>• Identifies/constructs basic two-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle).<br>• Classifies triangles and quadrilaterals by examining measures of sides and measures of angles.<br>• Identifies the relationship between the radius and diameter of a circle.<br>• Constructs circles, given either radius or diameter.<br>• Identifies similar and congruent geometric figures.<br>• Identifies/sketches/constructs models of basic three-dimensional shapes (i.e., rectangular solid, cube, cylinder, cone).<br>• Constructs geometric figures/designs, using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer). | • Uses previously developed geometry skills in progressively difficult and/or age-appropriate situations.<br>→<br>• Identifies/classifies polygons according to the number of sides.<br>• Constructs regular polygons (i.e., triangle, square, hexagon, octagon).<br>→<br>• Describes the terminology associated with the coordinate system (i.e., origin, axis, quadrant, coordinates, ordered pair).<br>• Plots a point on the coordinate plane when given its coordinates, and identifies the coordinates for a point graphed in the coordinate plane. |
| | | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | LENGTH | • Identifies common SI units of length (i.e., mm, cm, m, km).<br>• Estimates/measures length, using metric units and tools appropriate to the situation.<br>• Converts among SI units of length as required in applications.<br>• Estimates/measures/calculates the perimeter of figures bounded by line segments, using units and tools appropriate to the situation.<br>• Identifies the value of $\pi$ as the ratio of the circumference of a circle to its diameter.<br>• Develops/applies strategies for determining the circumference of a circle. | • Applies previously developed skills when measuring length in progressively difficult and/or age-appropriate situations.<br>→<br>• Develops/applies strategies for determining the perimeter of any regular polygon.<br>• Describes the Pythagorean relationship among sides of a right triangle, and applies the relationship to practical situations. |
| | | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | AREA | • Identifies common SI units of area (i.e., $cm^2$, $m^2$) and their application in practical situations.<br>• Approximates the area of two-dimensional geometric figures, using a square grid.<br>• Estimates/calculates area of the rectangle, square, triangle and circle, using units, tools and strategies appropriate to the situation. | • Applies previously developed skills when measuring area in progressively difficult and/or age-appropriate situations.<br>→<br>• Develops/applies appropriate strategies for determining the surface area of rectangular solids and cylinders. |

| | | MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|---|---|
| GEOMETRY AND MEASUREMENT (cont'd) | VOLUME | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | | • Identifies common SI units of volume (i.e., $cm^3$, mL) and their application in practical situations.<br>• Approximates the volume of rectangular solids using cubes.<br>• Estimates/calculates the volume of rectangular solids and cubes, using units, tools and strategies appropriate to the situation. | • Applies previously developed skills when measuring volume in progressively difficult and/or age-appropriate situations.<br>→<br>• Identifies the equivalence between cubic centimetres and millilitres.<br>• Develops/applies appropriate strategies for determining the volume of right cylinders. |
| | CAPACITY | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | | • Identifies common SI units of capacity (i.e., mL, L) and their application in practical situations.<br>• Estimates and measures capacity, using metric units and tools appropriate to the situation.<br>• Converts between mL and L. | • Applies previously developed skills when measuring capacity in progressively difficult and/or age-appropriate situations.<br>→ |
| | MASS | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | | • Identifies common SI units of mass (i.e., g, kg, t) and their application in practical situations.<br>• Estimates and measures mass, using metric units and tools appropriate to the situation.<br>• Converts between g and kg and between kg and t. | • Applies previously developed skills when measuring mass in progressively difficult and/or age-appropriate situations.<br>→ |
| | TIME | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | | • Uses a calendar, recognizing the relationship between days, weeks, months and years.<br>• Uses accepted standards for numeric dating.<br>• Estimates/measures/records time on the 12-hour and 24-hour clocks, using traditional and digital time pieces.<br>• Determines the interval between a pair of times.<br>• Determines the finishing time, given the starting time and duration. | • Applies previously developed skills when measuring time in progressively difficult and/or age-appropriate situations.<br>→ |
| | TEMPERATURE | | DEMONSTRATES ABILITY TO ESTIMATE AND MEASURE TEMPERATURE IN DAILY LIFE AND WORK-RELATED SITUATIONS.<br>• Estimates/measures temperature on the Celsius scale.<br>• Recalls important temperatures on the Celsius scale (i.e., boiling/freezing point of water, normal room/body temperature).<br>• Determines temperature change, including changes from below zero to above zero and vice versa. |
| | ANGLE | DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS. | |
| | | • Identifies an angle and the degree as a unit of measure.<br>• Identifies/estimates angles of 30°, 45°, 60°, 90°, 180° and 360°.<br>• Uses a protractor to measure and draw angles from 0° to 180°. | • Applies previously developed skills when measuring angles in progressively difficult and/or age-appropriate situations.<br>• Uses a protractor to measure and draw angles from 0° to 360°.<br>• Applies knowledge of the sum of the angles of a triangle in practical situations. |

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## STRANDS OF MATHEMATICS (continued)

**DATA INTERPRETATION AND DISPLAY**

| MATHEMATICS 16 | MATHEMATICS 26 |
|---|---|
| COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN PRACTICAL EVERYDAY SITUATIONS. | |
| • Distinguishes between a survey and census, also between a sample and population, identifying potential biases that may occur in surveys and samples.<br>• Interprets and analyzes data presented in tables/charts and bar/line/picture/circle graphs.<br>• Displays data in the form of tables/charts and bar/line/picture/circle graphs.<br>• Identifies the characteristics of a misleading graphical display.<br>• Determines/calculates appropriate measures of central tendency in practical situations (i.e., mean, median, mode, range).<br>• Conducts a survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing data:<br>– selects a suitable sample from a given population<br>– collects and organizes data using tally sheets/frequency tables<br>– displays data in the form of bar/line/picture/circle graphs<br>– makes inferences/generalizations about the population from which a sample was taken. | • Applies previously developed statistical skills in progressively difficult and/or age-appropriate situations.<br>→ |
| | • Explains the meaning of "probability" and gives examples of its use in today's society.<br>• Uses counting techniques such as tree diagrams to find the number of possible outcomes of an experiment.<br>• Expresses the probability of the occurrence of a single event related to practical experience or a simple experiment. |

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# LEARNING RESOURCES FOR MATHEMATICS 26

The following learning resources address goals and objectives of the Mathematics 26 program. The abbreviations for BASIC and SUPPORT resources referenced throughout the TRM are noted in brackets. A description of each learning resource is provided in Appendix A.

## STUDENT RESOURCES

### BASIC LEARNING RESOURCE

The learning resource listed below addresses the majority of the goals and learning objectives identified in the *Program of Studies/Curriculum Guide*.

Carli, Enzo, et al. *Consumer and Career Mathematics, Second Canadian Edition* (CCM). Agincourt, Ontario: Gage Educational Publishing Company, 1991.

### SUPPORT LEARNING RESOURCE

The learning resource listed below assists in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide*. Multiple copies (but not necessarily class sets) may be desired.

Cutting, Robert, et al. *Math You Need, Book 2* (MYN). Scarborough, Ontario: Nelson Canada, A Division of Thomson Canada Limited, 1982.

Lunney, Jeri, et al. *MathSense, Book 2* (MS). Scarborough, Ontario: Nelson Canada, A Division of Thomson Canada Limited, 1991.

### OTHER LEARNING RESOURCES

Other learning resources potentially useful in implementing the Mathematics 26 program are identified below. These resources have not undergone the standard review procedures of Alberta Education, and their listing is not to be construed as an explicit or implicit departmental approval for use. These titles are provided as a service only to assist local jurisdictions in identifying potential learning resources. The responsibility for evaluating these resources prior to selection and use rests with the local jurisdiction according to local policy.

Alberta Consumer and Corporate Affairs

- Booklets:
  - *2000 A.D., A Guide to Financial Awareness*
  - *Consumer Complaints*
  - *Moving Out*
  - *Taking Charge of Your Money*
- "Consumer Talk" Information Sheets:
  - *Buying a Used Car*
  - *Credit*
  - *Tenants: Living There*
  - *Tenants: Moving In*
  - *Tenants: Moving Out*
- Consumer Tip Sheets:
  - *How to Use Credit Responsibly*
  - *Protect Your Credit Rating*
- Keeping Your Balance Series:
  - *Putting Your Spending Plan on Paper*
  - *Take Care of Your Savings*

Canadian Bankers' Association

Booklets: *Credit Wise*
*Helping You Bank*

Canadian Imperial Bank of Commerce

Youth Presentation Package: *CIBC's Greatest Hits*

Dlabay, Les. *Decisions, Making Personal Economic Choices*. St. Paul, Minnesota: EMC Publishing Corporation, 1985.

Elrick, Thomas. *Forms in Your Life: A Student Workbook and Guide to Everyday Forms*. Toronto, Ontario: Globe/Modern Curriculum Press, 1981.

Lennox, Art. *Banking, Budgeting and Employment*. Phoenix, New York: Frank E. Richards Publishing Company Incorporated, 1984.

Lyng, Merwin, et al. *Applied Mathematics*. Markham, Ontario: Houghton Mifflin Canada Limited, 1989.

Revenue Canada Taxation
*Teaching Taxes Student's Workbook*
*Teaching Taxes Teacher's Update*
*Teaching Taxes Taxopedia*

Wool, John. *Useful Arithmetic, Volume 1*. Phoenix, New York: Richards Publishing Company, 1987.

Wool, John. *Useful Arithmetic, Volume 2*. Phoenix, New York: Richards Publishing Company, 1988.

## TEACHER RESOURCES

The teacher resources listed below assist in addressing major goals and learning objectives identified in the *Program of Studies/Curriculum Guide*.

Alberta Education: *Problem-Solving Challenge for Mathematics*. Edmonton, Alberta: Alberta Education, 1985.

Alberta Education: *Problem Solving in Mathematics: Focus for the Future*. Edmonton, Alberta: Alberta Education, 1987.

Board of Education for the City of Etobicoke. *Making the Grade, Evaluating Student Progress*. Scarborough, Ontario: Prentice-Hall Canada Inc., 1987.

Calgary Board of Education. *Metric Practice Guide for Teachers*. Edmonton, Alberta: Alberta Government Services, Metric Branch and Alberta Education, 1982.

Charles, R., et al. *How to Evaluate Progress in Problem Solving*. Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Easterday, K., et al. *Activities for Junior High School and Middle School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1981.

Hope, J., et al. *Mental Math in Junior High*. Palo Alto, California: Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: The National Council of Teachers of Mathematics, 1989.

## TECHNOLOGY AND MEDIA

There is a growing collection of technology and media resources (e.g., films, videos, computer software, kits, pamphlets) that support the development of specific learning objectives within the mathematics program. Some of these resources have been identified below. Teachers are encouraged to contact local media services, libraries and government agencies in obtaining these and other resources considered appropriate to the mathematics program.

### SUPPORT LEARNING RESOURCES

The computer software programs listed below assist in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide.*

*Math Strategies: Problem Solving.* Willowdale, Ontario: Science Research Associates (Canada) Limited, 1983.

*Money Manager.* Washington, D.C.: Computer Age Education Incorporated, 1984.

### OTHER LEARNING RESOURCES

Other computer software programs potentially useful in implementing the Mathematics 26 program are identified below. These resources have not undergone the standard review procedures of Alberta Education, and their listing is not to be construed as an explicit or implicit departmental approval for use. The responsibility for evaluating these resources prior to selection and use rests with the local jurisdiction according to local policy.

*Managing Lifestyles: Survival Math Skills.* Pleasantville, New York: Sunburst Communications Incorporated, 1984.

*Spreadsheets for Students.* Mississauga, Ontario: Copp Clark Pitman Limited, 1985.

*SRA Computer Drill and Instruction, Mathematics, Level D.* Willowdale, Ontario: Science Research Associates (Canada) Limited, 1981-83.

An annotated list of additional computer software available in mathematics is provided in Alberta Education's catalogue of *Computer Courseware Evaluations* (Curriculum Branch, Student Programs and Evaluation Division, Alberta Education, 1986) and annual supplements.

The video programs listed below also assist in addressing some of the learning objectives identified in the *Program of Studies/Curriculum Guide.* Most of these programs are available through ACCESS NETWORK and local resource/media centres.

*A History of Income Tax in Canada.* Revenue Canada Taxation.

*Mathways.* Kinetic, 1980.

*Math Wise.* Bloomington, Indiana: Agency for Instructional Television, 1981.

*Revenue for Canada.* Revenue Canada Taxation.

*Street Cents.* Alberta Consumer and Corporate Affairs/Canadian Broadcasting Corporation, 1990.

## ACCESS NETWORK

ACCESS offers a variety of resources and services to teachers. For a nominal dubbing and tape fee, teachers may obtain copies of educational programs directly from ACCESS NETWORK. ACCESS also offers a service called "Night Owl Dubbing". This allows educators to tape selected programs directly from their own televisions.

ACCESS publishes both an *Audio-Visual Catalogue* and a comprehensive schedule of programming, available on request.

For additional information, contact ACCESS NETWORK, Media Resource Centre, 295 Midpark Way S.E., Calgary, Alberta, T2X 2A8 (from outside of Calgary, telephone toll free, 1-800-352-8293; in Calgary, telephone 256-1100).

## REGIONAL AND URBAN RESOURCE CENTRES

There are a number of resource/media centres serving the needs of teachers within different geographical areas and jurisdictions across Alberta. Each centre carries audio and visual resources, and publishes its own catalogue listing the resources in its collection. These centres operate as libraries, and lend materials for specified time periods. For more information, contact the resource/media centre in your area.

### Regional Resource Centres

Zone I

Zone One Regional Resource Centre
P.O. Box 6536 / 10020 – 101 Street
Peace River, Alberta
T8S 1S3
Telephone: 624-3187

Zones II and III

Central Alberta Media Services (CAMS)
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: 464-5540 / 467-8896

Zone IV

Alberta Central Regional Education Services (ACRES)
County of Lacombe No. 14
Box 3220 / 5140 – 49 Street
Lacombe, Alberta
T0C 1S0
Telephone: 782-5730

Zone V

South Central Alberta Resource Centre (SCARC)
Westmount School
Box 90 / Wheatland Trail
Strathmore, Alberta
T0J 3H0
Telephone: 934-5028

Zone VI

Southern Alberta Learning Resource Centre (SALRC)
909 - Third Avenue North, Room #120
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: 320-7807

Urban Resource Centres

Bonnyville

Area Superintendent
Lakeland Public School District No. 5460
Postal Bag 1001
Bonnyville, Alberta
T9N 2J7
Telephone: 826-3145

Calgary

Supervisor
Education Media
Calgary Board of Education
3610 – 9th Street S.E.
Calgary, Alberta
T2G 3C5
Telephone: 294-8540

Calgary

Supervisor
Instructional Materials
Calgary Separate School Board
6220 Lakeview Drive S.W.
Calgary, Alberta
T3E 6T1
Telephone: 246-6663

Edmonton

Supervisor
Curricular Resources
Edmonton Catholic Schools
St. Anthony's Teacher Centre
10425 – 84 Avenue
Edmonton, Alberta
T6E 2H3
Telephone: 439-7356

Edmonton

Learning Resource Consultant
Edmonton Public School Board
Centre for Education
One Kingsway
Edmonton, Alberta
T5H 4G9
Telephone: 429-8320

**Medicine Hat**

Instructional Materials Centre Manager
Medicine Hat School District
601 First Avenue S.W.
Medicine Hat, Alberta
T1A 4Y7
Telephone: 526-1323

**Red Deer**

Coordinator of Instruction
Red Deer Public School Board
4747 – 53 Street
Red Deer, Alberta
T4N 2E6
Telephone: 343-1405

**Sherwood Park**

Director
Learning Resource Service
County of Strathcona
2001 Sherwood Drive
Sherwood Park, Alberta
T8A 3W7
Telephone: 464-8235

# CORRELATION OF BASIC LEARNING RESOURCE TO MATHEMATICS 16-26

A correlation of *Consumer and Career Mathematics, Second Canadian Edition* (1991) to the Mathematics 16–26 course sequence is provided below. The correlation illustrates how students will reinforce and extend their understanding of concepts and skills developed in Mathematics 16 through new thematic contexts and more extensive applications in Mathematics 26.

Topics in the textbook should be selected as appropriate in assisting students to develop prescribed content and solve practical problems at each grade level. Suggestions for using the textbook throughout four thematic units of instruction developed for Mathematics 26 are provided in a subsequent section of this manual (see Themes).

| CONSUMER AND CAREER MATHEMATICS, Second Canadian Edition (1991) | MATH 16 | MATH 26 |
|---|---|---|
| UNIT 1: MATHEMATICAL SKILLS | | |
| Chapter 1: Whole Numbers, Decimals and Fractions | | |
| Rounding Whole Numbers and Decimals | p. 4 | |
| Adding and Subtracting Whole Numbers/Decimals | p. 6 | |
| Multiplying Whole Numbers and Decimals | p. 8 | |
| Estimation | p. 10 | |
| Dividing Whole Numbers and Decimals | p. 12 | |
| Comparing and Renaming Fractions/Mixed Numbers | p. 14 | |
| Multiplying and Dividing Fractions/Mixed Numbers | | p. 16 |
| Adding and Subtracting Fractions/Mixed Numbers | | p. 18 |
| Chapter 2: Equations, Proportions and Percent | | |
| Addition and Subtraction Equations | | p. 24 |
| Multiplication and Division Equations | | p. 26 |
| Two-Step Equations | | p. 28 |
| Ratio and Proportion | p. 30 | |
| Writing Decimals and Fractions as Percents | p. 32 | |
| Writing Percents as Decimals and Fractions | p. 34 | |
| Finding a Percent of a Number | p. 36 | |
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# RELATIONSHIP OF MATHEMATICS 16-26 TO MATHEMATICS 14-24

## RATIONALE

As outlined in the *Guide to Education: Senior High School Handbook* (1991-92), Mathematics 16 is designed to allow students to meet the credit requirements of the Certificate of Achievement. Mathematics 26 is optional, allowing students to develop their knowledge and skills in mathematics more fully, and facilitates transfer to the General High School Diploma Program. In contrast, the Mathematics 14-24 program is designed to allow students to meet the credit requirements of the General High School Diploma.

| CERTIFICATE OF ACHIEVEMENT (80 credits) | | | *GENERAL DIPLOMA TRANSFER* |
|---|---|---|---|
| Grade 10 | Grade 11 | Grade 12 | |
| English 16 (3) | English 26 (3) | English 36 (3) | *English 23→33 (5,5)* |
| Mathematics 16 (3) | *Mathematics 26 (3) | | *Mathematics 24 (3,5)* |
| Science 16 (3) | *Science 26 (3) | | *Science 24 (3,5)* |
| Social Studies 16 (3) | Social Studies 26 (3) | | *Social Studies 23, 33 (5,5)* |
| Occupational 16 (3,5) | Occupational 26 (10) | Occupational 36 (10) | *Vocational 22, 32 (5, 10, 15, 20)* |

* Mathematics 26 and Science 26 – These courses are designed to prepare students for possible transfer to the General High School Diploma route. While not required as part of the Certificate of Achievement, I.O.P. students may choose to enrol in these courses and may apply the credits as part of their unspecified course requirements (13 credits).

Both Mathematics 16-26 and Mathematics 14-24 have been developed on the basis of their appropriateness to the developmental levels of students, and provide:

- potential for students to transfer to higher level mathematics courses
- an emphasis on the utility and application of concepts and skills in mathematics.

## COURSE DESIGN

The Mathematics 16–26 course sequence is heavily contextualized, focusing on thematic organization and situational learning experiences. Mathematics 14 is organized within the context of strands that are characteristic of the discipline. In contrast, Mathematics 24 is organized around themes that focus on practical life skills. Learning resources authorized for use within each course support instruction in a manner consistent with the way each course has been designed and organized.

## INSTRUCTIONAL EMPHASES

Instructional emphases within the Mathematics 16–26 and Mathematics 14–24 course sequences are consistent with one another, and focus attention on:

- integrating problem solving throughout all themes and strands of the curriculum
- making practical application of calculators and computer technologies
- developing and reinforcing computational skills through practical applications
- using a variety of methods of computation (i.e., paper-and-pencil algorithms, mental arithmetic, the calculator, estimation).

## PRESCRIBED CONTENT AND COGNITIVE DEMAND

Prescribed content in the Mathematics 16–26 course sequence is less extensive than prescribed content in the Mathematics 14–24 course sequence. The two programs also differ in the level of analysis and abstract thought processes expected of students. Applications in Mathematics 16–26 generally focus on the transfer of concepts and skills to daily life and work-related situations, whereas many of the applications in Mathematics 14–24 encourage students to apply concepts and skills in more abstract situations. Cognitive demands in the two programs will be further influenced by the needs and abilities of individual students.

The chart that follows outlines major similarities and differences in the prescribed content of six strands common to both Mathematics 16–26 and Mathematics 14–24.

### STRAND: Problem Solving

Problem-solving skills and strategies are explicitly emphasized and integrated throughout both course sequences. Students develop an understanding that problems can be solved in many ways, and will be expected to use appropriate strategies to solve routine problems that require the application of familiar techniques, as well as non-routine and open-ended problems.

Cognitive demands of the problems that students are expected to solve in the two course sequences differ in the level of analysis and abstract thought that are required. Problem solving in Mathematics 16–26 is heavily contextualized and focuses on concrete situations in daily life and the world of work. Problem solving in Mathematics 14–24 involves transfer, analysis and application in more complex and abstract situations.

### STRAND: Numeration

Both course sequences focus attention on the development of quantitative thinking and computational facility. Students are encouraged to develop a variety of strategies for performing computations that include the use of paper-and-pencil algorithms, mental arithmetic, estimation and the calculator.

In Mathematics 16–26, students will develop an understanding of fractions and integers at the concrete level through the use of models, manipulative materials and everyday examples. In Mathematics 14–24, students will develop and apply related concepts and skills at increasingly symbolic and abstract levels, and will study positive and negative fractions and integers.

### STRAND: Ratio, Proportion and Percent

Both course sequences will develop an understanding that ratio can be used to describe the relative sizes of two quantities, that rate is a ratio between quantities that have different units, and that percent is a ratio in which a quantity is compared to 100.

Learning objectives in Mathematics 16–26 focus attention on the development of related concepts and skills at the intuitive and concrete level within the context of everyday examples. Learning objectives in Mathematics 14–24 require an increasingly formal understanding of ratio and percent, and encourage independent application of skills in more complex situations.

**STRAND: Geometry and Measurement**

Both course sequences focus attention on the development of spatial and visualization skills, as well as an understanding of the properties of one-, two- and three-dimensional figures. Students are expected to estimate and measure length, area, volume, capacity, mass, time, angles and temperature, selecting SI units and tools appropriate to the situation.

Instructional content in Mathematics 16–26 emphasizes exploration, manipulative activities, informal reasoning and direct measurement. Mathematics 14–24 provides a more abstract analysis of geometric concepts and places increasing emphasis on the development of indirect measurement skills.

**STRAND: Algebra**

Both course sequences require students to recognize and use number patterns/relationships and critical thinking skills in order to make predictions and solve practical problems.

Mathematics 16–26 focuses attention on numerical patterns and relationships that are present in concrete situations, and the use of first-degree equations in one variable to describe these situations. Mathematics 14 provides a more formal approach to the use of algebraic expressions and equations. In addition to studying first-degree equations in one variable, students will investigate relationships between pairs of variables and the use of ordered pairs, equations and graphs to describe these relationships.

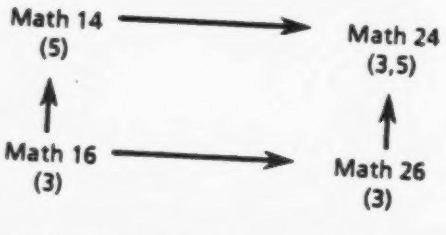
**STRAND: Statistics**

Both course sequences develop knowledge of the use of statistical data in practical situations, and assist students to make informed decisions that are based upon quantitative information. Students will gain first-hand experience in collecting, organizing, displaying and analyzing data in order to make valid inferences and predict probable outcomes.

Mathematics 16–26 focuses attention on collecting, organizing and displaying data, while Mathematics 14–24 places greater emphasis on the analysis and numeric manipulation of collected data. Mathematics 26 introduces experimental probability within contexts known to the student, while Mathematics 14 develops an understanding of theoretical probability.

## TRANSFERABILITY

Although prescribed content in Mathematics 16–26 is less extensive than prescribed content in Mathematics 14–24, the Mathematics 16-26 course sequence will develop concepts, skills and attitudes that facilitate potential transfer from Mathematics 26 to Mathematics 24. Program sequences and potential transfer points are illustrated below.



Additional information about transfer routes from the Integrated Occupational Program to the General High School Diploma Program is provided in the *Guide to Education: Senior High School Handbook* (1991-92).

# THEME A
# EARNING MONEY

## SUB-THEMES

- PAYING TAXES
- BUYING AND SELLING

## RATIONALE

This theme focuses attention on federal and provincial taxes placed on earnings and consumer goods/services, and on career opportunities that involve buying and selling. Thematic activities will assist students to develop appropriate strategies for:

- interpreting and completing a personal income tax return
- determining the cost (including sales tax) of articles bought and sold
- monitoring profit and/or loss in a simple retail business.

Students will apply previously developed knowledge of the wages, deductions and earnings associated with familiar jobs (see Mathematics 16: Earning Money) as they investigate factors that determine the amount of income tax paid. Learning objectives interrelate with activities in a subsequent theme, Math in the Workplace, and will enable students to monitor profits and/or losses in simulated entrepreneurial activities.

Students will be expected to consolidate and reinforce skills involving whole numbers, decimals, fractions, integers and percent as they perform calculations throughout the theme. Calculators should be used on a regular basis when performing routine computations.

Cooperative planning among teachers will ensure that activities within this theme complement activities undertaken by students in other subject areas (e.g., the occupational program). The community may offer appropriate resources for developing concepts and skills through first-hand investigation. Community-based investigations can relate to part-time jobs and actual work situations already experienced by students, and may involve studying the operation of a local retail business.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Assessment/Evaluation.

## THEMATIC OBJECTIVES

### PAYING TAXES

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- demonstrate an understanding of federal and provincial taxes placed on earnings and consumer goods/services
- identify factors that determine the amount of income tax paid
- interpret major parts of a personal income tax form and guide
- consult an income tax guide for explanations of the kinds of information to be provided on the numbered lines of a tax form
- identify the minimum annual income subject to taxation, and annual incomes at which taxation rates change
- identify and/or calculate allowable deductions from personal income
- determine the effect of a discretionary deduction (e.g., a registered retirement savings plan, a charitable or political contribution) on total tax paid
- interpret records of employment income, identifying gross earnings (including gratuities) and allowable deductions
- complete a personal income tax return using real or simulated records of employment.

### BUYING AND SELLING

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- calculate the selling price of an article when given the cost price and the percent markup
- calculate the percent markup on an article when given the cost price and the selling price
- calculate the sale price of an article when given the regular price and the discount rate (as a fraction or percent)
- calculate the total cost of an article (including sales tax) when given the basic price and the rate of taxation
- demonstrate an understanding of non-stock costs incurred in operating a retail business (e.g., rent, utilities, salaries)
- demonstrate an understanding of profit as the amount of money left after all costs associated with operating a business are paid
- interpret statistical data related to profit and/or loss in business and trade-related situations
- interpret and use appropriate measures of central tendency in practical situations related to buying and selling.

## CONTEXT FOR INSTRUCTION

### PROBLEM SOLVING

Students will be expected to:

- determine income, deductions, tax credits and income tax associated with local jobs in which they are typically employed
- consider the effect of discretionary deductions on total income tax payable
- solve problems related to the cost price, selling price and/or sale price of an article
- suggest methods of increasing profit in the operation of a simple retail business (e.g., increasing gross sales, decreasing costs).

### USE OF TECHNOLOGY

Students will be expected to:

- use a calculator when performing routine calculations related to income tax and the price of articles that are bought or sold
- work independently with prepared computer software as required to develop/maintain mathematical concepts and skills
- develop an understanding of applications of computer technology in related areas (e.g., preparing tax tables, maintaining personal records of earnings, assessing income tax returns, monitoring profit and/or loss in trade-related situations).

### COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform computations with a calculator on a regular basis
- round as appropriate during calculations (e.g., monetary amounts to the nearest cent)
- use mental arithmetic when appropriate in determining solutions to quantitative problems
- use estimation to check the reasonableness of calculations and solutions.

## SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

| | | | | |
|---|---|---|---|---|
| Number Systems and Operations | Ratio, Proportion and Percent | Algebra | Geometry and Measurement | Data Interpretation and Display |

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## NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals and fractions in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers and decimals as required
- perform basic operations with whole numbers, decimals, fractions and integers
- perform a sequence of operations in correct order
- determine decimal equivalents for commonly used fractions.

## RATIO, PROPORTION AND PERCENT

Students will be expected to:

- express whole number and fractional percents as ratios and decimals
- express ratios and decimals as percents
- estimate and calculate a percent of a number
- estimate and calculate what percent one number is of another.

## ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe practical situations
- apply equation-solving strategies to practical situations
- interpret formulas as word statements
- use substitution and equation-solving strategies to find the missing element in a formula.

## DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- read and interpret information presented in table, chart and graph form
- collect and organize information, using tally sheets and/or frequency tables
- display information in table, chart and graph form
- interpret and/or calculate appropriate measures of central tendency in practical situations
- interpret statements of probability in order to predict future outcomes/actions.

## INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with activities that may be undertaken by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

### ENGLISH

Students may:

- investigate the meaning of technical words used in taxation guides and forms
- read and interpret information provided and/or requested in taxation guides and forms
- complete the personal information section of an income tax return
- investigate the meaning of technical words used in operating a business (e.g., cost price, selling price, markup, discount).

### SOCIAL STUDIES

Students may:

- distinguish between federal and provincial jurisdiction in issues related to income tax
- relate the collection of taxes to the financing of roads, education and other services in society
- investigate direct and indirect forms of personal taxation
- explore entrepreneurial opportunities in the local community that involve buying and selling.

### OCCUPATIONAL COURSES

Students may:

- complete an employee's Tax Deduction Return
- maintain records of employment and remuneration
- identify compulsory and discretionary deductions on a pay cheque
- perform calculations as required to determine cost price, selling price, markup, discount, sale price and/or sales tax
- operate a simple retail business
- identify entrepreneurial opportunities that involve buying and selling.

### CALM

Students may:

- identify ways of reducing taxable income
- complete a TD1 taxation form and appropriate income tax return
- develop personal budgets that accommodate annual income tax payments coming due
- explore career opportunities that are appropriate to personal interests and abilities.

## COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing community resources, and for involving students in the community by way of meaningful activities linked to the mathematics program are provided below.

- Invite a human resources officer from a local business into the classroom to discuss various aspects of an employee's Tax Deduction Return.

  e.g.,
  - Why must all employees complete a TD1 taxation form?
  - What steps should an employee follow in completing a TD1 taxation form?
  - What is the function of the net claim code on a TD1 taxation form?
  - How are tax tables used in determining income tax deductions?

- Invite a local accountant or other knowledgeable person into the classroom to discuss and demonstrate appropriate strategies for completing an income tax return.

  e.g.,
  - What function is served by each of the major sections of an income tax return?
  - What is the minimum annual income subject to taxation?
  - What are the annual incomes at which taxation rates change?
  - What allowable deductions are available to most students filing an income tax return?

- Identify community agencies that provide information and/or assistance to individuals completing an income tax return. Discuss the advantages and/or disadvantages of accessing these services.

  e.g.,
  - local offices of Revenue Canada Taxation
  - law firms specializing in tax law
  - accounting firms
  - local businesses that specialize in completing income tax returns for individuals

- Invite a local retailer into the classroom to discuss various aspects of operating a simple retail business.

  e.g.,
  - What is the average markup on different types of consumer items?
  - What factors need to be considered in determining markup and selling price?
  - What factors need to be considered in planning a promotion or "sale" of merchandise?
  - What are "loss leaders", and how are they identified?
  - How can statistics be used to analyze consumer trends?
  - How is net profit or loss calculated?

- Provide opportunities for students to investigate employment and career opportunities in the local community that involve buying and selling.

## PAYING TAXES

### LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 18 – Income Tax

SUPPORT LEARNING RESOURCE

MYN: Unit 6 – Earning Money

*Math Strategies: Problem Solving* (computer software)

OTHER LEARNING RESOURCES

*2000 A.D., A Guide to Financial Awareness*
Becoming Tax Wise (pp. 16-21)

*A History of Income Tax in Canada* (video program)

*Revenue for Canada* (video program)

*SRA Computer Drill and Instruction, Mathematics, Level D*
(computer software)

*Teaching Taxes Student's Workbook*

*Teaching Taxes Taxopedia*

*Teaching Taxes Teacher's Update*

### SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Order copies of the following materials from the Public Affairs Office of Revenue Canada Taxation:

   - *Teaching Taxes Student's Workbook*
   - *Teaching Taxes Teacher's Update*
   - *Teaching Taxes Taxopedia.*

The *Student's Workbook* and *Teacher's Update* are updated annually, provide an orientation to the taxation form, and ask questions about typical employment and taxation scenarios. The *Taxopedia* provides background information and references about income tax.

These materials are available free of charge in classroom quantities, and should be ordered well in advance of instruction.

Calgary:
220 – 4th Avenue S.E.
Calgary, Alberta
T2G 0L1
Telephone: (403) 292-4249

Edmonton:
9700 Jasper Avenue
Edmonton, Alberta
T5J 4C8
Telephone: (403) 495-4173

2. Introduce the concept of taxation by viewing the following video presentations:

   - *Revenue for Canada*
   - *A History of Income Tax in Canada*.

   These videos may be borrowed free of charge from the Public Affairs Office of Revenue Canada Taxation. Teachers can make copies of these tapes if they wish.

   Other relevant films on taxation and income tax can be obtained from the National Film Board of Canada. An annotated list of available films is provided in *Teaching Taxes Teacher's Update*.

3. Develop strategies for determining income, deductions, tax credits and income tax within the context of local jobs in which students are typically employed. Refer to current tax forms and guides, and identify appropriate steps in completing an income tax return.

   e.g.,
   - Identification
   - Calculation of Total Income
   - Calculation of Taxable Income
   - Calculation of Total Non-Refundable Tax Credits
   - Summary of Tax and Credits

   Ask questions that will cause students to focus their attention on important parts of the income tax guide and return.

   e.g.,
   - What kind of information is provided on page 1?
   - On which page are deductions listed?
   - What is the amount of this year's basic personal tax credit?
   - On which line do you claim tuition fees?
   - On which page of the guide can you find information about claiming tuition fees?

4. Provide students with a sample T-4 slip. Discuss the type of information provided in each box, and how this information is used in completing an income tax return. A sample copy of a T-4 slip is provided in Resource 1: Statement of Remuneration (T-4 Slip).

5. Identify and discuss allowable deductions from income. Focus attention on those deductions students may wish to claim at present or in the near future.

   e.g., – Canada Pension contributions
   – Unemployment Insurance premiums
   – union dues
   – child care expenses
   – tuition fees
   – medical expenses

6. Discuss the concept of a "marginal tax rate". Model the sequence of calculations performed when computing the amount of tax payable from information provided in a table of marginal tax rates.

   Alberta Consumer and Corporate Affairs publishes a booklet entitled *2000 A.D., A Guide to Financial Awareness*. This booklet explains how marginal tax rate is determined and provides information on investments/tax returns. An excerpt from this booklet appears in Resource 2: Marginal Tax Rate.

7. Provide each student with a T-4 slip containing information relevant to familiar employment scenarios. Ask students to complete an income tax return, using information provided on the T-4 slip. Coach students as they complete the income tax return, providing oral instructions and demonstrations on the overhead.

   Encourage students to perform computations with calculators when completing income tax returns. A brief review of calculator skills may be necessary (see Use of Technology, "The Calculator").

8. Students may experience difficulty in distinguishing between a "balance due" or "refund" on the tax form. Remind students that:

   - TOTAL PAYABLE = total taxes to be paid
   - TOTAL CREDITS = total taxes already paid
   - it is necessary to compare TOTAL PAYABLE and TOTAL CREDITS in order to determine if a balance is due or a refund should be claimed.

9. Identify and discuss effective strategies for reducing taxable income. Ask students to identify strategies they might wish to use in lowering their future taxable incomes, and to justify/defend the strategies they choose (see Problem Solving, "A Framework for Solving Problems").

10. Select a simple calculation that is used in completing an income tax return (e.g., charitable donation credit). Ask students to write a formula or computer program that can be used to perform this calculation (see Use of Technology, "The Computer"; and Problem Solving, "Using Algebra to Solve Problems").

11. Consult with teachers of CALM in order to identify ways to integrate the content of this sub-theme with topics that may be studied in Career and Life Management. CALM students who complete the optional module, "Consumer and Investment Choices", will be given opportunities to:

    - complete a personal income tax return
    - identify strategies for reducing personal income tax
    - analyze the impact of investments on personal taxes.

## BUYING AND SELLING

### LEARNING RESOURCE CORRELATION

**BASIC LEARNING RESOURCE**

CCM: Chapter 2 – Equations, Proportions and Percent
Chapter 4 – Problem-Solving Skills and Strategies

**SUPPORT LEARNING RESOURCE**

MYN: Unit 5 – Spending Money
Unit C – Computing Skills You Need

MS: Paying Sales Tax (pp. 116-117)

**OTHER LEARNING RESOURCES**

*Applied Mathematics*
2–10: Marketing the Product
2–11: Discounts

*Mathways* (video program)
The Percent

*Spreadsheets for Students* (computer software)
Dynashop

### SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Provide opportunities for students to develop an understanding of technical vocabulary used in entrepreneurial activities that involve buying and selling.

   e.g.,
   - retail
   - wholesale
   - cost price
   - selling price
   - markup
   - overhead
   - discount
   - tax rate

   Discuss real life situations in which the words may be used, and ensure that each word becomes part of the students' active vocabulary. Encourage students to record words that are used throughout the theme in a personal "mathematics glossary".

2. Collect advertisements from local newspapers or store flyers that provide the regular price and sale price of merchandise. Ask students to calculate the rate of discount for the sale merchandise being advertised.

3. Invite one or more occupational teachers into the mathematics classroom to discuss and compare the "wholesale" and "retail" price of products sold in particular occupational areas (e.g., Hair Care, Service Station Services). Gather information about the wholesale price and retail price of products that are sold, and determine the margin of profit for each product. Record the results of investigation in a chart similar to the one illustrated below.

   **CLARIFICATION EXAMPLE**

   **WHOLESALE/RETAIL PRICE COMPARISON CHART**

   | **Product** | **Wholesale Price** (Cost Price) | **Retail Price** (Selling Price) | **Profit** (Markup) | **Margin of Profit** (Percent Markup) |
   |---|---|---|---|---|
   | | | | | |
   | | | | | |
   | | | | | |
   | | | | | |

4. Assist students to design (or modify) a computer program that calculates the selling price of an article when given cost price and percent markup (see Use of Technology, "The Computer").

   Provide instruction on using memory keys of the calculator when performing repeated calculations with the same rate of markup or discount (see Use of Technology, "The Calculator").

5. Invite a local Member of Parliament or other knowledgeable person to discuss the nature of federal and provincial sales taxes.

   e.g.,
   - Why are taxes introduced?
   - What are the differences between hidden and overt taxes?
   - What goods and services are taxed?
   - How are these taxes collected?

   Research the rate of sales tax (and types of goods/services taxed) in each province. Use this information to compare the total cost, including tax, on items purchased in different provinces.

   Appropriate software for determining sales tax and total cost on items purchased is provided in *Spreadsheets for Students*, Dynashop.

6. Coach students in the use of appropriate mental arithmetic skills and estimation strategies when determining the amount of sales tax to be paid in consumer situations (see Computational Facility and Estimation, "Developing Mental Arithmetic Skills").

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**CLARIFICATION/EXAMPLE**

**USING MENTAL ARITHMETIC/ESTIMATION IN CONSUMER SITUATIONS**

Suppose the rate of federal sales tax is 9%. Will the total cost (including sales tax) of a compact disc advertised at $15.98 be less than $20.00?

$15.98 is approximately $16.00
9% is close to 10%
10% of $16.00 is $1.60
$16.00 + $1.60 = $17.60.

The total cost of the compact disc is less than $20.00.

7. Ask a local entrepreneur or business education teacher to discuss various aspects of operating a retail business.

e.g.,
- items that constitute overhead
- non-stock costs that are incurred in operating the business
- factors that determine selling price of products sold
- reasons for promoting the sale of certain products through discounted prices
- marketing strategies that increase profits
- interpersonal and business skills that are used

Following the discussion, divide the class into small groups (three or four students) and work on a plan for producing and selling a product. Encourage students to consider a variety of factors related to operating a retail business (e.g., cost of supplies/advertising/wages, markup and selling price, intended market, anticipated profit) when formulating their business plans. Ask each group of students to present their business plan to other members of the class, explaining:

- how they would produce, advertise and market the product
- potential strengths and weaknesses in their business plans.

A sample checklist for evaluating group presentations is suggested below.

**CLARIFICATION/EXAMPLE**

| EVALUATION CRITERIA | Excellent 4 | Well Done 3 | Satisfactory 2 | Not Acceptable 1 |
|---|---|---|---|---|
| The group demonstrated knowledge of concepts and skills related to buying and selling. | | | | |
| The presentation was meaningful and well explained. | | | | |
| The presentation demonstrated evidence that research on operating a business had been conducted. | | | | |
| All students contributed to and participated in the presentation. | | | | |
| The group evaluated their plan for starting a small business. | | | | |

8. Investigate applications of statistics in buying and selling. Related activities may include:

- conducting a survey/poll in the community in order to determine consumer trends and preferences
- interpreting tables, charts, and graphs that provide information about profit and/or loss in particular types of business
- interpreting statements of probability made about the success of an entrepreneurial enterprise.

Ask questions throughout the theme that will encourage students to recognize how measures of central tendency are used in trade-related situations.

e.g., – Which measure of central tendency would a shoe salesperson be most interested in when ordering stock?
– What do we mean when we say an item is "mid-priced"?
– Which measure of central tendency would be used to determine "average" daily cash receipts?

# RESOURCE 1: STATEMENT OF REMUNERATION (T-4 SLIP)

**Revenue Canada**
**Taxation**

**T4**

**Supplementary**

**STATEMENT OF REMUNERATION PAID**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (C) EMPLOYMENT INCOME BEFORE DEDUCTIONS → | (D) EMPLOYEES PENSION CONTRIBUTION<br>CANADA PLAN | QUEBEC PLAN | (E) UI PREMIUM | (F) REGISTERED PENSION PLAN CONTRIBUTION | (G) INCOME TAX DEDUCTED | (H) UI INSURANCE EARNINGS | (I) CPP PENSIONABLE EARNINGS | (J) EXEMPT |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| BOX (C) AMOUNT ALREADY INCLUDES ANY AMOUNTS IN BOXES (K), (L), (M), (N), (O), AND (P) | TAXABLE ALLOWANCES AND BENEFITS ▶ | (K) HOUSING BOARD AND LODGING | (L) TRAVEL IN A PRESCRIBED AREA | (M) PERSONAL USE OF EMPLOYER'S AUTO | (N) INTEREST FREE AND LOW INTEREST LOANS | (O) OTHER TAXABLE ALLOW AND BENEFITS | (P) EMPLOYMENT COMMISSIONS |

| | | | | | |
|---|---|---|---|---|---|
| (Q) UNION DUES | (R) CHARITABLE DONATIONS | (S) PAYMENTS TO DPSP | (T) PENSION PLAN OR DEFERRED PROFIT SHARING PLAN REGISTRATION NUMBER | (A) PROVINCE OF EMPLOYMENT | (B) SOCIAL INSURANCE NUMBER |

**EMPLOYEE** SURNAME FIRST (in capital letters) USUAL FIRST NAME AND INITIALS AND FULL ADDRESS

SURNAME USUAL FIRST NAME AND INITIALS

→

| | |
|---|---|
| EMPLOYER NAME | |
| ACCOUNT NUMBER | EMPLOYEE NO. |

TO BE RETURNED WITH T4-T4A SUMMARY

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# RESOURCE 2: MARGINAL TAX RATE'

## MARGINAL TAX RATE

Different portions of your taxable income are taxed at different percentage rates. You pay a low rate of tax on the first portion of your taxable income, and progressively higher rates on subsequent portions, with marginal tax rate being the rate of tax applied to the last dollars of your taxable income for a given year.

Think of your taxable income as a staircase with each step having a different and usually higher rate of tax. Each step is referred to as a tax bracket. Suppose the structure of the federal tax rates is 17% on the first $27,500 of taxable income, 26% on the next $27,500 of taxable income, and 29% on any taxable income over $55,000. These federal rates are then combined with the provincial tax rate to determine the total taxes that you owe.

To calculate your marginal tax rate (MTR) for a recent year, examine the table on the following page. Locate the bracket into which your income will fit. For example, if your taxable income is $25,000, then your tax bracket will be between $10,707 and $27,500. At this income level, your marginal tax rate will be 25.92%.

To calculate the tax owing, the table shows that on the first $10,707 of your taxable income, you will pay $2,775, on the remaining $14,293 ($25,000 minus $10,707) you will pay 25.92% or $3704.75 ($14,293 × 25.92%).

Knowing your marginal tax rate allows you to assess accurately the tax impact of various financial strategies. For instance, at a marginal tax rate of 25.92%, a $500 child care expense which is tax deductible will reduce your income tax by $129.60 ($500 × 25.92%). An RRSP contribution of $3,000 will reduce your tax by $777.60 ($3,000 × 25.92%).

On the other hand, at a marginal tax rate of 25.92%, any additional income you earn will be taxed at a rate of $259.20 per $1,000. If your additional income is sufficient to move you to the next higher tax bracket, both your base tax and your marginal tax rate will increase.

Using current tax rates, determine your marginal tax rate and the impact of various deductions and/or additional income on taxes paid. Always consider your marginal tax rate when you are developing financial strategies.

## RESOURCE 2: MARGINAL TAX RATE (continued)

**MARGINAL TAX RATES FOR A RECENT YEAR**
**(Combined federal and Alberta rates)**

| | TAX PAYABLE | | | |
|---|---|---|---|---|
| Taxable Income | Basic Tax Payable for Tax Bracket | | Marginal Tax Rate | Marginal Income |
| To $3 569 | 17.51% | | | |
| In excess of: | | | | |
| $ 3 569 | $ 625 | + | 30.12% on next | $ 7 138 |
| 10 707 | 2 775 | + | 25.92% on next | 16 793 |
| 27 500 | 7 127 | + | 39.37% on next | 10 969 |
| 38 469 | 11 445 | + | 40.34% on next | 16 531 |
| 55 000 | 18 113 | + | 45.93% on remainder | |

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# THEME B
# BUDGETING AND BANKING

## SUB-THEMES

- USING CREDIT
- SAVING AND INVESTING

## RATIONALE

Many high school students are looking ahead to the day when they begin full-time employment. Future plans of students often include making major purchases (e.g., a car or home) that require careful thought and planning. This theme will provide opportunities for students to:

- recognize and appreciate the need for financial planning
- examine the cost of credit
- develop strategies for financing major purchases and saving/investing money.

Students will apply previously developed knowledge of banking services and budgeting strategies (see Mathematics 16: Budgeting and Banking) throughout this theme as they investigate the cost of credit and make plans for the future. Learning objectives interrelate with activities in a subsequent theme, Skills for the Consumer, and will contribute to an understanding of the costs incurred when establishing a personal residence and buying a vehicle.

Cooperative planning among teachers will ensure that investigations within this theme complement related topics that are studied by students in other subject areas. The local community offers a variety of resources that will assist students to plan for future spending, saving and use of credit.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation
- Assessment/Evaluation.

## THEMATIC OBJECTIVES

### USING CREDIT

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- demonstrate an understanding that credit enables the consumer to make purchases through the loan of someone else's money, that credit cards provide the consumer with instant loans, and that credit usually has a cost which is determined by the rate of interest paid
- identify sources of credit within the community (e.g., financial institutions, loan companies, retail businesses)
- describe the characteristics of different types of credit (e.g., single-payment loans, level-payment loans, credit cards, mortgages)
- relate features of a credit contract (e.g., interest rate, term of loan, minimum monthly payment) to the credit worthiness of the consumer (e.g., income, assets, debts)
- compare the actual cost of credit with different types of loans and different companies and/or creditors
- read and interpret information on credit card sales drafts and monthly statements
- calculate simple interest charged on a single-payment loan and/or a credit card balance
- determine the monthly payment on a level-payment loan, using a table
- calculate the total cost of repaying a loan and/or credit card balance, given the amount loaned, minimum monthly payment and term of the loan.

### SAVING AND INVESTING

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- demonstrate an understanding that a regular saving plan may assist in preparing for future expenditures
- describe the characteristics of various plans for saving and investing money (e.g., savings accounts, bonds, guaranteed investment certificates)
- compare different investment plans with respect to features such as term of deposit, rate of return, minimum deposit, transaction fees and penalty for early withdrawal
- calculate simple interest on investments for periods of time involving days, months and/or years
- demonstrate an understanding of compound interest as a series of simple interest calculations
- determine compound interest, using tables or a computer.

## CONTEXT FOR INSTRUCTION

### PROBLEM SOLVING

Students will be expected to:

- evaluate available sources and types of credit by considering the advantages/disadvantages of particular credit contracts
- compare potential investment opportunities, and design a plan for saving money that will accommodate future goals and needs
- compare the effects of simple and compound interest on money borrowed and money saved over varying periods of time.

### USE OF TECHNOLOGY

Students will be expected to:

- use calculators to determine simple interest on loans, credit card balances and investments
- work independently with prepared computer software as required to develop/maintain an understanding of mathematical concepts and skills
- develop an understanding of the use of computer technology in determining simple and compound interest on loans and investments, and in preparing account statements for the consumer
- use prepared computer software to determine simple and compound interest on money saved and/or money borrowed.

### COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator on a regular basis
- round as appropriate during calculations
- use mental arithmetic when appropriate to expedite solutions to quantitative problems
- estimate interest charged on loans and interest earned on investments
- use estimation to check the reasonableness of calculations and solutions to problems.

## SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

| Number Systems and Operations | Ratio, Proportion and Percent | Algebra | Geometry and Measurement | Data Interpretation and Display |
|---|---|---|---|---|

## NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals, fractions and integers in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers, decimals and integers in solving practical problems
- perform basic operations with whole numbers, decimals and integers
- perform a sequence of operations in correct order
- use fractions to represent parts of a whole
- convert fractions into decimals and vice versa.

## RATIO, PROPORTION AND PERCENT

Students will be expected to:

- express percents as ratios and decimals
- express ratios and decimals as percents
- perform basic arithmetical operations with ratios and percents
- estimate and calculate a percent of a number
- estimate and calculate what percent one number is of another.

## ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe practical situations
- apply equation-solving strategies to practical situations
- interpret formulas as word statements (e.g., $I = prt$)
- use substitution to find the missing element in a formula.

## DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- interpret and/or construct tables, charts and graphs that display information about money borrowed and money invested
- use interest and amortization tables in practical situations
- interpret and/or calculate appropriate measures of central tendency in practical situations.

## INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with budgeting and banking activities that are undertaken by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

### ENGLISH

Students may:

- identify personal goals and needs that need to be considered when developing a plan for saving and investing money
- develop an understanding of technical vocabulary used in credit and investment contracts
- interpret information given and requested on an application for credit (e.g., bank loan, charge account, credit card)
- interpret information provided through the media about current interest rates, credit plans and investment opportunities.

### SOCIAL STUDIES

Students may:

- identify factors that need to be considered when planning personal finances
- investigate the services offered by financial institutions in the local community
- consider the advantages/disadvantages associated with various forms of credit and investment opportunities
- interpret information provided through the media about current interest rates, credit plans and investment opportunities.

### OCCUPATIONAL COURSES

Students may:

- apply budgeting skills when selecting and costing materials used in construction and/or repair projects
- design a financial plan prior to pursuing an entrepreneurial activity.

### CALM

Students may:

- identify factors that need to be considered in managing personal finances
- prepare a plan for saving and investing money that accommodates future goals and needs
- investigate the services offered by financial institutions in the local community
- develop an understanding of procedures followed when obtaining credit and/or investing money.

## COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing community resources, and for involving students in the community by way of meaningful activities linked to budgeting and banking are provided below.

- Identify different types of financial institutions in the local area (e.g., banks, credit unions, loan companies). Discuss the services offered by these institutions.
- Visit a local financial institution and interview a loans officer. Ask this person to explain the features of different types of loans and/or other credit contracts offered by the institution.

  e.g.,
  - What are the costs associated with different types of loans offered by the institution?
  - Why do interest rates on loans vary?
  - What is the purpose of collateral and a co-signer in a loan agreement?
- Visit a local trust company, bank or credit union. Ask a staff member to explain the features of various investment opportunities offered by the institution.

  e.g.,
  - What different types of term deposits are available?
  - What are the current interest rates being paid on savings and investment accounts?
  - What are the benefits of investing in a registered retirement savings plan?
- Invite an accountant or financial planner to speak to the class about the importance of making a financial plan in order to accommodate future goals and needs. This person may discuss different ways of saving and investing money.
- Visit a local bank and observe the use of computer technology. Investigate the nature of the tasks performed by computers. What tasks do computers accomplish for the bank? What tasks do they accomplish for the consumer?

## USING CREDIT

### LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 11 – Consumer Credit

SUPPORT LEARNING RESOURCE

MYN: Unit 8 – Loans and Credit

MS: Unit 5 – Investing and Borrowing

*Money Manager* (computer software)

OTHER LEARNING RESOURCES

*Banking, Budgeting and Employment*
Part I – Banking, Budgeting

"Consumer Talk" Information Sheet: *Credit*

Consumer Tipsheets:
*How to Use Credit Responsibly*
*Protect Your Credit Rating*

*Credit Wise*

*Decisions, Making Personal Economic Choices*
Chapter 12 – Saving, Spending or Borrowing Money
Chapter 16 – Shopping for Financial Services
Chapter 17 – Shopping for Credit

*Forms in Your Life: A Study Workbook and Guide to Everyday Forms*
Form 12 – Application for a Savings Account
Form 20 – Application for a Charge Account

*Helping You Bank*

*Moving Out*
Credit (pp. 25–27)

*Spreadsheets for Students* (computer software)
Dynaloan

*Street Cents* (video program)

*Taking Charge of Your Money*
Using Credit (pp. 21–29)

*Useful Arithmetic, Volume 2*
Section IV – Banking Services

Youth Presentation Package:
*CIBC's Greatest Hits*

## SUGGESTED ACTIVITIES

The student learning resources identified on the preceding page provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. A number of useful publications on credit are available from Alberta Consumer and Corporate Affairs. The publications listed below are free, except where noted*, and can be ordered in classroom quantity from any one of the Alberta Consumer and Corporate Affairs offices listed in Appendix B.

   "Consumer Talk" Information Sheet:
      *Credit*

   Consumer Tipsheets:
      *How to Use Credit Responsibly*
      *Protect Your Credit Rating*
      *Putting Your Spending Plan on Paper*
      *What Creditors Can Do If You Don't Pay Your Debts*

   *Moving Out**

   *Taking Charge of Your Money*

   *Class sets must be ordered at cost from the Learning Resources Distributing Centre, 12360 – 142 Street, Edmonton, Alberta, T5L 4X5.

2. The Canadian Bankers' Association publishes numerous pamphlets and brochures on using credit that may be appropriate for classroom use.

   e.g.,
   - *Credit Wise*
   - *Helping You Bank*
   - *Will That Be Cash or Charge?*

   These publications are available free of charge, and can be ordered from:

   Consumer Education Advisor
   The Canadian Bankers' Association
   Box 348
   2 First Canadian Place
   Toronto, Ontario
   M5X 1E1

3. Additional sources of information on credit and financial planning are provided in Resource 1: Taking Charge. Ask students to write letters requesting information on related topics, and to share the information they receive with other members of the class. Teachers may wish to plan these activities in conjunction with an English class.

   Information on credit is also available at Consumer Corners, located in many public libraries and community agencies throughout the province.

4. Encourage students to use their calculators when determining interest and the amount of a loan/investment. A brief review of calculator procedures may be worthwhile (see Use of Technology, "The Calculator").

5. Ask students to identify an item they would like to purchase, but for which they would need to use credit. Consider the pros and cons of making this purchase. Use a comparative and contrastive web to:

   - decide if the purchase should be made at this time
   - identify problems that may occur if credit is used
   - develop a financial plan most appropriate to personal circumstances
   - identify alternative courses of action.

   A web that may assist students to consider alternatives has been provided in Resource 2: Comparative and Contrastive web.

6. Identify different types of financial institutions in the local area (e.g., trust companies, banks, credit unions). Investigate the services offered by these institutions through field visits or by inviting guest speakers into the classroom.

7. Invite a bank loans officer to speak to the class about:

   - different types of loans that are available
   - factors that cause interest rates on loans to vary
   - the need for collateral and/or a co-signer
   - the responsibilities of a co-signer
   - how a "credit rating" is established.

   Additional information on obtaining a loan and establishing a credit rating is provided in Resource 3: The Credit Choice.

8. Ask students to prepare a table that provides the simple interest and amount owed on a single-payment loan using variable interest rates. Students might use a computer to calculate interest and amount owed (see Use of Technology, "The Computer").

   **CLARIFICATION EXAMPLE**

**AMOUNT OWED ON A SINGLE PAYMENT LOAN OF $100 FOR ONE YEAR**

| Principal | Interest Rate | Simple Interest | Amount |
|---|---|---|---|
| $100.00 | 10% | $10.00 | $110.00 |
| $100.00 | 10.5% | $10.50 | $110.50 |
| $100.00 | 11% | $11.00 | $111.00 |
| • | • | • | • |
| • | • | • | • |
| • | • | • | • |

9. Assist students to identify potential "bugs" in this BASIC program designed to calculate simple interest.

```
10 REM      CALCULATE SIMPLE INTEREST
20 PRINT    "WHAT IS THE INTEREST RATE?"
30 INPUT    R
40 PRINT    "WHAT IS THE PRINCIPAL?"
50 INPUT    P
60 PRINT    "THE INTEREST IS:"
70 PRINT    R*P
80 END
```

Discuss with students how this program could be adapted to calculate simple interest for periods of time other than one year (see Use of Technology, "The Computer").

10. Use the table provided in Resource 4: Monthly Loan Payments, to compute the monthly payments for loans at different interest rates and over varying periods of time. Encourage students to determine the total cost of each loan.

11. Introduce students to the "Dynaloan" feature of *Spreadsheets for Students* (see computer software). Using Dynaloan, ask students to determine the monthly payments for a variety of loans at different interest rates and over varying periods of time.

12. Discuss how the following statements reflect differing views about the use of credit. Ask students to explain why they share or do not share the views expressed by each statement.

    - "Why should I pay now when I can pay later?"
    - "In the future, credit will create a cashless society."
    - "I never use credit. If I don't have the cash, I just don't buy it."
    - "I only use credit for large purchases."
    - "Credit is an evil."

13. Invite a financial counsellor from the local community to speak to the class about the use of credit. Ask this person to explain:

    - major types of problems encountered by people who use credit
    - strategies for avoiding problems associated with the use of credit.

# SAVING AND INVESTING

## LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 20 – Investments

SUPPORT LEARNING RESOURCES

MYN: Unit 7 – Savings and Interest

MS: Unit 5 – Investing and Borrowing

*Money Manager* (computer software)

OTHER LEARNING RESOURCES

*2000 A.D., A Guide to Financial Awareness*

*Banking, Budgeting and Employment*
Part I – Banking, Budgeting

*Decisions, Making Personal Economic Choices*
Chapter 12 – Saving, Spending or Borrowing Money
Chapter 16 – Shopping for Financial Services
Chapter 18 – Shopping for Personal Investments

*Helping You Bank*

Keeping Your Balance Series:
*Putting Your Spending Plan on Paper*
*Take Care of Your Savings*

*Moving Out*
Make the Most of Your Money (p. 24)
Planning Your Future (p. 44)

*Spreadsheets for Students* (computer software)
Dynasave

*Street Cents* (video program)

*Taking Charge of Your Money*
Putting Your Spending Plan on Paper (pp. 13 – 16)

*Useful Arithmetic Skills, Volume 2*
Section IV – Banking Services

Youth Presentation Package:
*CIBC's Greatest Hits*

## SUGGESTED ACTIVITIES

The student learning resources identified on the preceding page provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. A number of useful publications on saving and investing money are available from Alberta Consumer and Corporate Affairs. The publications listed below are free, except where noted*, and can be ordered in classroom quantity from any one of the Alberta Consumer and Corporate Affairs offices listed in Appendix B.

   *2000 A.D., A Guide to Financial Awareness**

   Keeping Your Balance Series:
      *Putting Your Spending Plan on Paper*
      *Take Care of Your Savings*

   *Moving Out**

   *Taking Charge of Your Money*

   *Class sets must be ordered at cost from the Learning Resources Distributing Centre, 12360 – 142 Street, Edmonton, Alberta, T5L 4X5.

2. The Canadian Bankers' Association publishes numerous pamphlets and brochures on saving and investing money that may be appropriate for classroom use.

   e.g., – *Helping You Bank*
            – *Securing a Better Future*

   These publications are available free of charge, and can be ordered by writing to the address provided on page 54.

3. Brainstorm and discuss reasons for developing a regular savings plan.

   Identify essential elements of a household budget as identified in Resource 5: Budget Planner. Ask students to simulate a "living on your own" situation and prepare a budget that includes a monthly savings plan.

4. Ask students to read Resource 6: An Emergency Fund for Your Family. Answer and discuss the questions in "Emergency! Emergency!".

   Encourage students to use appropriate problem-solving strategies in developing personal savings plans (see Problem Solving, "Using Strategies to Solve Problems").

5. Invite a representative from a bank/credit union to speak to the class about:

   - different types of savings plans that are available to the consumer
   - minimum deposits/term deposits
   - variable interest rates.

6. Discuss the purchase of bonds as part of a regular savings plan. Provide opportunities for students to investigate:

- different types of bonds that are available
- current interest rates paid on bonds
- the advantages/disadvantages of purchasing bonds.

Use the following chart to determine interest on a Canada Savings Bond paying a 9.75% rate of return. Ask students to:

- calculate the interest earned each year to maturity
- explain why the interest earned each year varies
- determine interest paid on amounts other than $100.

**Compound Interest Paid on a $100 Canada Savings Bond**

| Years after issue date | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Redemption value | $109.75 | $120.45 | $132.19 | $145.08 | $159.23 | $174.75 | $191.79* |

*Maturity

7. Investigate the effects of "compound interest" on money that is invested. Demonstrate how compound interest may be calculated as a series of simple interest payments. Encourage students to organize their work in a table when calculating compound interest.

CLARIFICATION/EXAMPLE

| Period | Principal | Interest Rate (per annum) | Time (in years) | Interest $I = prt$ | Amount $A = P + I$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | | | | | |

8. Discuss the meaning of the following statement:

"The account pays 12% per annum compounded quarterly."

Demonstrate alternative methods of calculating the interest earned throughout each interest period. Students should note that the rate (r) and time (t) must be in the same units when calculating interest.

**CLARIFICATION/EXAMPLE**

**TWO METHODS OF CALCULATING QUARTERLY INTEREST**

Find the interest earned on \$100 invested at 12% per annum, when compounded quarterly.

Method 1:
Convert the time into years.

$I = prt$

$I = \$100\ (0.12)\ (0.25)$

$I = \$3.00$

Interest for the first quarter is \$3.00

Method 2:
Convert the yearly interest rate to a quarterly interest rate.

$I = prt$

$I = \$100\ \left( \frac{0.12}{4} \right)\ (1)$

$I = \$100\ (0.03)\ (1)$

$I = \$3.00$

Interest for the first quarter is \$3.00

9. Assist students to design a simple computer program (or flow chart) for determining simple/compound interest paid by an investment (see Use of Technology, "The Computer").

Encourage students to adapt/modify the interest formula to suit situations that involve:

- periods of time other than one year
- daily/monthly interest rates.

10. Introduce students to the "Dynasave" feature of *Spreadsheets for Students* (see Computer Software). Using Dynasave, ask students to determine returns on a variety of investments using different interest rates and varying periods of time.

# RESOURCE 1: TAKING CHARGE[1]

## THERE IS MORE TO LEARN ABOUT TAKING CHARGE OF YOUR MONEY

There are excellent sources of information all around you. You can learn how to invest your savings, plan your retirement, and shop wisely. Want to know more about getting out of debt? Reducing expenses? Protecting your assets? How about learning different ways to plan your spending? All this information is available. Here is a list of some ways to find the information you want.

### TAKE A COURSE

Alberta Consumer and Corporate Affairs offers free programs on money management. Call the office nearest you for more information.

Community colleges also offer money management programs. For example, you might contact:

Consumer Education Project
Grant MacEwan Community College
Cromdale Campus
8020 - 118 Avenue
Edmonton, Alberta
T5B 0R8
Telephone: 477-0200

Consumer Education Program
Division of Continuing Education
Lethbridge Community College
3000 College Drive South
Lethbridge, Alberta
T1K 1L6
Telephone: 320-3343

Continuing education divisions of other community colleges, universities, and school boards often sponsor money management courses and workshops. Call the agencies in your area to see what is available.

### READ THE NEWSPAPERS AND MAGAZINES

Keep up-to-date with the latest in money management ideas by reading the national financial press. Three Canadian papers that carry financial news are the *Financial Post*, the *Financial Times*, and the *Globe and Mail*.

Many local Alberta papers also carry interesting and useful information on financial matters. Check your paper each day for relevant articles.

*Business Life, Canadian Business, Financial Post Magazine, Your Money,* and *Fortune* are magazines you might also like to read. Many libraries have copies. If you like what you see, order your own.

## MAKE USE OF THE SERVICES OFFERED BY AGENCIES

*Alberta Agriculture*
You can get dozens of information sheets and booklets from this department. Many stress money-saving ideas. Write and ask for the *Publications List*.

**Alberta Agriculture**
**Print Media Branch**
**J.G. O'Donoghue Building**
**7000 - 113 Street**
**Edmonton, Alberta**
**T6H 5T6.**

*Alberta Consumer and Corporate Affairs*
This department offers free debt counselling. The article "Credit: What to do if You Have Too Much of a Good Thing" describes the service. The department also offers publications on money and credit management.

*Alberta Energy*
Booklets on saving money by conserving energy are available from this department. Ask for the *Alberta Energy Saver Series.*

**Energy Conservation Branch**
**Alberta Energy**
**2nd Floor, Highfield Place**
**10010 - 106 Street**
**Edmonton, Alberta**
**T5J 3L8**

*Alberta Career Development and Employment*
This department offers career information, counselling, and workshops. Call the Career Hotline for more information.

**Career Hotline**
**Edmonton residents call 422-4266**
**All other Albertans dial 0 and ask the operator for Zenith 22140 (toll free)**

*Alberta Securities Commission*
Contact the Commission for information on securities and franchises or for copies of these booklets: *A Guide to Securities Investments, A Guide to Old Shares, A Guide to Franchising, An Introduction to Franchising in Alberta*. The Commission also periodically distributes Investor Alerts. For a nominal fee you can examine public files at the Commission's offices on companies which are reporting issuers. Among other things, these files contain historical, financial, and management information on publicly traded companies.

**Alberta Securities Commission**
**21st Floor**
**10025 Jasper Avenue**
**Edmonton, Alberta**
**T5J 3Z5**
**or**
**919 J. J. Bowlen Building**
**620 - 7 Avenue S.W.**
**Calgary, Alberta**
**T2P 0Y8**

## RESOURCE 1: TAKING CHARGE (continued)

*Alberta Women's Secretariat*
Among other activities, the secretariat collects and distributes information of interest to women. Write for these booklets: *Wills and Estates for Albertans, A Guide to Probate Procedure.*

Alberta Women's Secretariat
8th Floor, Kensington Place
10011 - 109 Street
Edmonton, Alberta
T5J 3S8

*The Canadian Life and Health Insurance Association*
Contact this Association if you have questions about life or disability insurance or if you want one of these booklets: *Planning for Success, Family Money Manager, A Guide to Buying Life Insurance, Sharpen Your Pencil, Where Will the Money Come From if You're Disabled, You and Your Group, Where is Everything, Planning for a Successful Retirement.*

The Life and Health Insurance Information Centre
2500, 20 Queen Street West
Toronto, Ontario
M5H 3S2
Call 1-800-268-8099 (toll free)

*Canadian Securities Institute*
Write to this industry sponsored educational organization for a list of publications and public courses.

Canadian Securities Institute
#2330, 355 - 4 Avenue S.W.
Calgary, Alberta
T2P 0J1

*Dial-A-Law*
This service is sponsored by Calgary Legal Guidance. Phone toll free and listen to recorded messages on law related matters. A written copy of the information will be sent to you on request. Some tapes related to money and credit management are: *Buying on Time, Sales Contracts, Social Allowance, Bankruptcy, Foreclosure, Financing a Home, Collection Agencies.* Call Monday to Friday between 8:30 a.m. and 4:30 p.m.

Dial-A-Law
Calgary residents call 234-9022
All other Albertans call 1-800-332-1091 (toll free)

*The Insurance Bureau of Canada*
Contact this Bureau for information about automobile or property insurance or if you want one of these booklets: *Car Insurance Explained, Home Insurance Explained.*

The Insurance Bureau of Canada
1105, 10080 Jasper Avenue
Edmonton, Alberta
T5J 1V9
Edmonton residents call 423-2212
All other Albertans call 1-800-232-7275 (toll free)

## RESOURCE 1: TAKING CHARGE (continued)

*Revenue Canada*
In addition to answering tax questions by telephone, Revenue Canada distributes dozens of useful pamphlets. Ask for a list of what is available.

Revenue Canada
Edmonton residents call 420-3510
Calgary residents call 292-4101
All other Southern Albertans call 1-800-332-1410 (toll free)
All other Northern Albertans call 1-800-232-1966 (toll free)

*Other Organizations*
Banks, credit unions, trust companies, securities dealers, chartered accounting firms, etc. often have booklets and information.

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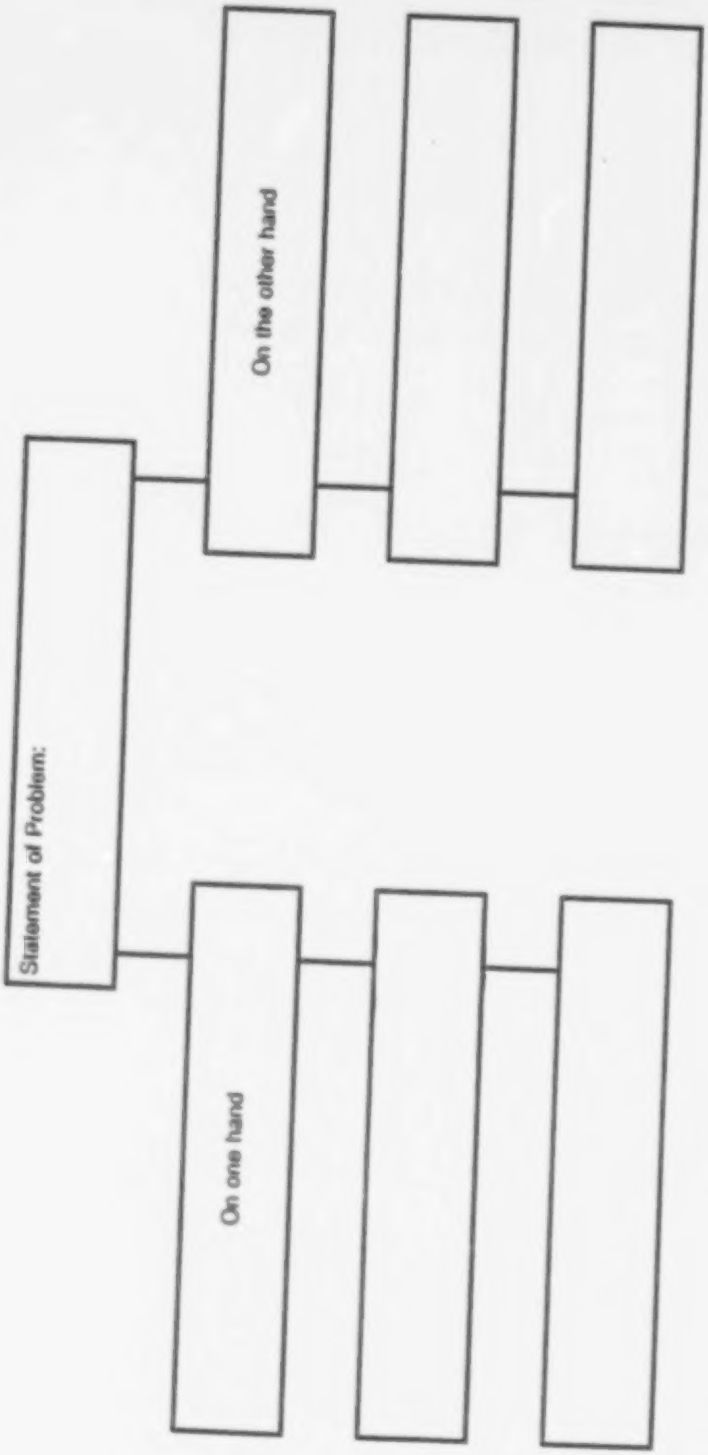
## RESOURCE 2: COMPARATIVE AND CONTRASTIVE WEB


On the other hand
Statement of Problem:
On one hand

# RESOURCE 3: THE CREDIT CHOICE[1]

## SECURITY FOR LOANS

In many instances, the only security required for a loan is your signature. Your signature is a symbol of your character, reputation and willingness to repay. In other instances, security may take the form of real estate, stocks and bonds or durable goods, such as a car.

When you offer tangible assets as security for a loan, the amount that you may borrow may be increased and/or the rate of interest charged may be lowered. Remember, however, that when you offer security, you risk losing it if you do not meet your obligation to repay.

In some provinces, lenders require the signatures of both spouses on loans which use, as security, such property as the family home or car. This requirement serves to protect the lender's security as well as making both spouses aware that assets considered to be family property under provincial family law legislation are being used to secure a loan.

Two types of secured loans will be discussed here – the chattel mortgage and the collateral mortgage.

### CHATTEL MORTGAGE

In legal terms most movable property, such as boats and cars, is referred to as a chattel. Accordingly, if you provide such property as security, you are required to sign a promissory note and a chattel mortgage which gives the lender the right to take possession of the property if you do not pay as agreed. The chattel mortgage contains a number of conditions, which may be the following: you must make payments as agreed, the property provided as security must not be taken out of the province permanently without the consent of the lender, the security must be kept free of other legal claims (liens) and the security must not be sold without the permission of the lender.

### COLLATERAL MORTGAGE

This is a means of using equity in real property, such as a house, as security for a loan. Equity is the difference between what you could sell the property for and what you still owe on it. Using the equity may help you obtain a better rate or a larger loan.

Some people have a high priority on free and clear ownership of their home. If you feel this way, you should consider using some other form of security rather than the equity in your home.

## THE COST OF CREDIT

If you see an ad promising "Nothing down! Easy terms!", be suspicious. Someone always has to pay. When it comes to consumer credit, it is you – the borrower – who covers the costs. You pay this in the form of interest, which is the fee, or charge, for using someone else's money. Interest is generally expressed in the form of a percentage rate.

## BUILDING A GOOD CREDIT RATING

If you are starting out, financially speaking, the first favour that you can do for yourself is to build the basis for a good credit rating. Here is how you can get started.

1. Open a savings account and make regular deposits. The idea is to save money toward the cost of purchasing what you will want in the future and to show that you are responsible and reliable about money.
2. Be prompt about payments for any obligations (e.g., your rent and your monthly utility bills).
3. When you borrow, borrow only what you need or can comfortably repay.
4. Arrange your loan repayment schedule with a view to repaying the loan as soon as possible.

## HOW THE COST OF CREDIT IS SET

Many factors influence the cost of credit to you, the borrower – the cost of money, the risk involved and other costs of doing business. Lenders obtain their money from either depositors (as, for instance, the banks do) or from other financial institutions (as finance companies do). These sources must be paid a return on the money they are lending to you.

The difference in interest rates between what the lender pays to borrow money and what the lender charges you for a loan is commonly referred to as the spread. The money earned by the lender because of the spread is expected to cover operating expenses and to provide a reasonable profit to the lender.

Another factor is the risk of non-repayment as assessed by the lender. Financial institutions must ensure that loans are repaid because they have a responsibility to their depositors and/or shareholders whose money is being used to make those loans. Thus the interest rate will vary with the degree of risk.

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[1] Reproduced with permission from *Credit Wise*, published by the Canadian Bankers' Association.

# RESOURCE 4: MONTHLY LOAN PAYMENTS

**TABLE FOR DETERMINING MONTHLY PAYMENTS**

| Interest Rate | One-Year | Two-Year | Three-Year | Four-Year |
|---|---|---|---|---|
| 10% | 0.08762 | 0.04614 | 0.03227 | 0.02536 |
| 10½% | 0.08815 | 0.04638 | 0.3227 | 0.02560 |
| 11% | 0.08838 | 0.04661 | 0.03250 | 0.02585 |
| 12% | 0.08885 | 0.04707 | 0.03321 | 0.02633 |
| 12½% | 0.08908 | 0.04731 | 0.03345 | 0.02658 |
| 13% | 0.08932 | 0.04754 | 0.03369 | 0.02683 |
| 13½% | 0.08955 | 0.04778 | 0.03394 | 0.02708 |
| 14% | 0.08979 | 0.04801 | 0.03418 | 0.02733 |
| 14½% | 0.09002 | 0.04825 | 0.03442 | 0.02758 |
| 15% | 0.09026 | 0.04849 | 0.03467 | 0.02783 |
| 15½% | 0.09049 | 0.04872 | 0.03491 | 0.02808 |
| 16% | 0.09073 | 0.04896 | 0.03516 | 0.02834 |
| 16½% | 0.09097 | 0.04920 | 0.03540 | 0.02860 |
| 17% | 0.09120 | 0.04944 | 0.03565 | 0.02886 |

Follow these steps when using the table to determine monthly payments on a loan:

1. Locate the row corresponding to the appropriate interest rate and the column corresponding to the desired amortization period.

2. Multiply the number in the chart corresponding to the appropriate interest rate and amortization period by the amount of the loan.

**CLARIFICATION/EXAMPLE**

The monthly payment on a $10 000 loan at 13% over 3 years is:
0.03369 x $10 000 = $336.90

# RESOURCE 5: BUDGET PLANNER[1]

## BUDGET DETAILS

Transfer the totals from boxes A, B, and C to the appropriate lines on the following page.

**A MONTHLY UTILITIES**

| | | |
|---|---|---|
| POWER | | |
| WATER/SEWAGE/WASTE | | |
| TELEPHONE | local | |
| | long distance | |
| NATURAL GAS | | |
| CABLE TV/PAY TV | | |
| TOTAL MONTHLY UTILITIES | | |

**B MONTHLY PERSONAL ALLOWANCE AND RECREATION**

| | | |
|---|---|---|
| PERSONAL ALLOWANCES (monthly amount for each family member for lunches, grooming, hobbies, tobacco, alcohol, etc.) | | |
| | | |
| | | |
| | | |
| | | |
| FAMILY RECREATION | | |
| TOTAL MONTHLY PERSONAL ALLOWANCES AND RECREATION | | |

**NOTES**

**C IRREGULAR AND ANNUAL EXPENSES (All figures on this chart should be annual not monthly amounts.)**

| | | |
|---|---|---|
| CLOTHING (list annual amount for each family member) | | |
| | | |
| | | |
| | | |
| | | |
| INSURANCE (if you pay directly) | vehicles | |
| | life | |
| | property | |
| | other | |
| MEDICATION/MEDICAL FEES | | |
| DENTAL/OPTICAL | | |
| EDUCATION | tuition | |
| | books & supplies | |
| TAXES (if you pay directly) | property | |
| | income | |
| LICENSES | vehicles | |
| | other | |
| MAINTENANCE | vehicles | |
| | home & garden | |
| | furnishings | |
| GIFTS/FESTIVITIES | Christmas | |
| | other | |
| TRAVEL/HOLIDAYS | annual holiday | |
| | other travel | |
| CONTRIBUTIONS/DONATIONS | | |
| MEMBERSHIPS/SEASON TICKETS | | |
| OTHER (subscriptions, items you plan to purchase next year, etc.) | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| TOTAL IRREGULAR AND ANNUAL EXPENSES | | |
| MONTHLY REQUIREMENT FOR IRREGULAR AND ANNUAL EXPENSES (divide total by 12) | | |

# RESOURCE 5: BUDGET PLANNER (continued)

## MONTHLY TAKE-HOME INCOME

| | |
|---|---|
| WAGES/SALARY | |
| WAGES/SALARY | |
| FAMILY ALLOWANCE/PENSION | |
| OTHER REGULAR MONTHLY INCOME | |
| **TOTAL MONTHLY TAKE-HOME INCOME** | |

## MONTHLY BUDGET

| | | |
|---|---|---|
| **MONTHLY SAVINGS** | EMERGENCY FUND | |
| | | |
| | | |
| | **TOTAL MONTHLY SAVINGS** | 1 |

| | | | |
|---|---|---|---|
| **MONTHLY LIVING EXPENSES** | FOOD (plus other grocery story items) | | |
| | HOUSING | 1st mortgage or rent | |
| | | 2nd mortgage | |
| | | | |
| | UTILITIES (total from box "A" on opposite page) | | |
| | HOUSEHOLD INCIDENTALS (dry cleaning, newspapers, etc.) | | |
| | TRANSPORTATION | gasoline | |
| | | bus fare | |
| | | parking | |
| | PERSONAL ALLOWANCES AND RECREATION (total from box "B" on opposite page) | | |
| | OTHER (child support, alimony, child care, household help, etc.) | | |
| | | | |
| | | | |
| | MONTHLY REQUIREMENT FOR IRREGULAR AND ANNUAL EXPENSES (total from box "C" on opposite page) | | |
| | **TOTAL MONTHLY LIVING EXPENSES** | | 2 |

| | | |
|---|---|---|
| **MONTHLY CREDIT PAYMENTS** | | |
| | | |
| | | |
| | **TOTAL MONTHLY CREDIT PAYMENTS** | 3 |

| | |
|---|---|
| **TOTAL MONTHLY BUDGET (add lines 1, 2, and 3)** | |

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## RESOURCE 6: AN EMERGENCY FUND FOR YOUR FAMILY[1]

### An Emergency Fund for Your Family

LIFE IS NEVER AS SMOOTH AS YOU MIGHT LIKE. A SUDDEN DEATH, A SERIOUS ACCIDENT, a strike, an unexpected downturn in the economy; any of these could put a serious strain on your income. Even a small but unavoidable over-expenditure in one budget category could begin a destructive cycle of borrowing from a second category to pay the first, from a third category to pay the second, and so on. Looking on the bright side, you might need money for an unexpected opportunity.

> **••• use the fund for real emergencies — not for expenses such as car insurance, holidays •••**

Having an emergency fund is one way of meeting these financial challenges. Here is how to set up such a fund. First, decide on the total amount of money you would like to have in your fund. A general guideline is: set aside an amount of money equal to what your family would need to live on for three months. When you are calculating this amount, include food, housing, and transportation costs. Also include money for the monthly payments to which your family is committed.

You might need more or less than three months' living expenses in your particular fund. It depends on the emergencies your family is likely to face and how much money you can count on in each case. Answering the questions in the display box titled *"Emergency! Emergency!"* will help you decide if three months' living expenses is the right amount for your family. Once you have decided how much money you want in your fund, start saving. Even if you can set aside only a few dollars every month, it will add up in time. Keep your emergency fund in a savings account or in readily accessible investments such as Canada Savings Bonds.

There are three other important points about emergency funds. Situations change. Review your fund once each year to be sure the amount you have set aside is adequate. Second, if you must withdraw money from your fund, rearrange your budget so you can put the money back into the fund. Finally, use the fund for real emergencies – not for expenses such as car insurance, holidays, and Christmas. These expenses can be anticipated. That's what financial planning is all about.

## RESOURCE 6: AN EMERGENCY FUND FOR YOUR FAMILY (continued)

**Emergency! Emergency!**

Think of all the financial emergencies your family might have to deal with in the future. Calculate how much money you could count on in each case. Answering the questions below will get you started. Your answers might show that you would be short of money and headed for problems if something bad happened. One way to avoid problems is to set up an emergency fund.

1. If you were to go on strike, how much strike pay would you receive? For how long?

   __________________________________________

2. If you lost your job, would you be able to collect unemployment insurance? How much? For how long?

   __________________________________________

3. If you had an accident or illness that kept you from working for a prolonged period, would you still receive an income? How much? For how long?

   __________________________________________

4. If you were to die suddenly, would the family have immediate access to sufficient cash for day-to-day expenses and funeral arrangements? How much money would they need immediately?

   __________________________________________

5. What is the deductible on your car insurance policy or policies? Your household policy?

   __________________________________________

6. What emergencies other than those already mentioned might befall your family?

   __________________________________________

7. Is there another income earner in your family? What is that person's monthly net income?

   __________________________________________

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# THEME C
# SKILLS FOR THE CONSUMER

## SUB-THEMES

- ESTABLISHING A PLACE OF YOUR OWN
- BUYING A VEHICLE

## RATIONALE

Many students may soon wish to establish a place of their own and/or buy a vehicle. This theme focuses attention on strategies that will enable students to maximize the value they receive for money spent, and is intended to develop a process for solving problems and making decisions in real life situations that involve:

- "moving out" and establishing a residence of their own
- buying and paying for a vehicle.

Students will apply previously developed consumer skills (see Mathematics 16: Skills for the Consumer) throughout this theme as they investigate costs associated with establishing a residence of their own and buying a vehicle. Learning objectives interrelate with activities in a prior theme, Budgeting and Banking, and will assist students to understand the cost of credit and importance of planning for major purchases.

Abundant opportunities exist for students to reinforce basic computation and estimation skills. Students will apply these skills to situations involving decimals, fractions, ratios and percents. Measurement skills will be used in making quantitative comparisons among competing consumer products. Problem-solving activities will place emphasis on developing strategies for making informed consumer choices, and will require students to gather, compare and assess information about competing consumer goods and services.

Cooperative planning among teachers will ensure that skills developed within this theme complement consumer topics studied by students in other subject areas. The local community offers a variety of resources that may be used in developing consumer skills through situational and concrete learning experiences.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed in this theme can be found in:

- Problem Solving
- Computational Facility and Estimation
- Situational and Concrete Approaches
- Assessment/Evaluation.

## THEMATIC OBJECTIVES

### ESTABLISHING A PLACE OF YOUR OWN

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify various perspectives that need to be considered in making informed choices about housing alternatives (e.g., location, size, amenities, cost)
- gather, compare and assess information about different types of rental housing that are available (e.g., apartment, duplex, single detached dwelling)
- demonstrate an understanding that there are "move-in" costs associated with establishing a rental home (e.g., utility hook-up fees, damage deposits)
- explain the benefits, limitations and costs of different types of insurance intended for the consumer who chooses a rental home
- determine the total monthly cost of maintaining a rental home, considering fixed costs (e.g., rent, insurance) and variable costs (e.g., utilities, telephone)
- plan the furnishings for at least one room in a home, using scale drawings
- estimate and/or calculate the cost of decorating at least one room in a home, selecting appropriate floor/wall/window coverings and furniture.

### BUYING A VEHICLE

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- identify various perspectives that need to be considered when planning to buy a new or used vehicle (e.g., personal needs, financial resources, quality and cost of vehicle)
- gather, compare and assess information about different types of vehicles advertised in the marketplace
- compare the costs of various payment options that may be available for the purchase of a vehicle
- determine the monthly payment and total cost of a vehicle loan, using prepared loan schedules
- demonstrate an understanding of different types of vehicle insurance (e.g., liability, collision), and of factors that may affect insurance premiums (e.g., age and gender of driver, driving record, type of coverage, deductible amount, type and age of vehicle)
- demonstrate an understanding that a vehicle's value may depreciate, and that the rate of depreciation will vary according to vehicle type
- estimate and/or maintain a record of the monthly costs of owning and operating a vehicle, considering factors such as loan payments, insurance, depreciation, maintenance and fuel costs.

## CONTEXT FOR INSTRUCTION

### PROBLEM SOLVING

Students will be expected to:

- consider the costs, advantages and disadvantages associated with alternative forms of rental housing and different types of vehicles advertised in the marketplace
- gather, compare and assess information about competing products and services in order to make informed consumer decisions.

### USE OF TECHNOLOGY

Students will be expected to:

- use a calculator to make cost comparisons among competing products and services, and to determine costs associated with renting a home and/or buying a vehicle
- develop an understanding of how computer technology is used in maintaining driver records, monitoring insurance claims, and establishing insurance premiums
- use prepared computer software as required to:
  - maintain/reinforce basic concepts and skills in numeration and measurement
  - compare the costs of various payment options that may be available when financing the purchase of a vehicle.

### COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator on a regular basis
- round as appropriate during calculations
- use mental arithmetic when appropriate to expedite solutions to quantitative problems
- use estimation skills in a variety of consumer situations (e.g., to make price comparisons, to estimate total cost of a product or service).

## SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

| | | | | |
|---|---|---|---|---|
| **Number Systems and Operations** | **Ratio, Proportion and Percent** | **Algebra** | **Geometry and Measurement** | **Data Interpretation and Display** |

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## NUMBER SYSTEMS AND OPERATIONS

Students will be expected to:

- read, write and order whole numbers, decimals and fractions in applications
- identify place value from 0.01 to 1 000 000
- round whole numbers and decimals as required
- perform basic operations with whole numbers, decimals and fractions
- perform a sequence of operations in correct order
- multiply and divide whole numbers/decimals by multiples of 10
- recall decimal equivalents for one-half, thirds, quarters, fifths and tenths
- convert fractions into decimals.

## RATIO, PROPORTION AND PERCENT

Students will be expected to:

- compare two quantities in the same unit by writing a ratio
- generate equivalent ratios
- calculate rates/unit rates by writing ratios that involve numbers with different units
- write proportions that describe practical problem situations
- calculate the unknown value in a proportion
- express percents as ratios and/or decimals and vice versa
- estimate and calculate a percent of a number
- estimate and calculate what percent one number is of another.

## ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions/equations that describe practical situations
- apply equation-solving strategies to practical situations involving whole numbers and decimals
- interpret formulas as word statements
- use substitution to find the missing element in a formula.

## GEOMETRY AND MEASUREMENT

Students will be expected to:

- estimate and measure length/capacity, using SI units and tools appropriate to the situation
- convert among SI units of length/capacity as required in applications
- calculate perimeter and area using units and tools appropriate to the situation
- construct scale drawings using appropriate ratios.

## DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- interpret and/or construct tables, charts and graphs that display information about consumer products and services
- interpret and/or calculate appropriate measures of central tendency in practical situations
- conduct a consumer survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing information.

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## INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with consumer topics studied by students in other subject areas. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

### ENGLISH

Students may:

- develop an understanding of persuasive language and propaganda devices frequently used in the promotion of consumer products and services
- gather, compare and assess information about competing products and services advertised in the marketplace
- develop an understanding of technical vocabulary used in rental agreements, sales contracts and insurance policies.

### SCIENCE

Students may:

- identify basic principles associated with the functioning of sub-systems in a vehicle, and simple maintenance procedures that contribute to the efficient performance and general safety of a vehicle
- develop an understanding of recent technologies in the automobile industry (e.g., safety features, fuel efficiency, pollution control devices)
- investigate the effects of our use of fossil fuels on the environment, and the need for developing alternative sources of energy.

### SOCIAL STUDIES

Students may:

- consider the advantages and disadvantages of alternative forms of transportation and/or housing available in the local community
- investigate safety and environmental legislation established for the automobile industry (e.g., seat belt laws, emission control standards)
- examine the advantages and disadvantages associated with different types of loans and/or purchase agreements
- develop an understanding of the need for public liability insurance.

### OCCUPATIONAL COURSES

Students may:

- estimate and measure length, area and capacity as required in determining materials required for a decorating project
- interpret "directions for use" that are provided on consumer product labels (e.g., litres of paint required to paint a wall, rolls of wallpaper required to decorate a room)
- develop an understanding of the strategies and costs associated with maintaining a home and/or vehicle
- interpret and construct scale drawings of construction and/or repair projects in the home.

## COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing community resources, and for developing consumer skills through situational and concrete learning experiences in the community are provided below.

- Gather, compare and assess information on competing goods and services available in the local community. Establish an information base relevant to this theme by:
  - investigating different types (and costs) of rental housing that are available
  - investigating the price of new and used vehicles that are advertised for sale.
- Visit a local car dealership. Investigate the cost of new and used vehicles, and ask a salesperson to explain how the price of a new vehicle is determined (e.g., base price plus options).
- Invite representatives from local government agencies to discuss consumer issues related to renting a home or purchasing a vehicle (e.g., a representative from Alberta Consumer and Corporate Affairs might be asked to explain the rights and responsibilities of landlords and tenants as stated in "The Landlord and Tenant Act").
- Invite a knowledgeable person from the insurance industry to describe:
  - different types of vehicle and home insurance
  - factors that may affect insurance premiums.
- Invite a knowledgeable person from the automotive industry to describe:
  - strategies for monitoring vehicle fuel consumption and economy
  - costs associated with maintaining a vehicle.
- Consult consumer reports and car magazines for performance and safety information on new and used vehicles.
- Consult local utility companies. Investigate the hookup fees and monthly costs associated with various utilities that may be required when establishing a new home.
- Invite an interior decorator (or other knowledgeable person) to discuss strategies for decorating and furnishing a home on a limited budget.
- Invite a loans officer from the local bank to answer questions regarding payment options and other factors that need to be considered when purchasing a vehicle.

  e.g.,
  - different types of car loans and corresponding payment schedules
  - loan security (e.g., collateral, cosigner)
  - liens and encumbrances registered against a vehicle
  - current depreciation rates for different types of vehicles

## ESTABLISHING A PLACE OF YOUR OWN

### LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 15 – Renting and Decorating a Home

SUPPORT LEARNING RESOURCE

MYN: Unit 3 – Finding Your Own Place
Unit 4 – Running Your Own Place
Unit 9 – Insurance

*Money Manager* (computer software)

OTHER LEARNING RESOURCES

"Consumer Talk" Information Sheets:
*Tenants: Living There*
*Tenants: Moving In*
*Tenants: Moving Out*

*Decisions, Making Personal Economic Choices*
Chapter 10 –Getting and Using Information
Chapter 13 – Consumer Rights and Responsibilities
Chapter 15 – Shopping for Housing and Transportation
Chapter 19 – Shopping for Insurance

*Managing Lifestyles: Survival Math Skills* (computer program)
Budget for Success

*Moving Out*
Housing (pp. 4–11)
Money (pp. 12–24)

*Street Cents* (video program)

### SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Alberta Consumer and Corporate Affairs will provide materials relevant to topics studied in this sub-theme. The publications listed below are free, except where noted*, and can be ordered in classroom quantity from any one of the Alberta Consumer and Corporate Affairs offices listed in Appendix B.

   *Consumer Complaints* (booklet)

   "Consumer Talk" Information Sheets:
   *Tenants: Living There*
   *Tenants: Moving In*
   *Tenants: Moving Out*

   *Moving Out* (booklet)*

   *Class sets must be ordered at cost from the Learning Resources Distributing Centre, 12360 – 142 Street, Edmonton, Alberta, T5L 4X5.

   These materials will assist students in developing:

   - an awareness of some of their rights and responsibilities as tenants and consumers
   - personal strategies for dealing independently with tenant and consumer problems.

2. Simulate real-life situations that require students to develop and apply problem-solving strategies in "finding a place of their own". Assist students to define related problems, develop and carry out problem-solving plans, and choose among alternatives using suggestions and ideas provided in Problem Solving, "Using Strategies to Solve Problems". Ask students to provide reasons for the choices they make, and evaluate the action plans that they and others formulate.

   Mathematics teachers are encouraged to consult with CALM teachers regarding program coordination and the use of related resource materials and student activities.

3. Use classified advertisements in the newspaper to compare rental rates on apartments and semi-detached/detached housing in the local area. Sample advertisements are provided in Resource 1: Finding Rental Housing Through the Newspaper.

   Ask students to underline/circle pertinent facts in each advertisement, and determine:

   - the approximate cost of utilities (e.g., water & sewer, electricity, natural gas, telephone)
   - additional costs that may be incurred (e.g., parking, damage deposit, cable television).

   Encourage students to evaluate the appropriateness of various alternatives in rental housing by considering a variety of factors.

   e.g., – facilities/amenities offered
   – comparative costs
   – personal finances available.

4. Collect classified advertisements for comparable apartments or semi-detached/detached houses. Using these advertisements, ask students to determine:

   - the range of rental rates
   - the mean, median and mode of rental rates
   - the "best" measure of central tendency for this information.

5. Ask students to research costs that may be incurred when "moving in" to a place of their own (e.g., damage deposits, utility hookups, moving expenses, furnishings). A variety of related expenses are discussed in Resource 2: Moving-In Costs. Additional information and classroom activities are provided in:

   - *Moving Out* (pp. 12–13)
   - *Tenants: Moving In*

6. Invite a home insurance salesperson to discuss:

   - different types of home insurance designed for tenants
   - factors that determine premium rates
   - the cost of appropriate insurance coverage.

   Read the information provided in Resource 3: A Guide to Property and Automobile Insurance Claims. Compare major types of insurance protection discussed in this article.

   e.g., – actual cash value
   – replacement value
   – stated value.

7. Obtain additional information on property insurance by contacting the following agencies:

   The Insurance Bureau of Canada
   1105, 10080 Jasper Avenue
   Edmonton, Alberta
   T5J 1V9
   Phone: 423-2212 or 1-800-232-7275

   Insurance Brokers Association of Alberta
   505, 10240 - 142 Street
   Edmonton, Alberta
   T5N 3W6
   Phone: 482-6340 or 1-800-232-7300.

8. Gather information about the cost of tenant's insurance by contacting two or more local insurance companies. Compare costs and determine if there are differences in the coverage or services offered by these companies.

9. Ask students to simulate a "living on your own" situation and approximate their total monthly expenses. Students should consider factors such as:

   - rent
   - utilities
   - insurance premiums
   - food/clothing
   - transportation
   - entertainment
   - purchase of home furnishings.

   Organize and record the results of this activity using the "Budget Planner" provided in *Moving Out* (pp. 14–15).

10. Provide opportunities for students to investigate the cost of furnishing/redecorating their future home. Ask each student to:

   - prepare a scale drawing of one room that illustrates the work to be done (e.g., paint, wallpaper, floor covering) and furnishings that need to be purchased
   - determine the cost of redecorating this room through the use of advertisements/store flyers/catalogues.

   Students may wish to use the centimetre grid paper provided in Resource 4 when preparing scale drawings.

## BUYING A VEHICLE

### LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 12 – Buying a Car

SUPPORT LEARNING RESOURCE

MYN: Unit 2 – Automotive Math
Unit 8 – Loans and Credit
Unit 9 – Insurance

*Money Manager* (computer software)

OTHER LEARNING RESOURCES

*Consumer Complaints*

"Consumer Talk" Information Sheets:
*Advertising*
*Buying a Used Car*
*Car Repairs*
*Shop Around*

*Decisions, Making Personal Economic Choices*
Chapter 10 – Getting and Using Information
Chapter 13 – Consumer Rights and Responsibilities
Chapter 15 – Shopping for Housing and Transportation
Chapter 19 – Shopping for Insurance

*Forms in Your Life: A Student Workbook and Guide to Everyday Forms*
Form 17 – Application for a Car Loan
Form 18 – Application for Car Insurance

*Moving Out*
Credit (pp. 25–27)
Transportation (pp. 28–31)

*Street Cents* (video program)

### SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Alberta Consumer and Corporate Affairs publishes a variety of useful materials relevant to topics studied in this sub-theme. The publications listed below are free, except where noted*, and can be ordered in classroom quantity from any one of the Alberta Consumer and Corporate Affairs offices listed in Appendix B.

   *Consumer Complaints* (booklet)

   "Consumer Talk" Information Sheets:
   *Advertising*
   *Buying a Used Car*
   *Car Repairs*
   *Shop Around*

   *Moving Out* (booklet)*

   *Class sets must be ordered at cost from the Learning Resources Distributing Centre, 12360 – 142 Street, Edmonton, Alberta, T5L 4X5.

   These materials will assist students in developing:

   - an awareness of some of their rights and responsibilities when purchasing an automobile
   - personal strategies for dealing independently with problems encountered when purchasing an automobile.

2. Make a list of some pros and cons related to buying and owning a motor vehicle. Consider factors such as:

   - transportation needs
   - initial cost of the vehicle
   - maintenance and repair costs
   - insurance premiums
   - personal finances.

   Compare the cost of owning and operating a vehicle to the cost of alternative methods of transportation (e.g., public transportation, bicycle, taxi).

3. Investigate the cost of new/used vehicles through the use of classified advertisements in the newspaper, or by visiting local car dealerships.

   Compare the cost of a vehicle purchased from a private individual to the cost of a comparable vehicle (e.g., similar make, model, age and condition) purchased from a car dealership. Discuss advantages and disadvantages of making each type of purchase.

4. Have students work in small groups to research the purchase prices, repair costs, insurance costs and loan options on:

   - a new car
   - a used car
   - a new motorcycle.

   Compare costs related to each of these transportation alternatives, and display the results of investigation in the form of charts and graphs.

5. Invite a loans officer from a local bank to discuss various factors related to financing a new and/or used vehicle.

   e.g.,
   - various types of car loans that are available
   - collateral required for a car loan
   - financial responsibilities of a co-signer
   - appropriate down payments
   - interest rates/amortization periods/monthly payments
   - liens or other encumbrances that may be registered against a vehicle
   - current automobile depreciation rates.

   Additional information on loan options and credit is provided in *Moving Out* (pp. 25–29).

6. Ask students to simulate situations in which they:

   - "shop" for a vehicle appropriate to their needs
   - arrange to purchase a vehicle through the use of credit.

   Useful tips for selecting a vehicle are provided in *Buying a Used Car.* Assist students to arrange credit for their purchase by considering personal resources and related factors.

   e.g.,
   - the amount of the down payment
   - an appropriate amortization period
   - affordable monthly payments
   - the total cost of the loan.

   Students may wish to use Resource 5: Schedule for Determining Monthly Automobile Payments, when arranging a loan option appropriate to personal circumstances.

7. Provide opportunities for students to compare the cost of financing the purchase of a vehicle with different creditors in the local community.

   **CLARIFICATION EXAMPLE**

| Source of Credit | Annual Interest Rate | Actual Cost of Credit on Vehicle Purchase |
|---|---|---|
| | | |
| | | |
| | | |

8. Invite a representative from the Motor Vehicles Branch or an automobile insurance salesperson to discuss:

   - minimum insurance requirements for vehicle operators
   - factors that affect insurance premiums (e.g., age and driving record of driver, insurance coverage, age and type of vehicle).

   Ask students to use the tables provided in Resource 6: Automobile Insurance Rates, to determine the cost of various types of insurance coverage.

9. Obtain additional information on automobile insurance by contacting the following agencies:

   - The Insurance Bureau of Canada
   - Insurance Brokers Association of Canada.

   Addresses and phone numbers for these agencies are provided in the sub-theme "Establishing a Place of Your Own" (see p. 81).

10. Provide opportunities for students to estimate (or maintain a record of) the weekly/monthly costs associated with owning and operating a vehicle. When estimating costs, encourage students to consider:

   - fixed costs
     e.g., – loan payments
     – insurance premiums
     – depreciation
     – vehicle registration and driver licensing
   - variable costs
     e.g., – fuel
     – routine maintenance
     – other repairs.

   If possible, invite the Automotive Services teacher or another knowledgeable person in the community to discuss the cost of routine vehicle maintenance (e.g., oil change, lubrication, engine tune-up, tire rotation).

   An analysis of vehicle operating expenses is provided in the Canadian Automobile Association's booklet *Car Costs*. This booklet provides useful suggestions for planning related student activities, and can be obtained from the local Alberta Motor Association Office, or by contacting:

   Alberta Motor Association
   Consumer Information Service
   P.O. Box 3740, Station "D"
   Edmonton, Alberta
   T5L 4J5
   Phone: 474-8714 or 1-800-222-6578.

11. Through consultation with the English teacher, arrange to have students examine and possibly complete application forms for vehicle loans, vehicle registration and vehicle insurance.

   Obtain copies of appropriate forms from local agencies, or use the specimen forms provided in *Forms in Your Life*:

   - Form 17 – Application for a Car Loan
   - Form 18 – Application for Car Insurance.

   Discuss the meaning of common words, phrases and abbreviations that are used on these forms.

## RESOURCE 1: FINDING RENTAL HOUSING THROUGH THE NEWSPAPER

### MAKE 124 St. YOUR STREET

**BELMONT**
11025 – 124 St.
1 & 2 bedroom suites, in a clean, well maintained building. Very quiet, Intercom, and Security System.

**498-0844**

**KINGSTON**
11020 – 124 St.
Spacious 1 bedroom suites in a concrete building with lobby & elevator. Very clean and quiet.

**498-8119**

**HEATHERTON**
11140 – 124 St.
1 & 2 bedroom suites available. Clean, quiet, centrally located. New paint, and carpets. Secure building!

**498-0863**

**ASHER PROPERTY MANAGEMENT**
PROVIDING FINE RENTAL ACCOMMODATION FOR OVER 35 YEARS

**154 Shared Accommodation**

1 BDRM. incls. groceries & utils. $400. Clareview. 357-5907

DIRT CHEAP!
1 bdrm. lower units. 1 bath. Londonderry area. Phone 561-6338. 4200

FEMALE to share 3 bdrm. townhouse with f/p, 5 appls., 1½ baths, $275/mo. incl. utils. $150/dd. Riverbend (quiet area). 357-7913

M/F roommate needed to share 5 bdrm. house, on West End. 498-6202

M/F to share 3 bdrm. home. Must be neat, clean, N/S. $260/mo. utils incl. After 3:30 pm phone 562-0229.

MILLWOODS-n/smoker. New furn. New house. Call Bill: 841-8878.

NEAR Heritage Mall, M or F, furn. 2 bdrm. apt., water bed, cable t.v., parking w/plug in. $275/mo. 562-2396

**158 Rooms, Furnished**

$175 – Rooms. Students or working non-drinkers. Phone: 357-6269 or 480-9318.

ELLIOTT HOTEL. 2115-115 Ave.
From $200/mo. Phone 562-3565

DUBLIN HOTEL. 16204-110 St.
$60/week. 455-6266

PARK SQUARE
1165 sq. ft., 2 bdrm apt. 1½ baths, 3 appl. Sauna, exercise room. Free cable, all adult building. 10053-78 St. 561-3102561-1262 evngs.

GOOD DEAL!
Cozy 1 bdrm. in ceda[illegible]s. c/w satellite & cable free, elevator & limited U/G heated parking. Security entrance & more!
CEDAR ROYAL APTS.
31 Bellmead Cres.
Ph: 424-6526, evenings 841-8154

**172 Houses, Unfurnished**

19187 - 31 St. bach. suite, $200. Utilities included. 561-5448 or 462-6306

2 BDRM bsmt, priv. entr. $325, same dd. + utils. Westmount. 498-5990

2 BDRM bung., fin. bsmt. fenced, sgle. gar $600. 21 Beacon St. 841-4040

AVAIL. IMMED. 3 bdrm. large, central, 97 St. 111 Ave. area. 357-0032

BEAUTIFUL NEW
3 bdrm home, stove & fridge incl, sgle gar, nicely landscaped, oak cabinets & deck. $650/mo., $650 dd. 357-8030, apply 5-9 at 810 Grove Ave.

BELVEDERE - 4 BDRM., 2 BATHS, ½ duplex, close to all schools and LRT. Fresh paint and carpet, Immaculate, references req'd. Feb. 1st. $650/mo. Ph. 841-6467

CASTLEDOWNS - 4 bdrm., 2 baths, gar., sundeck, 2 appl. $685. 561-5333

CLAREVIEW, 2 bdrm., f/p, 2 appls., huge fenced yard, gar., deck. No dogs. Working. $595/mo. 561-8626, 562-8646

NEWLY decor. 3+1 bdrm., 3 appls., dble gar. 149 St. & 98 Ave. $850/mo. 357-2886 days, 498-5705 evgs./wkends.

QUIET STREET, 2 bdrm. main floor, living/dining/bdrms. carpeted, fridge & stove, 3 piece bath w/tub. $430/mo. incl. util. $250 dd. 357-5887 days. 562-9753 evenings.

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# RESOURCE 2: MOVING-IN COSTS

## Moving In: What is the Cost?[1]

Randy moved in with a friend. He brought along a few sheets and towels, and some old dishes which were donated to "the cause" by his parents. He coughed up $200 for his share of the rent and $30 for groceries. That's all.

David and Keith each spent $800 when they moved into their apartment. They each paid their share of the security deposit, the first month's rent, and the utility hook-up charges. More money went to purchasing food, cooking utensils, and towels to supplement what they had brought from their parents' homes.

Linda had been planning and saving for her own apartment for years. When she moved in, she did it with style. In addition to expenses such as the security deposit and first month's rent, she spent money on new furniture and a small stereo component set. Chalk up $4,025 for Linda!

Three different price tags. It is simply not possible to tell you how much it will cost to move into your own place. With a bit of research, however, you can prepare an estimate that will apply in your specific situation. Keep this dollar figure in your mind when you are planning your move.

The adjacent chart lists common moving in expenses. An explanation of each item follows.

### Moving In Costs

| Item | | Your Share |
|---|---|---|
| Moving Expenses | | |
| First Month's Rent and Security Deposit | | |
| Utility Hook-Up Charges | telephone | |
| | power | |
| | gas | |
| | water | |
| | hook-up deposits | |
| | cable | |
| Personal Property Insurance | | |
| Staple Food Items | | |
| First Week's Groceries (or until pay day) | | |
| Kitchen Utensils and Dinnerware | | |
| Linens and Bedding | | |
| Furniture | | |
| Other | | |
| Total Moving In Costs | | |

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## RESOURCE 2: MOVING-IN COSTS (continued)

### Moving Expenses

How do you plan to move your possessions from your current living space to your new home? If you plan to rent a truck for the move, calculate the rental fee and cost per kilometre, as well as the cost of gas and insurance.

If you have a friend or relative who will lend a truck or help you move, count the cost of gas. Also consider the cost of whatever food and drink you offer to your friends in exchange for helping you move.

### First Month's Rent and Security Deposit

You will have to pay the first month's rent and the security deposit when you move in. If you do not know what rent and security deposits are likely to cost, check around. Ask friends who are living on their own, inquire at apartment buildings in which you would like to live, and read the advertisements in your local newspapers.

The security deposit, also called a damage deposit, is held by the landlord in case of damage to the premises, unusual cleaning required, or non-payment of rent. Although the full security deposit plus interest will be refunded to you if no charges are incurred, it represents a significant initial cost to moving into your place. Security deposits usually vary from $100 to a maximum of one month's rent. The laws governing landlord and tenant matters are outlined in further detail in **The Landlord and Tenant Act**.

### Utility Hookups

Having utilities hooked up can be costly. A telephone connection costs about $25 to $41 depending where you live and what items need to be installed. The cost of renting a phone increases when you get a colored set or push-button dialing. Power and water hookup costs vary from place to place. Call your local town or city office for these costs. No hookup fee is charged for gas in most areas although a deposit may be required from first-time account holders.

Utility and telephone companies may also ask for a deposit between $50 and $300 on "first-time hookups". Check ahead and have money set aside for this expense.

A word of warning! If the utilities and telephone are connected in your name, you are legally responsible for payment. If someone else charges but does not pay for long distance calls and does not pay their share of the utilities, the debt is still your responsibility.

## RESOURCE 2: MOVING-IN COSTS (continued)

### Personal Property Insurance

As you move out it would be advisable to insure your personal property against loss or damage by fire, theft, or other perils.

Insurance companies provide this type of coverage as part of a packaged policy issued either to homeowners (Homeowner's Form) or to tenants (Tenant's Package Form). The policy is divided into two main parts. Part one covers the house (in the Homeowners Form) and the contents (your personal property such as furniture, wardrobe, sports equipment, record or tape collection, sound system, musical instruments, camera, etc.). Part two covers your legal liability. As a tenant, the policy will cover your legal liability to your landlord should you accidentally cause damage to the premises by fire, explosion, or smoke.

Unless you purchase a replacement value endorsement, your claim for loss will be settled on the basis of the depreciated value of your property and not on the replacement cost.

For details or further information consult an insurance agent.

### Staple Food Items & First Week's Groceries

Do not forget to allow money for staple food items such as flour, sugar, condiments, and beverages as well as for your first week's groceries. Your friends and parents may help you set a reasonable figure. Set aside enough money for groceries between moving in and your next pay cheque.

### Other Expenses

Do your best to scrounge kitchen utensils, dinnerware, linens, bedding, and furniture from friends and relatives. You will have to buy whatever is not donated. Make a list of the things you need, then go "window shopping" to check prices.

PEAS
CORN
MEAT
MILK
MILK
2%
Cottage Cheese

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[1] Reprinted with permission of Alberta Consumer and Corporate Affairs.

# RESOURCE 3: A GUIDE TO PROPERTY AND AUTOMOBILE INSURANCE CLAIMS[1]

Some time or another you will claim on an insurance policy, even if you are the most skillful driver or the most careful homeowner. Of course you will expect to receive the money you deserve and you will deserve to receive it without any fuss.

You might even be involved in a liability claim, for or against you.

## DO YOU HAVE THE INSURANCE YOU NEED?

Too often it is not until after people suffer a loss that they discover they do not have the insurance they need.

There are three main types of insurance protection: actual cash value, replacement value, and stated value. Which type is best for you?

- Actual cash value is provided by most insurance policies. When you make a claim, actual cash value is determined by taking the current cost of the article for which you are making a claim and subtracting depreciation for each year since the item was new. Depreciation is what is allowed for wear and tear, meaning that an article is worth less as the years go by. For example:

  A refrigerator purchased 12 years ago for $500 may now cost $1,200. Under an actual cash value settlement, you may only receive $400, the $1,200 cost of the refrigerator minus 12 years' depreciation.

- Replacement value coverage protects your loss at today's price. Following is an example:

  Suppose your refrigerator is destroyed in a kitchen fire. You paid $500 for it 12 years ago. A new refrigerator of the same make and model costs $1,200 today. Under replacement value coverage, you replace the refrigerator at no added expense.

  For this type of protection you must actually replace the article. Be prepared to show your receipt to the claims adjuster. Replacement value coverage is not normally available for motor vehicles, although some insurers do offer this protection on new vehicles for an additional premium.

- Stated value coverage under the Valued Automobile Endorsement provides compensation for your loss based on the item's appraised value. This appraised value is established at the time you buy the policy. For example:

  You have an antique car. It is appraised at $10,000. If your car is totally destroyed, your company will pay you $10,000, no more, no less.

Most homeowner policies limit coverage for such items as jewelry, furs, and other personal property. Check the amount of protection in your policy. If you have items that are worth more than the specified limit, or rare items such as antiques, art works, or jewelry, ask your agent to arrange for special coverage. Check your policy annually to make sure you have enough protection.

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## RESOURCE 4: CENTIMETRE GRID PAPER

# RESOURCE 5: SCHEDULE FOR DETERMINING MONTHLY AUTOMOBILE PAYMENTS

This table can be used to determine the monthly payments on an automobile loan. Follow these steps:

- select the row corresponding to the interest rate and the column corresponding to the amortization period of the loan
- multiply the number that appears in this row and column by the amount of money being loaned.

| Interest Rate | One-Year | Two-Year | Three-Year | Four-Year |
|---|---|---|---|---|
| 10% | 0.08762 | 0.04614 | 0.03227 | 0.02536 |
| 10½% | 0.08815 | 0.04638 | 0.3227 | 0.02560 |
| 11% | 0.08838 | 0.04661 | 0.03250 | 0.02585 |
| 12% | 0.08885 | 0.04707 | 0.03321 | 0.02633 |
| 12½% | 0.08908 | 0.04731 | 0.03345 | 0.02658 |
| 13% | 0.08932 | 0.04754 | 0.03369 | 0.02683 |
| 13½% | 0.08955 | 0.04778 | 0.03394 | 0.02708 |
| 14% | 0.08979 | 0.04801 | 0.03418 | 0.02733 |
| 14½% | 0.09002 | 0.04825 | 0.03442 | 0.02758 |
| 15% | 0.09026 | 0.04849 | 0.03467 | 0.02783 |
| 15½% | 0.09049 | 0.04872 | 0.03491 | 0.02808 |
| 16% | 0.09073 | 0.04896 | 0.03516 | 0.02834 |
| 16½% | 0.09097 | 0.04920 | 0.03540 | 0.02860 |
| 17% | 0.09120 | 0.04944 | 0.03565 | 0.02886 |

**EXAMPLE**

The monthly payment on a $10 000 loan at 13% over 3 years is:
0.03369 x $10 000 = $336.90

# RESOURCE 6: AUTOMOBILE INSURANCE RATES

The simulated tables provided below can be used to determine the cost of various types of automobile insurance sold by the ABC Insurance Company. The premiums are payable annually, and represent the cost of insuring a male driver between the ages of 16 and 25 through this company.

**ANNUAL PREMIUMS FOR BODILY INJURY AND PROPERTY DAMAGE LIABILITY INSURANCE**

| Driving Record* | Liability Limits | | | | |
|---|---|---|---|---|---|
| | $200 000 | $300 000 | $500 000 | $1 000 000 | $2 000 000 |
| 6 | 260 | 268 | 277 | 292 | 318 |
| 5 | 318 | 329 | 340 | 358 | 390 |
| 4 | 421 | 435 | 449 | 474 | 515 |
| 3 | 480 | 506 | 522 | 551 | 599 |
| 2 | 612 | 633 | 653 | 688 | 749 |
| 1 | 666 | 688 | 710 | 749 | 815 |
| 0 | 833 | 861 | 888 | 936 | 1019 |

**ANNUAL PREMIUMS FOR COLLISION INSURANCE**

| Driving Record* | Car Rating Group:** 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Deductible: $100 | $250 | $100 | $250 | $100 | $250 | $100 | $250 | $100 | $250 | $100 | $250 | $100 | $250 | $100 | $250 |
| 6 | 120 | 102 | 138 | 117 | 159 | 135 | 186 | 158 | 219 | 186 | 259 | 220 | 325 | 277 | 390 | 332 |
| 5 | 145 | 123 | 167 | 142 | 192 | 163 | 225 | 192 | 265 | 226 | 314 | 267 | 394 | 335 | 472 | 401 |
| 4 | 181 | 154 | 208 | 177 | 239 | 203 | 281 | 239 | 331 | 281 | 391 | 332 | 491 | 417 | 589 | 500 |
| 3 | 210 | 179 | 242 | 206 | 278 | 236 | 327 | 278 | 385 | 327 | 455 | 287 | 571 | 485 | 685 | 582 |
| 2 | 253 | 215 | 290 | 247 | 334 | 284 | 392 | 333 | 462 | 392 | 546 | 464 | 685 | 582 | 821 | 698 |
| 1 | 295 | 250 | 339 | 288 | 389 | 331 | 457 | 389 | 538 | 458 | 637 | 541 | 799 | 679 | 958 | 815 |
| 0 | 337 | 286 | 387 | 3298 | 445 | 378 | 522 | 444 | 615 | 523 | 728 | 618 | 913 | 776 | 1095 | 931 |

**PREMIUMS FOR COMPREHENSIVE INSURANCE**

| Deductible: | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $25 | 49 | 45 | 65 | 77 | 90 | 107 | 133 | 160 |
| $50 | 42 | 48 | 55 | 65 | 77 | 90 | 114 | 136 |

* The "Driving Record" represents the number of years with a clear driving record.

** The "Car Rating" is based upon the value of the car, its damagability, and the cost of repairs for that particular model.

# THEME D
# MATH IN THE WORKPLACE

## SUB-THEMES

- DESIGNING AND BUILDING
- MONITORING INVENTORY/CASH FLOW

## RATIONALE

This theme focuses attention on mathematical concepts and skills frequently used in the workplace. Thematic activities will provide opportunities for students to:

- develop strategies for planning and completing work-related projects undertaken in occupational courses
- develop an understanding of procedures used in monitoring inventory and cash flow in a simple retail business.

Students will apply previously developed measurement skills and knowledge of scale drawings (see Mathematics 16: Math in the Workplace) throughout this theme as they plan and complete work-related projects. Learning objectives interrelate with activities in a prior theme, Earning Money, and will contribute to an understanding of strategies used to monitor profits and/or losses in entrepreneurial activities.

An emphasis on estimation prior to engaging in activities that involve actual measurement will help students to establish the size of standard units of measure, relative to each other and to real objects. This approach will also enhance student ability to select measurement tools that are appropriate in a variety of situations. Projects that involve scale drawings and models will assist students to understand how geometry and measurement are used to gather, organize and display information about the physical world.

Cooperative planning among teachers will ensure that activities within this theme focus attention on mathematical skills used by students in completing work-related projects in their occupational courses. The local community provides additional opportunities for students to develop learning objectives through relevant activities related to the workplace.

Teachers are encouraged to reference the "Program Emphases and Methodology" section of this manual when planning for instruction. Strategies particularly relevant to the learning objectives addressed within this theme can be found in:

- Problem Solving
- Situational and Concrete Approaches
- Assessment/Evaluation.

## THEMATIC OBJECTIVES

### DESIGNING AND BUILDING

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- select units and tools of measure that are appropriate in everyday and work-related situations
- estimate length, area, volume, capacity, mass, time, temperature and angle measure, using strategies appropriate to the situation
- measure length, capacity, mass, time, temperature and angles within acceptable degrees of accuracy
- use formulas to measure indirectly perimeter, area and volume in practical situations
- demonstrate an understanding of the Pythagorean relationship among the sides of a right triangle, and its application in practical situations
- read and interpret the calibrations on meters, scales and gauges that are frequently used in everyday and work-related situations
- read, interpret and construct scale drawings and models in practical situations (e.g., blueprints, technical drawings, project plans)
- make plans for a simple construction or repair project by preparing scale drawings/models, identifying appropriate steps/procedures, and estimating/costing materials.

### MONITORING INVENTORY/CASH FLOW

The concepts, skills and attitudes addressed throughout this sub-theme will enable students to:

- demonstrate an understanding that inventory control systems are used to monitor goods sold and stock on hand, and may help to determine profits and/or losses in a business
- describe the use of manual inventories (i.e., physical counts) and computerized equipment in maintaining an inventory of goods sold and stock on hand
- interpret and complete various forms used in monitoring movement of stock in a business (e.g., bills of lading, bills of sale, packing slips, invoices, back orders, purchase orders)
- describe appropriate strategies for handling cash transactions in a business (e.g., recording sales, making change, balancing daily cash receipts, completing a cash summary report)
- describe appropriate strategies for handling credit transactions in a business (e.g., receiving a cheque, processing credit card sales).

## CONTEXT FOR INSTRUCTION

### PROBLEM SOLVING

Students will be expected to:

- use estimation and measurement to solve practical problems in work-related situations, selecting units and tools that lend a degree of accuracy appropriate to the task
- develop and apply formulas for finding perimeter, area and volume in practical situations
- gather, organize and display information about the physical world by constructing scale drawings and scale models
- monitor surpluses/shortages in stock, and balance daily cash receipts in simulated business situations.

### USE OF TECHNOLOGY

Students will be expected to:

- use a calculator to perform computations when:
  - solving perimeter, area and volume problems
  - determining actual measurements from a scale drawing
  - processing cash and/or credit sales transactions
- develop an understanding of applications of computer technology when:
  - constructing geometric patterns and scale drawings
  - monitoring inventory and cash flow in a business
- use prepared computer software as required to:
  - maintain/reinforce basic concepts and skills in ratio, proportion, measurement and geometry
  - develop effective problem-solving strategies
  - construct geometric figures, patterns and scale drawings.

### COMPUTATIONAL FACILITY AND ESTIMATION

Students will be expected to:

- maintain basic computational algorithms
- perform calculations with a calculator, on a regular basis
- use mental arithmetic when appropriate to expedite solutions to quantitative problems
- estimate length, area, volume, capacity, mass and angle measure in order to:
  - determine quantity/size of materials required for a project
  - check the results of actual measurement
- use estimation to approximate
  - the quantity and cost of materials used in a construction or repair project
  - the total cost of merchandise purchased and/or change due in a sales transaction.

## SUPPORTING STRANDS

Concepts, skills and attitudes within each of the shaded strands below are used throughout this theme. Teachers are encouraged to refer to the *Program of Studies/Curriculum Guide* (see Program of Studies/Presentation of Content, Column Four) for strategies that may be effective in addressing prescribed learning objectives within each of the strands.

| | | | | |
|---|---|---|---|---|
| Number Systems and Operations | **Ratio, Proportion and Percent** | **Algebra** | **Geometry and Measurement** | **Data Interpretation and Display** |

## RATIO, PROPORTION AND PERCENT

Students will be expected to:

- calculate rates/unit rates by writing ratios that involve numbers with different units (e.g., 1 cm : 5 m)
- identify proportions as statements about equivalent ratios
- write proportions that describe practical problem situations
- calculate the unknown value in a proportion
- express ratios as decimals and percents
- estimate and calculate what percent one number is of another
- estimate and calculate any one of the missing elements in practical problems involving percent.

## ALGEBRA

Students will be expected to:

- use variables to write mathematical expressions and/or equations that summarize patterns and relationships discovered through concrete experiences
- develop and apply formulas for finding perimeter, area and volume
- use substitution and equation-solving strategies to find the missing element in a formula.

## GEOMETRY AND MEASUREMENT

Students will be expected to:

- identify/sketch/construct basic two- and three-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle, rectangular solid, cube, cylinder, cone)
- construct circles, given either radius or diameter
- identify similar and congruent geometric figures
- construct geometric patterns/designs, using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer)
- estimate and measure length, capacity, mass, time, temperature and angles, using SI units and tools appropriate to the situation
- convert among SI units of length, capacity, mass and time as required in applications
- identify common SI units of area and volume, and their applications in practical situations
- develop and apply strategies for determining the:
  - perimeter of figures bounded by line segments
  - circumference of circles
  - area of rectangles, squares, triangles and circles
  - surface area and volume of rectangular solids and cubes
  - volume of right cylinders
- apply the Pythagorean relationship among sides of a right triangle in practical situations.

## DATA INTERPRETATION AND DISPLAY

Students will be expected to:

- read and interpret information displayed in tables, charts and graphs
- collect and organize information using tally sheets and/or frequency tables
- display information in table, chart and graph form
- interpret and/or calculate appropriate measures of central tendency in practical situations
- interpret statements of probability in order to predict future outcomes/actions.

## INTEGRATION ACTIVITIES

Teachers are encouraged to identify ways to integrate the content of this theme with competencies required of students in other subject areas. Projects undertaken in occupational courses are particularly relevant to this theme, and frequently require students to estimate, measure and interpret technical drawings. The references provided below are intended to facilitate curricular integration by establishing a base for cooperative planning among teachers.

### SCIENCE

Students may:

- use a variety of measuring tools and gauges
- collect data related to an investigation by measuring length, area, volume, capacity, mass, time, temperature and/or angles
- develop and apply formulas in practical situations (e.g., mechanical advantages, Ohm's law).

### SOCIAL STUDIES

Students may:

- apply knowledge of geometric patterns, relationships and grid systems when reading and interpreting maps
- gather, organize and display information about the physical world by constructing scale drawings and scale models.

### OCCUPATIONAL COURSES

Students may:

- use a variety of tools, meters and gauges to measure length, area, volume, capacity, mass, time, temperature and angles
- read and interpret tables, charts and/or graphs in order to complete a procedure or task
- interpret and/or construct technical drawings (e.g., scale drawings, scale models, patterns, templates)
- determine the quantity and cost of materials required to complete a project
- develop strategies for maintaining inventory in an occupational area
- complete forms used to monitor movement of stock in a business (e.g., bills of sale, invoices, purchase orders)
- develop strategies for handling cash and credit sales transactions
- consider factors contributing to the success or failure of a local business.

Activities suggested within this theme make reference to projects outlined in the *Student Workbook* for occupational courses in:

- Automotive Services
- Building Services
- Business Services
- Fashion and Fabric Services
- Food Services
- Service Station Services
- Warehouse Services.

## COMMUNITY PARTNERSHIP OPPORTUNITIES

The local community may offer resources that contribute to the development of learning objectives within this theme. Suggestions for utilizing community resources, and for developing work-related skills through situational and concrete learning experiences in the community are provided below.

- Visit local businesses and industries in order to investigate:
  - practical applications of estimation and measurement
  - different tools (e.g., meters, scales, gauges) used in measurement
  - applications of scale drawings, scale models, patterns and templates
  - inventory control systems in common use
  - forms used to monitor the movement of stock (e.g., bills of sale, packing slips, invoices, back orders)
  - methods of handling cash and/or credit sales transactions
  - applications of computer technology in the workplace.

  Possible community contacts include automotive repair shops, construction sites, food service industries, department stores and architectural/drafting firms.

- Invite local trades and craft people to discuss applications of measurement and geometry in their respective areas of service.

  e.g., – carpenters, painters, tile/carpet layers
  - draftspersons, interior designers, architects
  - graphic artists
  - garment and drapery fabricators
  - food service operators.

- Construct a scale drawing or scale model of a facility within the local community.

  e.g., – a floor plan of a house or public building
  - a technical drawing for a piece of furniture
  - a model of a community park.

- Estimate the quantity of materials required to maintain and/or construct a community facility.

  e.g., – litres of paint required to restore a local building
  - materials required to fence a yard or park
  - square metres of floor covering required to refinish a floor.

- Interview local trades/craft/business people and discuss factors that may contribute to the success or failure of different types of entrepreneurial activities in the local community.

## DESIGNING AND BUILDING

### LEARNING RESOURCE CORRELATION

BASIC LEARNING RESOURCE

CCM: Chapter 2 – Equations, Proportions and Percent
Chapter 3 – Measurement and Geometry
Chapter 4 – Problem-Solving Skills and Strategies

SUPPORT LEARNING RESOURCES

MYN: Unit B – Measurement Skills You Need

MS: Unit 3 – Repairing and Renovating

*Math Strategies: Problem Solving* (computer software)

OTHER LEARNING RESOURCES

*Applied Mathematics*
Part 1 – Reading Industrial Measurements
Part 2 – Algebra in the Workplace
Part 3 – Applied Geometry

*Mathways* (video program)
Areas
Volumes

*Math Wise* (video program)
Measuring 01: Measuring Instruments
Measuring 02: Formulas
Comparing 04: Proportions

*SRA Computer Drill and Instruction, Mathematics, Level D* (computer software)

### SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Provide opportunities for students to investigate relevant applications of estimation and measurement in their occupational courses and in the workplace.

   e.g., – Interview local trades/crafts/business people in order to determine units and tools of measure that are used in work-related situations. Structure and record the results of investigation using the tables and charts provided in Resource 1: Units and Tools of Measure, and Resource 2: Applications of Length, Area, Volume and Mass.

- Borrow, collect and examine measuring tools (e.g., steel rules, micrometers, calipers) in common use in occupational courses. Plan activities in mathematics class that will enable students to develop appropriate strategies for using these tools through "hands-on" experience. Activities that may assist students to use the micrometer and calipers in practical situations are provided in Resource 3: Reading Micrometers and Calipers.

2. Develop strategies that will assist students to interpret both decimal and fraction calibrations on various meters/scales/gauges that are used in work-related situations. Illustrate different measurement scales, using an overhead projector, and coach students in determining intermediate values on each scale through the process of interpolation.

   Activities that require students to read the scales on a variety of measuring instruments are provided in *Applied Mathematics* (see Chapter 1 – Reading Measurements), and in another section of this manual (see Situational and Concrete Approaches: Resource 11: Meters, Scales and Gauges).

3. Reinforce perimeter, area and volume concepts through concrete experiences with visual and manipulative materials (e.g., geoboards, tiles, cubes, geoblocks). Hands-on experiences will improve students' perception of spatial relationships that are inherent in two- and three-dimensional shapes. Appropriate activities for developing area and volume concepts are provided in Situational and Concrete Approaches (see Resource 9: Understanding Area, and Resource 10: Three-Dimensional Patterns), and in the video program *Mathways* (Areas, Volumes).

   Compare the concept of perimeter to "distance around", the concept of area to "surface covered", and the concept of volume to "space occupied". Ask students to identify work-related situations where perimeter, area and volume measurements are used. Sample worksheets that require students to apply perimeter, area and volume concepts are provided in Resource 4: Using Perimeter, Area and Volume.

4. Develop perimeter, area and volume formulas as a consequence of patterns and relationships discovered through investigation. Ask questions that will help students to deduce their own strategies for determining perimeter, area and volume based on the results of manipulation and construction. The video program *Math Wise* (02: Formulas) illustrates the development and application of perimeter, area and volume formulas.

   Ask students to make a list of algebraic formulas/equations used in their occupational courses. Remind students of other relationships that may be described by writing algebraic formulas/equations (e.g., SL = TL/CF where SL = scale length, TL = true length, CF = conversion factor).

   Suggestions for developing and applying algebra skills in work-related situations are provided in Problem Solving, "Using Algebra in Problem-Solving Situations".

5. Provide students with technical drawings and blueprints used in their occupational courses. Ask questions that will assist students to read and interpret these drawings.

   e.g.,
   - What is the distance between A and B on the drawing?
   - What is the actual distance between A and B?
   - What scale is used?
   - How can the scale be represented as a ratio?
   - How many metres of baseboard are required for this room?

   Sample technical drawings typical of those encountered in some work situations are provided in Resource 5: Technical Drawings.

6. Ask students to design and construct a geometric pattern or tessellation (see Resource 6: Tessellations). Encourage students to:

   - draw a preliminary sketch of a design/tessellation
   - identify an appropriate scale
   - make a materials list (that includes geometric tools required)
   - construct a model of the pattern/tessellation.

   Students may wish to construct a geometric pattern or tessellation on the computer using LOGO (see Use of Technology, "The Computer").

7. Identify specific projects undertaken by students in their occupational courses (e.g., sewing, woodwork). Assist students to design a flow chart that outlines major steps in planning, completing and evaluating a sample project (see Resource 7: A Planning Model).

   Ask students to prepare detailed plans for a project of particular interest. Plans should include:

   - a scale drawing or model
   - steps and/or procedures to be followed
   - the quantity and cost of materials required.

   Sample guidelines that may be used by students when planning a project are provided in Resource 8: Planning a Project.

8. Provide opportunities for students to plan a redecorating project in their home. Ask students to prepare:

   - a scale drawing
   - a materials list
   - an estimate of the cost of the project.

   Teachers may wish to structure this project by using the sample activities provided in Resource 9: A Redecorating Project.

9. Interview and/or survey people employed in a variety of career fields in order to determine applications of mathematical skills in the workplace. As students investigate career opportunities, focus attention on:

   - mathematical skills that are required
   - other educational and training requirements
   - potential remuneration for an entry-level position
   - the need for lifelong learning.

   Sample interview guides and survey charts that may be adapted for use when conducting interviews and surveys are provided in Resource 10: Career Opportunities Interview Guide, Resource 11: Job Survey Chart, and Resource 12: Conducting a Career Survey.

   Coach students in the use of appropriate statistical procedures when gathering, organizing, summarizing and reporting information. Provide opportunities for students to share the results of investigation with their peers.

# MONITORING INVENTORY/CASH FLOW

## LEARNING RESOURCE CORRELATION

**BASIC LEARNING RESOURCE**

CCM: Chapter 5 – Probability

**SUPPORT LEARNING RESOURCE**

MYN: Unit A – Basic Skills You Need

*Math Strategies: Problem Solving* (computer software)

**OTHER LEARNING RESOURCES**

*Math Wise* (video program)
- Locating/Interpreting 01: Organizing Information
- Locating/Interpreting 02: Graphs
- Locating/Interpreting 03: Maps, Charts, and Tables
- Predicting 01: Probability
- Predicting 02: Choosing a Sample
- Predicting 03: Averages

*SRA Computer Drill and Instruction, Mathematics, Level D* (computer software)

*Useful Arithmetic Skills, Volume 2*
- Section I – Money Problems
- Section II – Math in the Marketplace

## SUGGESTED ACTIVITIES

The student learning resources identified above provide a variety of instructional activities that support learning objectives within this sub-theme. The activities that follow complement those provided in the student learning resources. Teachers should be selective in their use, and consider students' interests/abilities and preferred methods of learning in planning appropriate instructional activities.

1. Ask students to read the information provided in Resource 13: Taking Inventory. Discuss the need for monitoring inventory, and different types of inventory control systems used in the workplace.

   e.g.,
   - manual inventories (i.e., physical counts)
   - computerized (i.e., point-of-sale) inventory control.

2. Invite a local business proprietor into the classroom to discuss various aspects of maintaining an inventory control system.

   e.g.,
   - reasons for maintaining inventory control
   - methods of taking a physical/manual inventory
   - applications of point-of-sale computer technology
   - advantages/disadvantages of computerized systems over manual systems.

3. Provide students with a simulated inventory and retail price list (see Resource 14: Daily Inventory Form). Using the inventory and price list, ask students to:

   - determine the total retail value of inventory
   - calculate the cost price of each inventory item (using a 40% markup)
   - determine potential profit on the sale of all inventory.

   Encourage students to use the memory keys on the calculator when performing related computations (see Use of Technology, "The Calculator").

4. Ask students to compile an inventory of assets within some area of the school.

   e.g., – an occupational shop or classroom
   – the science laboratory
   – the school store or cafeteria
   – the gymnasium.

   Students may wish to use the guidelines and form provided in Resource 15: Compiling an Inventory of Assets, for this activity.

5. Discuss the use of stock cards in maintaining a record of sales, purchases and a current balance of stock on hand. A sample stock card and related activity is provided in Resource 16: Stock Cards.

6. Provide opportunities for students to interpret and complete different forms used in monitoring movement of stock in a business (see Resource 17: Using Business Forms).

   Ask students to contact local businesses and collect specimen forms used for ordering, receiving and delivering merchandise.

   e.g., – order forms such as purchase orders
   – receiving forms such as invoices, bills of lading, and packing slips
   – delivery forms such as bus bills, courier bills and packing slips.

7. Investigate appropriate strategies for handling cash and credit transactions in a business.

   e.g., – recording sales
   – making change
   – handling credit cards and cheques
   – balancing cash.

   Interview the business education teacher and/or local retailers, and ask questions about procedures in common use. Related information and activities are provided in:

   - Resource 18: Completing Sales Slips
   - Resource 19: Handling Cash and Making Change
   - Resource 20: Processing Credit Card Sales
   - Resource 21: Approving Personal Cheques
   - Resource 22: Balancing Daily Cash Receipts
   - Resource 23: Completing a Cash Summary Report.

8. Compare the cost of various sources of credit in the local business community (e.g., bank cards, department store charge cards, bank loans). Discuss advantages and disadvantages of each source of credit.

**CLARIFICATION/EXAMPLE**

| Sources of Credit | | Annual Interest Charge | Borrow $1000.00. Calculate the cost of this credit after one year. |
|---|---|---|---|
| Bank Cards | | | |
| | | | |
| Department Store Charge Cards | | | |
| | | | |
| Bank Loans | | | |
| | | | |

# RESOURCE 1: UNITS AND TOOLS OF MEASURE

Visit a local business and investigate tools and units of measure that are used. Record your observations in the chart below.

Workplace Visited ______________________ Date ______________

| Types of Measuring Tools | Name of Units of Measurement | Metric | Imperial | Other |
|---|---|---|---|---|
| e.g., ruler | mm, cm, m | ✓ | | |
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## RESOURCE 2: APPLICATIONS OF LENGTH, AREA, VOLUME AND MASS

Visit a local business that supplies materials. Identify items that are usually sold by length, area, volume and mass. Record your observations in the chart below.

| Item | Sold by: | | | | Unit of Measurement | Metric/ Imperial |
|---|---|---|---|---|---|---|
| | Length | Area | Volume | Mass | | |
| e.g., apples | | | | ✓ | kg | Metric |
| e.g., gasoline | | | ✓ | | L | Metric |
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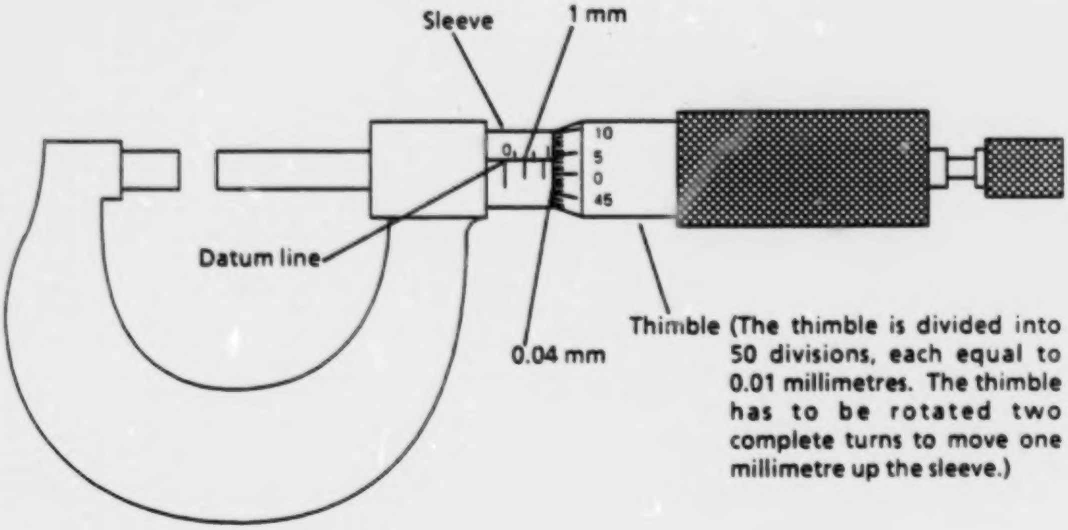
# RESOURCE 3: READING MICROMETERS AND CALIPERS

## READING A METRIC MICROMETER



(The thimble is divided into 50 divisions, each equal to 0.01 millimetres. The thimble has to be rotated two complete turns to move one millimetre up the sleeve.)

To read a metric micrometer:

1. Determine the number of exposed millimetre graduation marks on the sleeve (each mark represents 1 mm).
2. Add 0.50 mm if a half-millimetre graduation mark is exposed.
3. Obtain the number of 0.01 mm from the thimble by reading the graduation mark (on the thimble) which is most nearly in line with the datum line (on the sleeve). Each graduation is multiplied by 0.01 mm.
4. Determine the sum of all individual readings.

Example:

Determine the reading from the micrometer illustrated above.

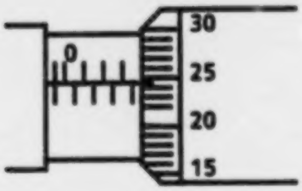
Solution:

1. 2 exposed mm graduation marks (2 × 1 mm) = 2.00 mm
2. 1 exposed 0.5 mm = 0.50
3. The datum line aligns with the fourth graduation on the thimble scale.
   Record 0.04 mm (4 × 0.01) = 0.04
4. Complete reading = 2.54 mm

## RESOURCE 3: READING MICROMETERS AND CALIPERS (continued)

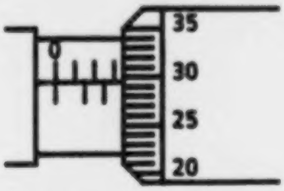
Exercises:

Record each of the readings for the metric micrometer settings below. Express your answers in a. millimetres and b. centimetres.



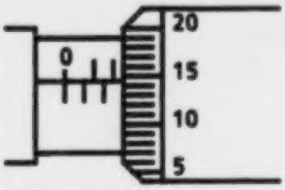
1. (a) ____

(b) ____



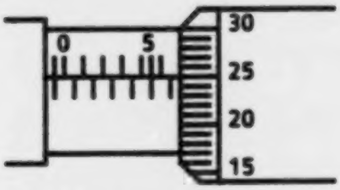
2. (a) ____

(b) ____



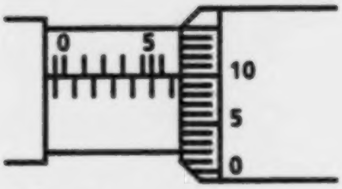
3. (a) ____

(b) ____


0
5
30
25
20
15

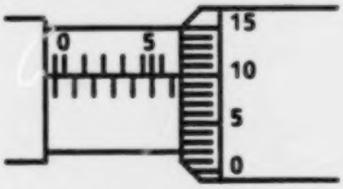
4. (a) ____

(b) ____



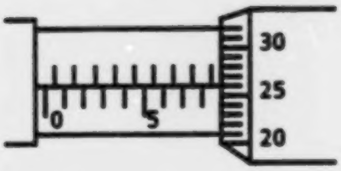
5. (a) ____

(b) ____



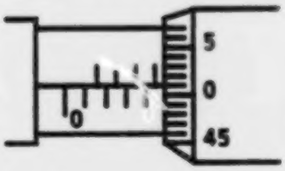
6. (a) ____

(b) ____



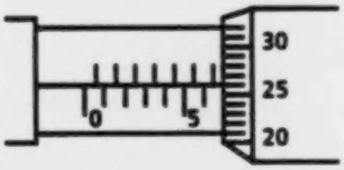
7. (a) ____

(b) ____



8. (a) ____

(b) ____



9. (a) ____

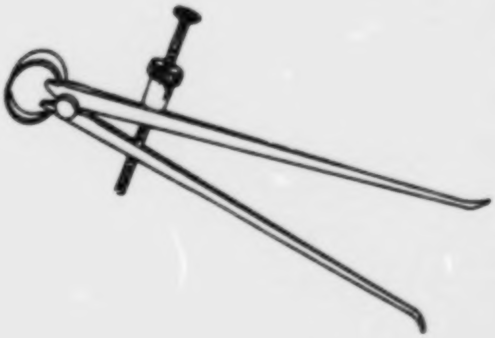
(b) ____

## RESOURCE 3: READING MICROMETERS AND CALIPERS (continued)

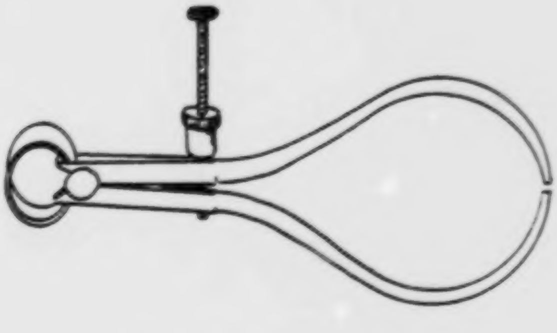
### MEASURING WITH CALIPERS

Calipers are used to determine the size of an object. As they do not have a scale of their own, they are not considered accurate. To obtain a measurement, the distance between the caliper points is measured with a ruler.

INSIDE CALIPERS

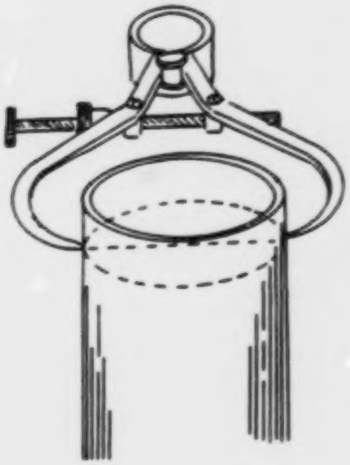


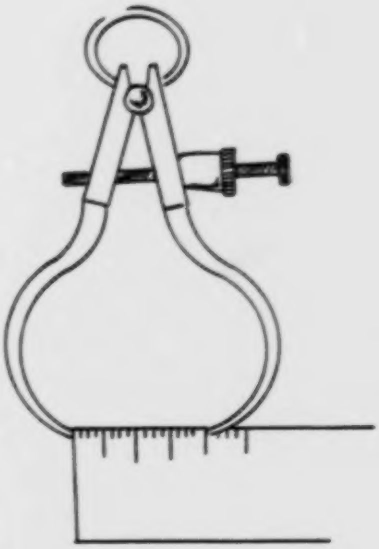
OUTSIDE CALIPERS



When using calipers to determine the size of an object, follow these steps:

1. Adjust the calipers so they lightly touch the surface of the object to be measured.
2. Place the points of the calipers along the scale of a ruler.
3. Record the measurement of the object.





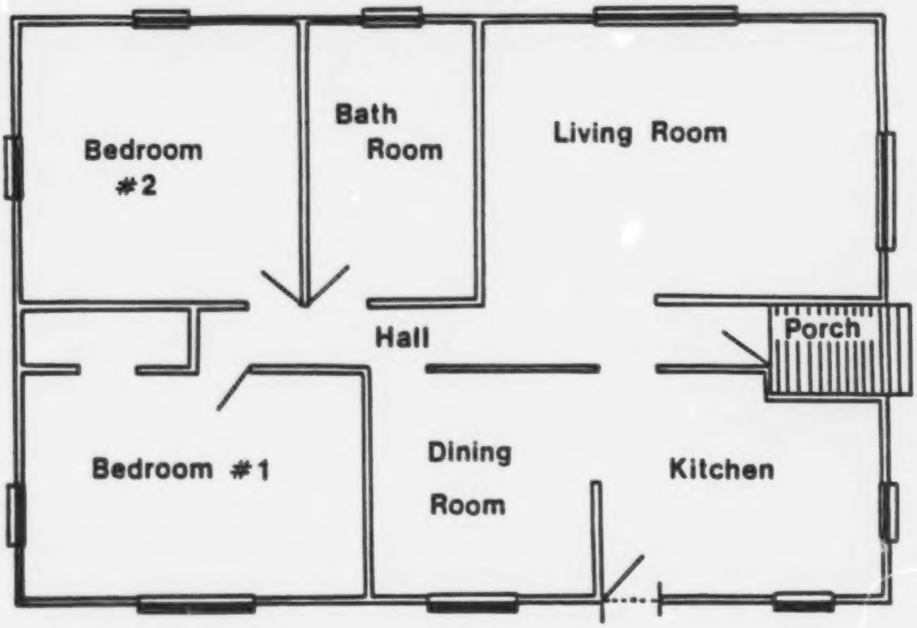
## RESOURCE 4: USING PERIMETER, AREA AND VOLUME[1]

1. Locate newspaper advertisements for two different types of fencing material. Calculate the cost of fencing a yard 30 m × 10 m with each material. Clip and mount your ads to the reverse side of this page. Show your calculations.

| | |
|---|---|
| Material 1: ______________ | Material 2: ______________ |
| Cost: ______________ | Cost: ______________ |

2. Compare the total cost of each. Which would you then select for your home and why?
______________________________
______________________________

3. Using the floor plan provided, determine:
   - the dimensions of each room
   - the perimeter of each room
   - the area of each room.


Bedroom #2
Bath Room
Living Room
Hall
Porch
Bedroom #1
Dining Room
Kitchen

Scale: 1 cm = 1 m

4. Find an advertisement displaying appropriate carpet for this house. Calculate the cost of carpeting the living room, dining room, hallway, bedrooms and closet.

5. Find a wallpaper advertisement. Calculate the cost of wallpapering the kitchen and bathroom, assuming that walls are a standard height.

## RESOURCE 4: USING PERIMETER, AREA AND VOLUME (continued)

6. Of all the building materials used, concrete is by far the most common. To order concrete, the volume of the slab must be calculated. Ready mixed concrete is ordered in cubic metres (m$^3$). 10% of the volume should be added for differences in thickness.

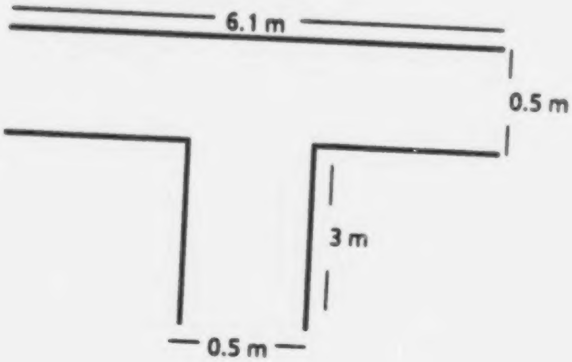
Volume = length × width × thickness

Given that the normal thickness of concrete for a garage pad/driveway/sidewalk is 0.1 m, find the volume of concrete needed for the following:

a. A double garage pad 6.7 m × 7.3 m. (Be sure to add 10% for the difference in thickness.)

b. A single garage pad 4.3 m × 6.7 m.

c. The sidewalk as illustrated:



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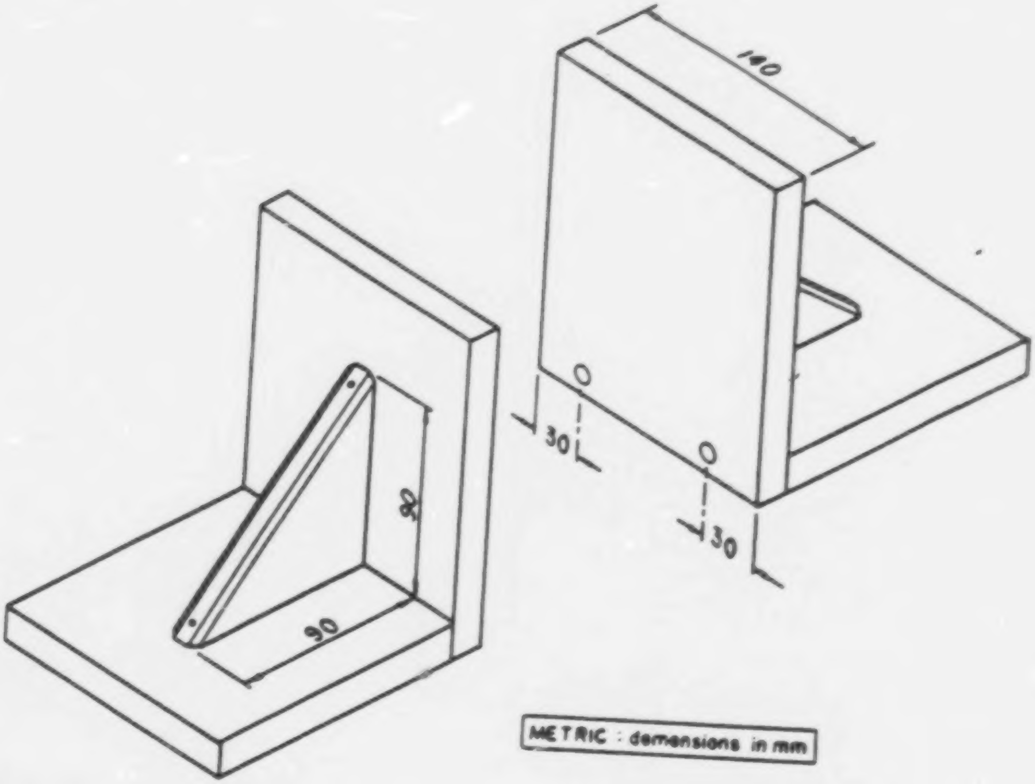
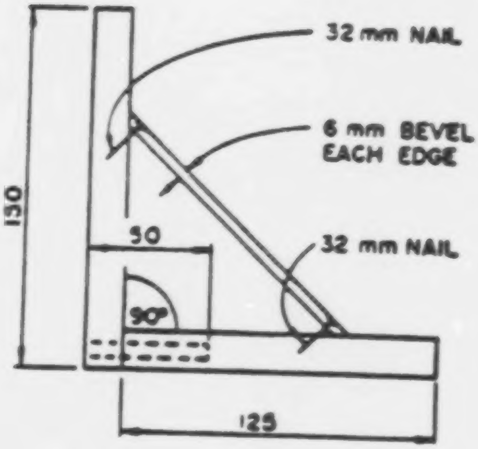
[1] Reprinted with permission of Kitchener-Waterloo Record.

# RESOURCE 5: TECHNICAL DRAWINGS

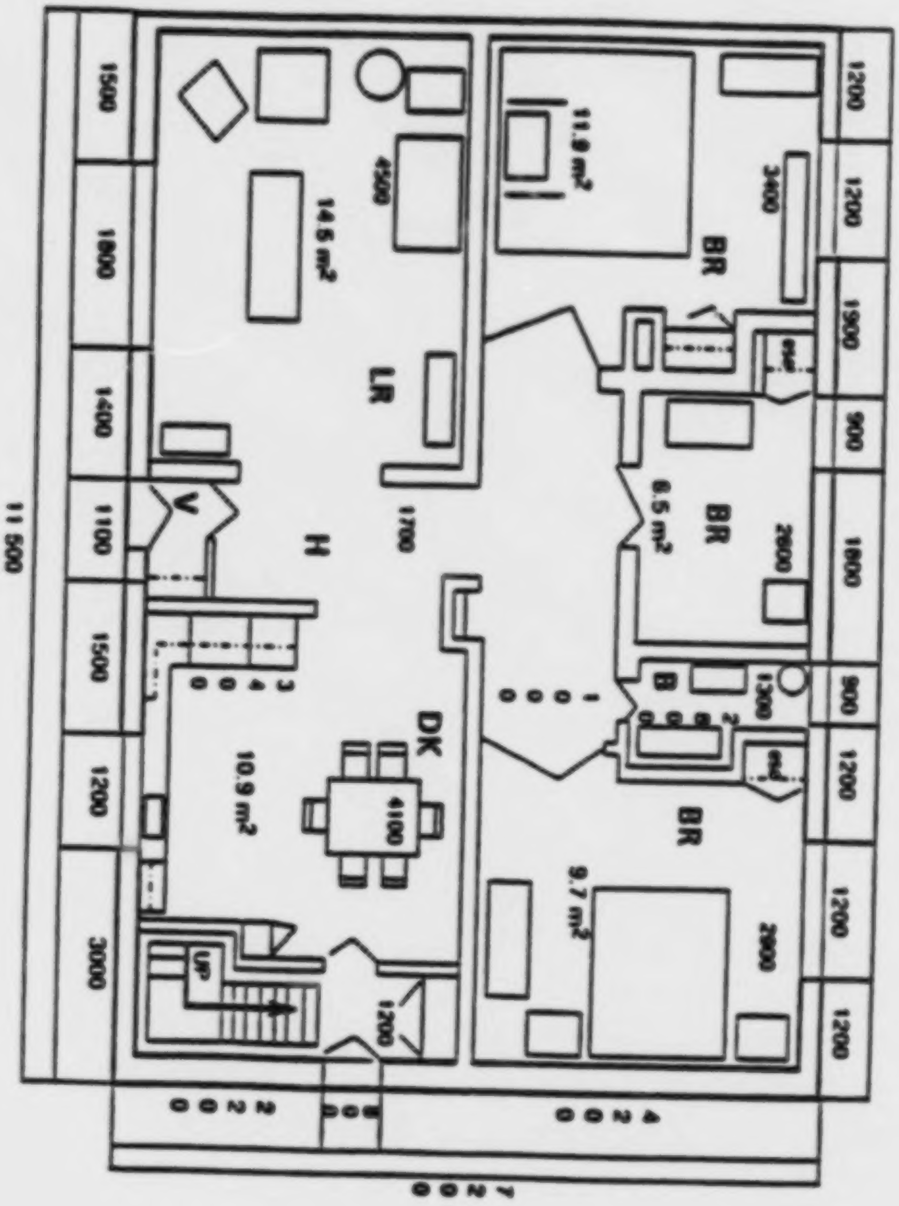
## BOOKENDS


140
30
30
90
90

METRIC : dimensions in mm


32 mm NAIL
6 mm BEVEL EACH EDGE
150
50
90°
32 mm NAIL
125

## LINEAR DIMENSIONING AND FLOOR PLANS


LR
14.5 m²
4500
H
1700
V
DK
10.9 m²
4100
UP
1200
BR
11.9 m²
3400
BR
8.5 m²
2600
BR
9.7 m²
2900
B
1300
1500
1800
1400
1100
1500
1200
3000
11 500
1200
1200
1900
900
1800
900
1200
1200
1200
2200
800
4200
7200

NOTE: All dimensions are given in millimetres unless otherwise indicated

# RESOURCE 6: TESSELLATIONS

## EXPLORATIONS WITH TESSELLATING POLYGONS[1]

By Barbara Kaiser

### ACTIVITY 1: TWO-DIMENSIONAL SHAPES

Supply each student with a set of polygons. Each shape should be cut from sturdy paper, for example, bristol board, and should be about the size of the palm of your hand. Review the names of the shapes before beginning to work with them.

First, have students sort the shapes into two piles – those they think will tile a floor and those they think will not. Next, students select a tile shape and trace it repeatedly on a large sheet of paper, for instance, newsprint, to show a design that can be made from that shape. At this point students discuss their patterns and see the need to set two limitations: (a) the tiles should not overlap, and (b) no spaces should be left between tiles. It is also important to discuss how to handle the edge of the paper. It does not matter here if the tiles go off the paper – floor tiles often have to be cut around the edges of a room. The important factor is whether the shape can tile the middle of the paper without overlapping or leaving spaces.

Some shapes offer more challenge than others. Encourage your students to persevere until they find a way to pattern with each tile shape or until they are absolutely convinced that no pattern is possible for a particular shape. Be sure to allow sufficient time for students to explore several variations with the shapes (see figure 1). Some students will find it helpful to trace and cut out eight to ten copies of the more difficult shapes and move them around to find a tiling design. The shapes can then be glued onto the newsprint.

Pattern blocks work well for this activity. However, you may wish to include other shapes to supplement the variety available in the pattern-block container. Patterns can be recorded using both isometric and regular dot paper. Students can color in their designs to illustrate patterns that they find. A design using only one regular polygon (one having equal sides and equal angles) is called a regular tessellation.

Fig. 1 Tessellations using the rectangle.

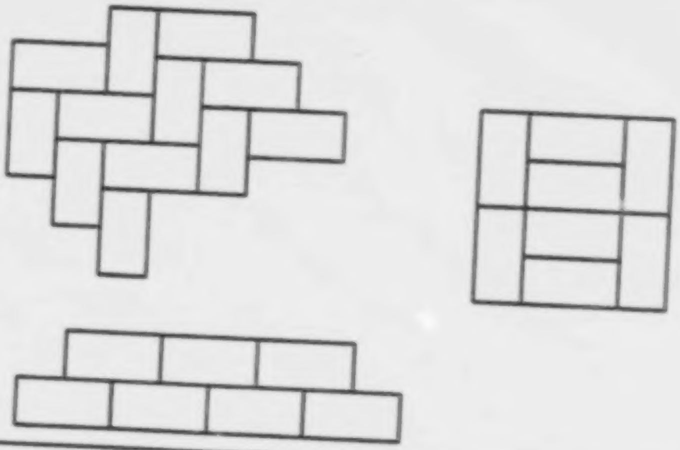
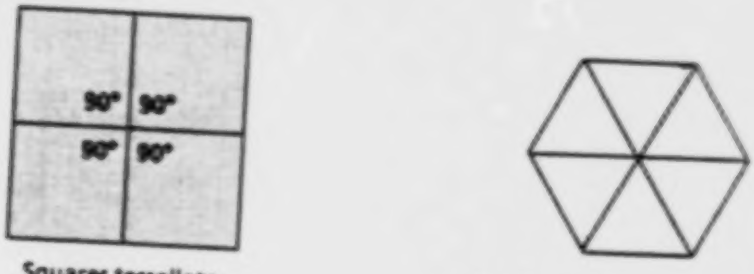
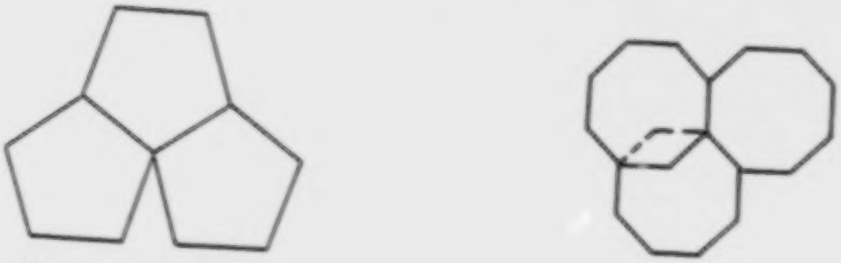


Fig. 2 Not all shapes tessellate.



Squares tessellate

Equilateral triangles tessellate



Pentagons leave gaps

Octagons overlap

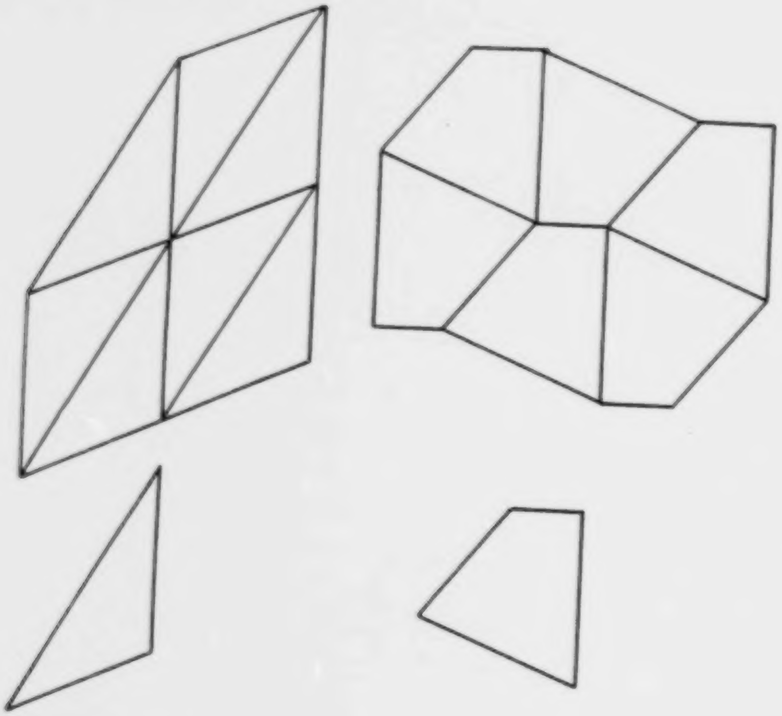
*Extensions*

1. a. Explore further the reason why a shape will or will not tile. This study will furnish an excellent application for skills of angle measurement. Have students discover that the sum of the angles around any point, or vertex, where the shapes meet must equal 360 degrees. Next, go back and look at each regular polygon and its corresponding tiling pattern. The angles in a square are each 90 degrees. Since four right angles make 360 degrees, four squares will meet at a vertex with no gaps or overlapping. The equilateral triangle has 60-degree angles, so six of these will meet at a vertex. The angles in a regular hexagon are each 120 degrees, and three fit at a vertex. However, three regular pentagons with 108-degree angles total only 324 degrees and so leave a gap in the pattern. Three regular octagons with 135-degree angles total 405 degrees and overlap around the vertex (see figure 2).

   b. It is also interesting to explore shapes other than regular polygons. The sum of the angles in any triangle is 180 degrees, and so by careful arrangement six angles will form 360 degrees at a vertex. Similarly, all quadrilaterals have angles that sum to 360 degrees, and thus all quadrilaterals can tile a floor (see figure 3).

## RESOURCE 6: TESSELLATIONS (continued)

Fig. 3 All triangles and quadrilaterals can tessellate.



The sum of the angles around every vertex is 360° (i.e., a+b+c+d=360°).

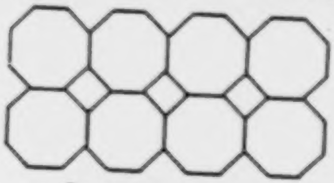
2. Challenge students to create patterns using two or more regular polygons; include the equilateral triangle, square, hexagon, octagon, and dodecagon. For these patterns use a standard length for the sides of each polygon. Designs composed of two or more regular polygons are called semiregular tessellations. Eight semiregular tessellations exist that use regular polygons. They can be organized and named according to the different polygons that meet at the vertex. In the following list, each polygon is indicated by its number of sides, for example, 4 indicates a square, 12 a dodecagon. Thus, the eight semiregular tessellations are as follows (see figure 4):

    8-4-8
    12-3-12
    6-4-3-4
    6-3-3-3-3
    6-3-6-3
    12-4-6
    4-4-3-3-3
    12-3-4-3

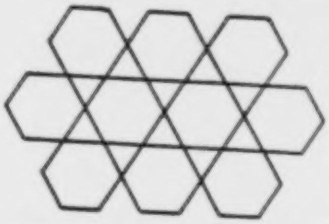
    Variations: 3-3-6-6; 3-4-4-6; 4-3-4-3-3.

3. Have students make a collection of different tiling patterns that they find outside the classroom. These could include various shapes of paving stones and patio blocks, floor and wall coverings, and so on. An interesting variation would be to have the students record the tessellations they find by doing rubbings of each. Rubbings can be made by using a sheet of paper large enough to cover a few of the tiles and a crayon. Remove the paper wrapper and lay the crayon flat on it side as it is rubbed over the tile.

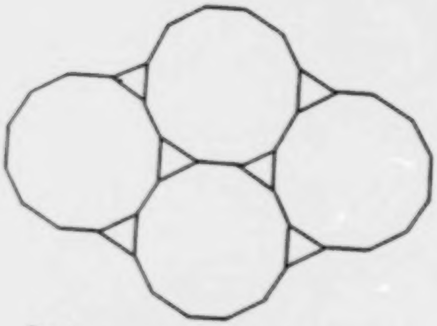
Fig. 4 Semiregular tessellations. Numbers indicate the number of sides of the polygons that meet at a common vertex.



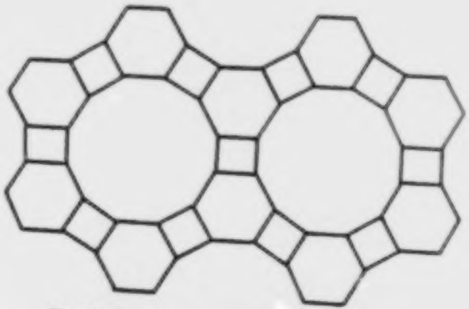
Octagons, squares
(8-4-8)



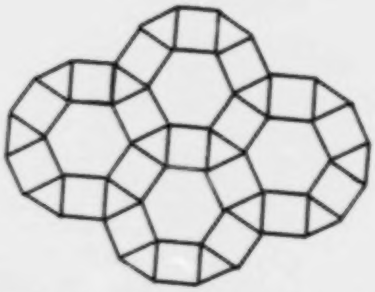
Hexagons, equilateral triangles
(6-3-6-3)



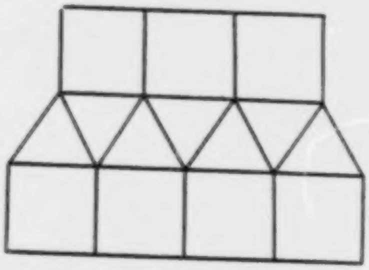
Dodecagons, equilateral triangles
(12-12-3)



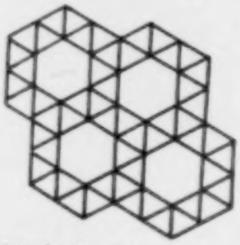
Dodecagons, squares, hexagons
(12-4-6)



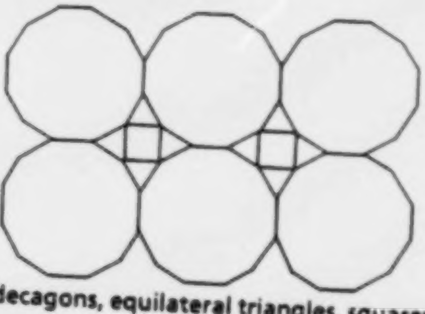
Dodecagons, equilateral triangles, squares
(12-12-3)



Squares, equilateral triangles
(4-4-3-3-3)



Equilateral triangles, hexagons
(3-3-3-3-6)



Dodecagons, equilateral triangles, squares
(12-3-4-3)

### ACTIVITY 2: LETTERS OF THE ALPHABET

Certain alphabet letters can be designed to tessellate. Cut out the letters C, L, S, and T from graph paper and let your students try tiling with them. With a little care, each letter can be shaped so it will tessellate. This is an excellent opportunity for students to apply their knowledge of transformations (slides, flips, and turns) as they describe how each pattern has been made. Again, encourage a variety of tiling patterns.

*Extension*

Let students try to design other letters using graph paper and then draw their tiling patterns. Which letters will tessellate? Could any digits be drawn so that they will tessellate?

### ACTIVITY 3: ESCHER-TYPE SHAPES

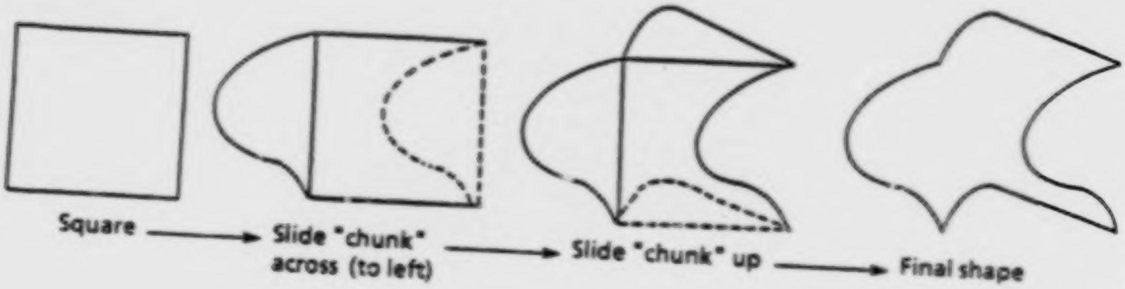
These shapes can prove fascinating for your students. Many articles and books have been written about designing this type of tessellating pattern (e.g., Zurstadt [1984]). In general terms, you begin with a shape you know will tessellate, for example, a rectangle, hexagon, square, or parallelogram. Cut out a "chunk" from the shape, making sure you start and end on the same side. Slide this chunk directly across the shape and attach it (with tape) to the opposite side (see figure 5). You can continue cutting out and attaching chunks using each pair of opposite sides in the figure. This new shape will tessellate.

Once students have created their own unique tessellating shape, they should try to imagine what their shape represents – a fish, a creature's head, an ice cream sundae, and so forth. Have the students position their shape on a piece of paper and cover the page by tessellating the shape. Next have them color in the design to illustrate what they have imagined their shape to be. Since this activity does take some time to do well, it is a good idea to use a fairly small piece of paper.

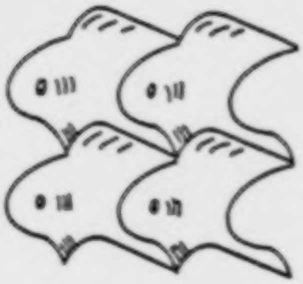
*Extension*

This type of tessellating design can be generated in more complex ways (Haak, 1976). Some students might find it interesting to look at the work of M.C. Escher in more depth.

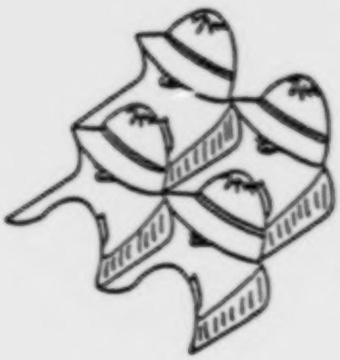
Fig. 5 Creating a unique tessellating shape.



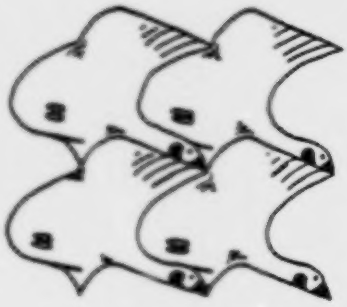
Three designs using this tessellating shape:



School of fish



Group of people



Flock of birds

## CONCLUSION

To conclude the unit, refer to the original unit problem. Compare the students' responses with what they discovered during these activities. Most students will be very surprised by the number and variety of shapes that would successfully tile a floor. Allow students to reflect on what they have learned. A discussion can be generated by asking such questions as the following:

1. If so many tiling shapes are possible, why do manufacturers mainly produce square tiles rather than other shapes? (They are easier to package in cartons.)
2. What advantages might some shapes have over other shapes? (Interlocking shapes are less prone to slide out of position.)

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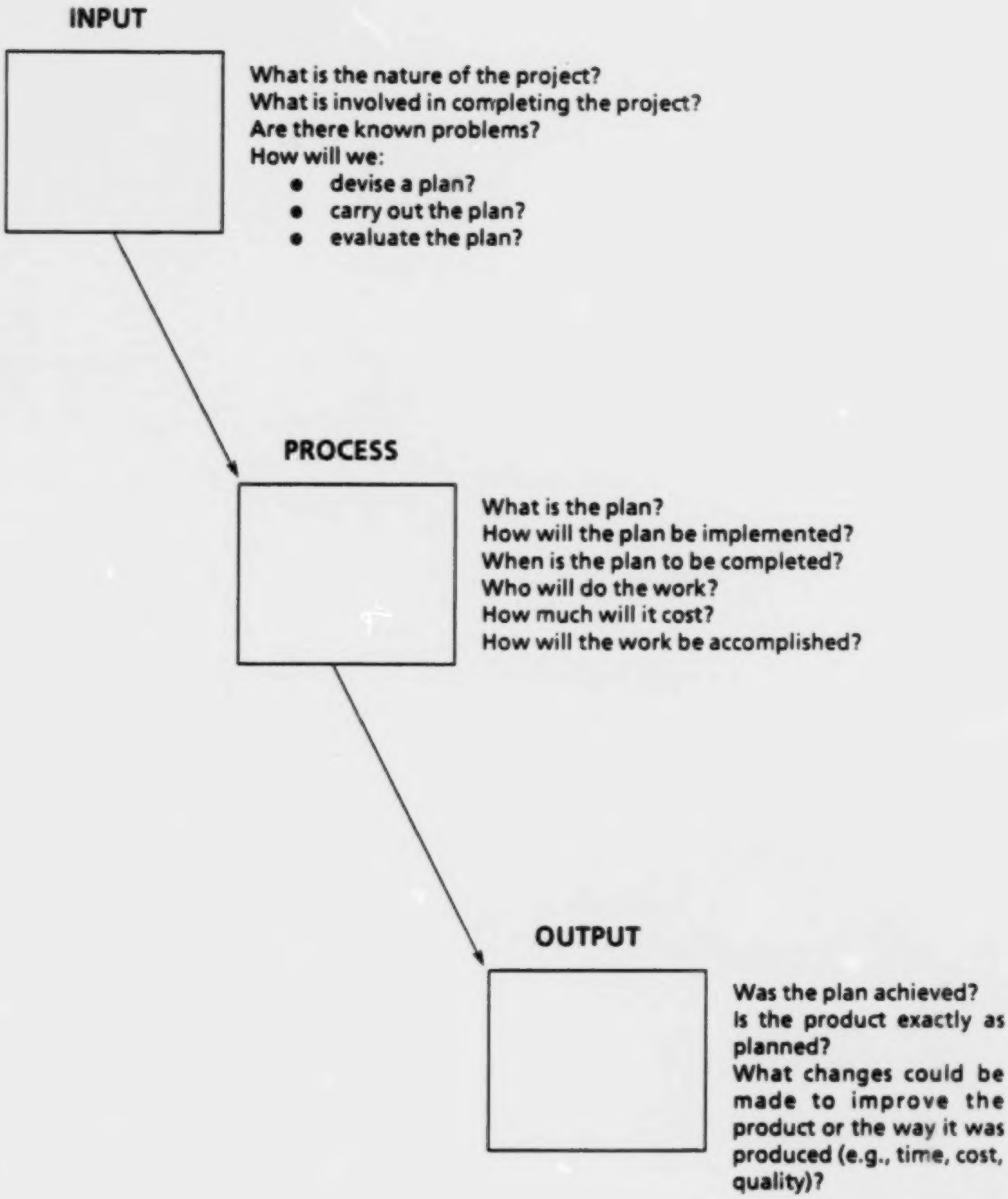
# RESOURCE 7: A PLANNING MODEL



**INPUT**

What is the nature of the project?
What is involved in completing the project?
Are there known problems?
How will we:

- devise a plan?
- carry out the plan?
- evaluate the plan?

**PROCESS**

What is the plan?
How will the plan be implemented?
When is the plan to be completed?
Who will do the work?
How much will it cost?
How will the work be accomplished?

**OUTPUT**

Was the plan achieved?
Is the product exactly as planned?
What changes could be made to improve the product or the way it was produced (e.g., time, cost, quality)?

# RESOURCE 8: PLANNING A PROJECT

## PREPARING A SCALE DRAWING

Select an appropriate project. Prepare a scale drawing in the space below that depicts different views of the project (e.g., top, side, front). Indicate the scale that was used in preparing the drawing.

Now that you have prepared a working drawing for your project, you can proceed to the next stage of the planning procedure.

### IDENTIFYING APPROPRIATE STEPS AND PROCEDURES

Working alone or as a member of a small group, identify and describe appropriate steps that must be taken to complete the project. List the steps in their correct sequence.

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________

What safety precautions must be taken throughout various steps of project construction?

- ______________________
- ______________________
- ______________________
- ______________________
- ______________________

In the chart below, list the equipment and tools that are required to complete the project.

| Equipment | Tools |
|---|---|
|  |  |

## ESTIMATING AND COSTING MATERIALS

Prepare a list of the type and quantity of materials that will be required to complete the project. Visit a local store or use a catalogue to determine the cost of materials. Record information in the chart below.

| Quantity | Materials Description | Cost |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

Total Cost of Materials $________________

Estimate how long it will take you or your group to complete the project. Determine an appropriate labour charge that is based on your estimate of person-hours required to complete the project.

____________________________________________________________

____________________________________________________________

____________________________________________________________

# RESOURCE 9: A REDECORATING PROJECT

A. Determine the following measurements of a room in your house (e.g., your bedroom, living room or kitchen).

length of room
width of room
width/location of doorway
width/location of closet doors
depth of closets
width/location of windows

B. Using an appropriate scale, make a scale drawing of this room on centimetre grid paper. Show the location of the door, closets, windows and furniture in the room.

SCALE: 1 cm = ________

## RESOURCE 9: A REDECORATING PROJECT (continued)

C. Assume you have been given $1000 to redecorate this room. Your redecorating may include painting, wallpapering, and carpeting, as well as replacing curtains, light fixtures and furniture.

Select materials that you wish to use in redecorating the room. You may use catalogues and newspaper advertisements, or decide to "shop" for the materials you wish to use.

Shopping Tips:

- Be very careful to base quantities of paint, wallpaper and carpet required on actual room dimensions. Check quantities and prices very carefully as these materials are expensive. Plan to have the least possible amount of waste material.

- Before selecting furniture, ensure that the size will fit the location in which it will be placed. Furniture can be drawn to scale and placed on the scale drawing of your room to make sure that pieces fit.

Use the following table to help you record the materials used, the quantities required, and their cost. Be sure not to spend more than $1000.

| ITEM | STORE | QUANTITY | UNIT COST | TOTAL COST |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | TOTAL | |

# RESOURCE 10: CAREER OPPORTUNITIES INTERVIEW GUIDE

Name of person interviewed: ______________________ Occupation: ______________

Student: ______________________________ Date: ______________

1. What are the entry-level requirements in this occupation?

2. What mathematical skills are required?

3. Are there other educational and/or training requirements?

4. Can the work be performed by a handicapped person? (Please specify type of handicaps that may interfere with this type of work.)

5. a. How long have you been working in this career field?

   b. How long have you been the manager/proprietor of this business or industry?

6. What additional knowledge and skills did you need to become a manager/proprietor?

7. What steps did you follow in preparing for and obtaining your present position?

8. Would you recommend this career to me? Why or why not?

## RESOURCE 10: CAREER OPPORTUNITIES INTERVIEW GUIDE (continued)

9. If I decide to enter this career, are there other entry-level requirements of which I should be aware? Please specify what they are.

10. a. Is joining a union mandatory? ☐ Yes ☐ No

    b. What are the advantages/disadvantages of becoming a member of a union?

| Advantages | Disadvantages |
|---|---|
| | |
| | |
| | |
| | |

    c. Which union is associated with this occupation?

    d.
    - How much are the union fees?
    - How frequently are they paid?

11. a. What is the average remuneration for an entry-level employee in this occupation? (Hourly? Weekly? Monthly? Annually?)

    b. Are there opportunities to increase income? ____ How? (e.g., overtime, further education, length of service, previous experience.)

12. a. Describe general working conditions in this occupation.

    b. What do you like most about these working conditions?

    c. What do you like least about these working conditions?

# RESOURCE 11: JOB SURVEY CHART

Student's Name: ____________________

Occupation: ____________________

Major tasks performed: ____________________

## 1. WAGES

State the highest, lowest and average wage offered to an entry-level employee in this occupation according to your survey.

- Highest entry-level wage ____________________
- Lowest entry-level wage ____________________
- Average entry-level wage ____________________

(indicate if wages are hourly, weekly, monthly or annual rates)

## 2. WORKING CONDITIONS

List positive and negative aspects of working conditions in this occupation.

| Positive Aspects | Negative Aspects |
|---|---|
| | |
| | |

## 3. TRAINING REQUIREMENTS AND MATHEMATICAL SKILLS NEEDED

## 4. OCCUPATIONAL CHOICE (Select (a) or (b) and complete the sentence.)

This occupation:

a. interests me as a possible career choice because:

b. does not interest me as a possible career choice because:

## RESOURCE 12: CONDUCTING A CAREER SURVEY

Visit a local shopping centre, mall or other place of business and conduct a career survey using some adaptation of the sample survey sheet provided on page 133.

When conducting the survey, use appropriate methods of gathering, organizing and presenting information.

e.g.,
- select a suitable sample from the population being surveyed
- using tally sheets and/or frequency tables, collect and organize data
- display data by constructing a suitable graph (e.g., bar, line, picture or circle graph).

Once information has been gathered, organized and presented, use the guidelines provided on page 134 to analyze data and make inferences about the population from which the sample was taken.

e.g.,
- calculate appropriate measures of central tendency
- use survey results to determine the probability of particular responses
- identify biases that may have occurred in the survey and sample taken
- describe misleading characteristics of any graphs that were constructed.

**Prior to completing this activity, permission to conduct a survey should be obtained from the appropriate persons (e.g., shopping mall manager, the local police).**

## SAMPLE SURVEY SHEET

Record people's responses according to the following key:
1 = Most important reason
2 = Less important reason
3 = Least important reason.

1. You chose your present work because:
   - a. you enjoy the work? ☐
   - b. you wanted to earn "big" money? ☐
   - c. you like the hours of work? ☐
2. Are you:
   - a. happy with the work you do? ☐
   - b. bored with the work you do? ☐
   - c. wanting a new job? ☐
3. Does your work require you to use mathematical skills:
   - a. on a daily basis? ☐
   - b. frequently, but not every day? ☐
   - c. on an infrequent basis? ☐
4. Would you prefer:
   - a. to be employed? ☐
   - b. to be self-employed? ☐
   - c. to be employed and have a small business? ☐

Additional Information:

Gender of Respondent

Male ☐
Female ☐

Age of Respondent

15 – 25 ☐
26 – 40 ☐
Over 40 ☐

Thank you for participating in this survey.

## ANALYSIS OF RESPONSES

1. For each of the four questions, determine the frequency of "1", "2" and "3" responses.

2. For each of the four questions, calculate the percent of "1", "2" and "3" responses .

3. Based on your calculations, answer the following questions:

   a. Why did most people choose their present work?

   ________________________________________

   ________________________________________

   b. Are most people happy or bored with the work they do?

   ________________________________________

   ________________________________________

   c. To what extent did the people surveyed use mathematical skills in their work?

   ________________________________________

   ________________________________________

   d. What percent of the people surveyed want a new job? Suggest reasons.

   ________________________________________

   ________________________________________

   e. Given the choice, would people prefer employment, self-employment or some combination of both?

   ________________________________________

   ________________________________________

   f.
   - What was the average age of the people surveyed? ____________________
   - What percent of the people surveyed were 15-25 years of age? ____________________
   - Over 40 years of age? ____________________

   g.
   - What percent of those surveyed were male? ____________
   - Female? ____________

# RESOURCE 13: TAKING INVENTORY

The term INVENTORY is used to describe a list of articles and their estimated value. When an individual or a company takes an inventory, the task is to quantify and record the details of all items in a given place.

At various times during the year, each business operator takes inventory. Taking inventory enables a business operator to identify:

- stock on hand
- goods sold
- damaged goods.

After taking an inventory, details may be compared to an earlier inventory. This information helps to identify business profit.

Retailers take frequent inventories. The process enables them to identify:

- goods in storage
- goods on the floor
- value of goods
- goods missing.

The inventory control system is a very important part of any retail operation.

Taking inventory is simply a system of accounting, and like most accounting tasks, the information and calculations may be handled manually or with the aid of a computer. Some retail operators may use a combination of methods.

Manual inventory taking is a slow process. It may require one or more workers to count packages and cartons contained in each bin, rack, pallet, etc., in the store. The manual count can then be checked against a handwritten record of stored goods, or against a computer printout.

Computerized inventory taking may be performed in several ways. One method uses a computer printout of commodities. Spot checks of particular items are counted and compared to the computer's output. Newer inventory control systems use bar-codes to record an item into stock and to remove the item from an inventory list. This method is both fast and accurate, but may be costly because of the equipment involved.

Goods may be counted, weighed or inventoried by volume. Large items or items that are classified as "valuable" may require that serial numbers be recorded.

# RESOURCE 14: DAILY INVENTORY FORM

A. A partial daily inventory for a small convenience store is provided below. Enter the transactions in the daily inventory form on the following page. Keep a running balance. Materials received are marked "in".

(Beginning balance: Eggs – 15 dozen, Bread – 10 loaves, Milk – 20 litres, Chips – 35 packages.)

| DAY | TRANSACTION |
|---|---|
| 1 | 10 doz. eggs<br>15 loaves bread (in)<br>20 L milk |
| 2 | 10 doz. eggs (in)<br>16 loaves bread<br>14 L milk (in)<br>22 pkg. chips |
| 3 | 7 doz. eggs<br>5 loaves bread<br>25 pkg. chips (in) |
| 4 | 15 loaves bread (in)<br>3 L milk<br>22 pkg. chips |
| 5 | 10 doz. eggs (in)<br>13 loaves bread<br>15 L milk (in)<br>12 pkg. chips |
| 6 | 9 doz. eggs<br>2 loaves bread<br>20 pkg. chips (in) |
| 7 | 10 doz. eggs<br>10 loaves bread (in)<br>9 pkg. chips |

**CLARIFICATION/EXAMPLE**

| Date | Eggs | | |
|---|---|---|---|
| | In | Out | Balance |
| Beginning Balance | 15 | | 15 |
| 1 | | 10 | 5 |
| 2 | 10 | | 15 |

B. Use the retail price list that follows in order to determine:

- the retail value of inventory after one week
- the cost price of inventory if the average markup is 40%.

Price List:

eggs – $2.10/dozen
bread – $1.09/loaf
milk – $.99/litre
chips – $.75/package

# RESOURCE 14: DAILY INVENTORY FORM (continued)

| BEGINNING OF THE MONTH BALANCE | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Date | Product 1 | | | Product 2 | | | Product 3 | | | Product 4 | | |
| | In | Out | Bal. | In | Out | Bal. | In | Out | Bal. | In | Out | Bal. |
| Beginning balance | | | | | | | | | | | | |
| 1 | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | |
| 11 | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | |
| 16 | | | | | | | | | | | | |
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| 26 | | | | | | | | | | | | |
| 27 | | | | | | | | | | | | |
| 28 | | | | | | | | | | | | |
| 29 | | | | | | | | | | | | |
| 30 | | | | | | | | | | | | |
| 31 | | | | | | | | | | | | |

## RESOURCE 15: COMPILING AN INVENTORY OF ASSETS

1. Using the form provided on the following page, compile an inventory of the assets in your classroom.
2. Estimate the value of each item listed in your inventory.
3. Total the estimated values of assets in your classroom.
4. Compare your inventory with those of your classmates. Are the inventory counts identical? How does your estimated value of assets compare to estimates made by others?
5. Obtain permission to compile an inventory of another area within the school (e.g., science laboratory, school store).
6. Using a copy of the form provided on the following page, and working in groups of two or three, compile an inventory of assets in this area of the school.
7. Obtain an appropriate catalogue and price the items you have inventoried.
8. Compare the wholesale and retail prices of three or more items on your inventory.

| Item | Wholesale Price | Retail Price |
|---|---|---|
| | | |
| | | |
| | | |

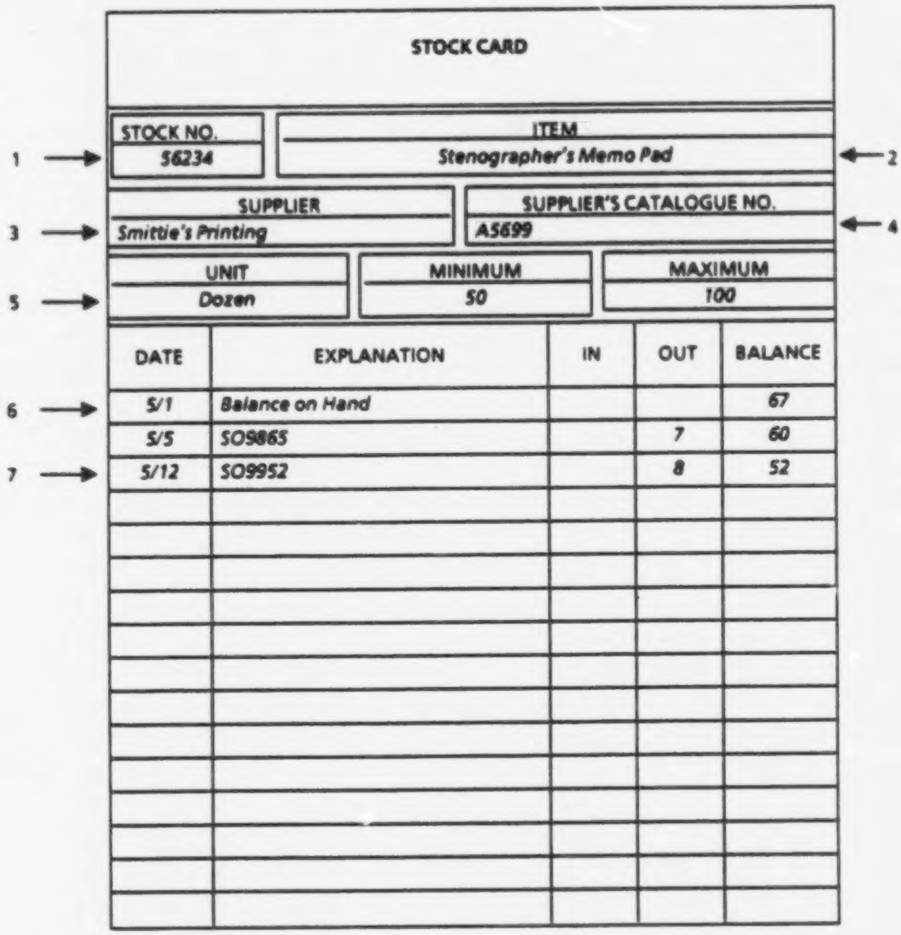
9. Total the value of assets in this area of the school.

## INVENTORY OF ASSETS

| Asset | Value |
|---|---|
| FURNITURE | |
| | |
| | |
| | |
| | |
| | |
| FIXTURES | |
| | |
| | |
| | |
| | |
| | |
| SUPPLIES | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| TOTAL ASSETS | |

## RESOURCE 16: STOCK CARDS

Some businesses use a perpetual inventory system to make sure that there is enough merchandise on hand to meet the needs of customers. With this system, all inventory items are recorded as they are sold. The stock clerk uses a stock card to record sales, purchases, and the current balance of an inventory item.



**STOCK CARD**

| | |
|---|---|
| STOCK NO. | *56234* |
| ITEM | *Stenographer's Memo Pad* |
| SUPPLIER | *Smittie's Printing* |
| SUPPLIER'S CATALOGUE NO. | *A5699* |
| UNIT | *Dozen* |
| MINIMUM | *50* |
| MAXIMUM | *100* |

| DATE | EXPLANATION | IN | OUT | BALANCE |
|---|---|---|---|---|
| *5/1* | *Balance on Hand* | | | *67* |
| *5/5* | *SO9865* | | *7* | *60* |
| *5/12* | *SO9952* | | *8* | *52* |
| | | | | |
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## RESOURCE 16: STOCK CARDS (continued)

Legend:

1. Stock number assigned to an item.
2. Description of item.
3. Name of the supplier from whom the item is normally purchased.
4. Number assigned to an item in the supplier's catalogue.
5. Unit by which an item is sold, and the minimum and maximum stock levels.
6. Balance on hand taken from the last line of previously completed stock card.
7. Date of the sale, shipping order (SD) number, number of items taken out of inventory, and new balance (e.g., previous balance on hand − inventory taken out = new balance).

### STUDENT ACTIVITY

Assume you are a stock clerk at Office Products & Supplies Company in Provost, Alberta. Record the following information on the sample stock card provided on page 142.

Stock No.: 8935
Item: Electronic Calculator
Supplier: Prod Manufacturing Co.
Supplier's Catalogue No.: 355888
Unit: Each
Min.: 36
Max.: 72
Balance on June 1: 66

| | | |
|---|---|---|
| June 5 | Sold 8, Shipping Order | 918395 |
| June 13 | Sold 13, Shipping Order | 938760 |
| June 21 | Sold 5, Shipping Order | 978814 |
| June 25 | Sold 6, Shipping Order | 983122 |

## RESOURCE 16: STOCK CARDS (continued)

### STOCK CARD

| STOCK NO. | ITEM |
|---|---|
| | |

| SUPPLIER | SUPPLIER'S CATALOGUE NO. |
|---|---|
| | |

| UNIT | MINIMUM | MAXIMUM |
|---|---|---|
| | | |

| | | | | |
|---|---|---|---|---|
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## RESOURCE 17: USING BUSINESS FORMS

A variety of forms are used to monitor movement of stock in a business. These forms can be divided into the following groups:

- order forms (e.g., purchase orders)
- receiving forms (e.g., invoices, bills of lading, packing slips)
- delivery forms (e.g., bus bills, courier bills, packing slips).

### PURCHASE ORDERS

Purchase orders, or PO's, are written requests for goods or services. They list in detail each item or service that is being requested for purchase. Purchase orders contain the following information:

- date of transaction
- transaction authorization number
- name and address of buyer
- description of items/services
- buyer's signature.

The buyer's signature on a purchase order indicates that, upon safe and proper delivery, he or she agrees to pay for the items received as ordered.

### INVOICES

Invoices are documents used by businesses to bill customers for goods or services received. Invoice forms are usually in multiple copies, one of which may be used as the packing slip. An invoice usually includes the following information:

- purchaser's name and address
- seller's name and address
- where item is to be shipped
- stock numbers
- item descriptions
- unit prices
- total price of all items shipped
- method of shipment
- date the invoice was generated
- payment conditions.

### BILLS OF LADING

Bills of lading itemize articles to be shipped by a carrier. These forms identify articles in the shipment by quantity, weight, rate, payee, consignee and consignor. This information is also used by the shipper for billing purposes. Bills of lading are used by transporting companies for the shipment of heavy or quantity items when speed of delivery is not crucial.

## RESOURCE 17: USING BUSINESS FORMS (continued)

### PACKING SLIPS

Packing slips indicate items that have been ordered and shipped. They must be checked thoroughly. Items ordered but short shipped (not included) must be noted and recorded. This provides a check for both the shipper and receiver of the goods supplied for billing, inventory and payment purposes.

### BUS BILLS

These bills are used when shipping merchandise through a bus system. Using a bus usually results in overnight service to a customer. The items can be sent either prepaid or collect but either method of shipping requires some basic information before the carrier will accept them. The basic information required includes:

- who will pay the transportation costs (prepaid or collect)?
- number of pieces
- total weight
- consignee's name and address (receiver of the goods)
- shipper's name and address (sender of the goods).

### COURIER BILLS

Courier bills are similar to bus bills. They are usually used for lighter items requiring speedy delivery. Delivery can range from in-town services only to international services.

## COMPLETING PURCHASE ORDERS

Use the information provided below to complete the purchase order.

> The Farmer's Feed and Grain, Your Town, Your Province, is buying supplies from Uptown Drug Supply Company, Box 101, Other Town, Other Province. Supplies are needed by February 15, 19-- and include two 50 kg bags of Medimix at 25¢ per kilogram, one case of 24 plastic tubes at 85¢ per tube, and ten one-litre bottles of hog wormer at $1.00 per bottle.
>
> The freight is prepaid and the supplies will be shipped by commercial truck. The goods are required by the 15th of the following month. The purchase order number is 412.

**Note:** Print legibly to avoid errors when the order is being processed.

**PURCHASE ORDER**

NO. ____________

TO ________________________________________ DATE ________ 19___

ADDRESS ________________________________________

SHIP TO ________________________________________

ADDRESS ________________________________________

| SHIP VIA | F.O.B. | TERMS | DATE REQUIRED |
|---|---|---|---|
| | | | |

| | QUANTITY | STOCK NUMBER/DESCRIPTION | PRICE | PER | EXTENSION | |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | | | | | | |
| 10 | | | | | | |
| | | | TOTAL | | | |

**IMPORTANT**
OUR ORDER NUMBER MUST APPEAR ON
INVOICES, PACKAGES AND CORRESPONDENCE
ACKNOWLEDGE IF UNABLE
TO DELIVER BY DATE REQUIRED

________________________________________

Buyer ________________________________

ORIGINAL

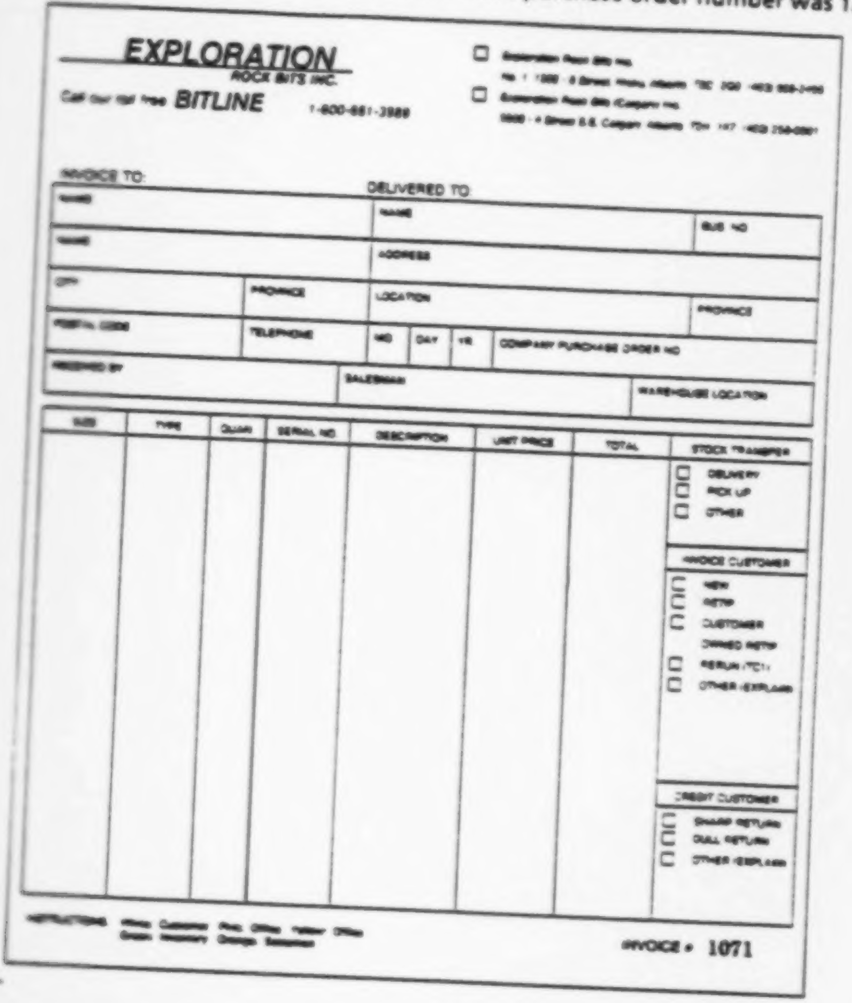
### COMPLETING INVOICES

Complete the invoice for the following sale from the Calgary warehouse.

Two new drill bits were sold without any trade-in (return):
- one 12 cm TSM rockbit #AG2305 ($235.00)
- one 13 cm TSM rockbit #AF1101 ($258.00).

The order was shipped by your delivery truck to Rig #6, Krystal Water Wells, 10526 - 100 St., in Bottleville, Alberta.

The invoice must be sent to Antos Water Well Drilling, Box 1735, Biton, Alberta, T9E 2R7, telephone number 222-3333, and the customer's purchase order number was 123456.



**EXPLORATION**
ROCK BITS INC.
Call our toll free **BITLINE** 1-800-661-3988

☐ [illegible]
☐ [illegible]

INVOICE TO:

NAME
NAME
CITY
PROVINCE
POSTAL CODE
TELEPHONE
RECEIVED BY

DELIVERED TO:

NAME
BUS NO
ADDRESS
LOCATION
PROVINCE
MO
DAY
YR
COMPANY PURCHASE ORDER NO
SALESMAN
WAREHOUSE LOCATION

| SIZE | TYPE | QUAN | SERIAL NO | DESCRIPTION | UNIT PRICE | TOTAL |
|---|---|---|---|---|---|---|
| | | | | | | |

STOCK TRANSFER
☐ DELIVERY
☐ PICK UP
☐ OTHER

INVOICE CUSTOMER
☐ NEW
☐ RETIP
☐ CUSTOMER OWNED RETIP
☐ RERUN (TC)
☐ OTHER (EXPLAIN)

CREDIT CUSTOMER
☐ SHARP RETURN
☐ DULL RETURN
☐ OTHER (EXPLAIN)

INVOICE # 1071

---

Invoice from Exploration Rock Bits Inc. Reprinted with permission.

BEST COPY AVAILABLE

### COMPLETING BILLS OF LADING

Use the following information and the name of a local trucking company to complete the bill of lading below.

> Five engines are being shipped today to Engine Rebuilders in Regina, Saskatchewan. Each engine weighs approximately 110 kg. The company you work for will pay the transportation costs. Your company's name is Acme Automotive Supplies, 9999 Right Street, Stony Plain, Alberta, T9T 3G3.

COMBINATION SHORT FORM OF STRAIGHT BILL OF LADING – EXPRESS SHIPPING CONTRACT ADOPTED BY RAIL FREIGHT AND EXPRESS CARRIERS SUBJECT TO THE JURISDICTION OF THE CANADIAN TRANSPORT COMMISSION ISSUED AT SHIPPER'S REQUEST

Received, subject to the classifications and tariffs in effect on the date of issue of this Original Bill of Lading or, received subject to the Rules for the Carriage of Express and Non-Carload Freight Traffic and tariffs in effect on the date of issue of this original Shipping Contract (bill of lading), goods described below, in apparent good order, except as noted (contents and conditions of contents of packages unknown), marked, consigned and destined as indicated below, which said Company agrees to carry to its usual place of delivery at said destination, if on its road, otherwise to deliver to another carrier on the route to said destination. It is mutually agreed as to each carrier of all or any of said goods over all or any portion of said route to destination, and as to each party at any time interested in all or any of said goods, that every service to be performed hereunder shall be subject to all the terms and conditions (which are hereby incorporated by reference and have the same force and effect as if the same were severally, fully, and specifically set forth herein)

NAME OF CARRIER  POINT OF ORIGIN  SHIPPING DATE  CARRIER'S NO

CONSIGNEE  FROM

(MAIL ADDRESS – NOT FOR PURPOSES OF DELIVERY)

DESTINATION  PROV/STATE  COUNTY OF

ROUTE  CAR INITIAL  CAR NO TRAILER NO CONTAINER NO

| PIECES/ PACKAGES | DESCRIPTION OF ARTICLES AND SPECIAL MARKS | WEIGHT (SUBJECT TO CORRECTION) | RATE |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

If charges are to be prepaid, write or stamp here, "to be prepaid"

RECEIVED $________

TO APPLY IN PREPAYMENT OF THE CHARGES ON THE PROPERTY DESCRIBED HEREON

AGENT OR CASHIER

| FOR CARRIER'S USE CHARGES | | |
|---|---|---|
| ADVANCE AND/OR BEYOND | $ | |
| MISC | $ | |
| BASIC | $ | |
| PIECE | $ | |
| VALUE | $ | |
| TOTAL | $ | |

WHERE REQUIRED BY THE TARIFF SHIPPER MUST COMPLETE THE FOLLOWING.

| Total Number of Pieces/ Packages | Dimensions of Shipment | Total Cubic Feet | Dimensional Weight | Total Weight | Number X.L. Pieces/packages |
|---|---|---|---|---|---|

SHIPPER SHOW AMOUNT OF C.O.D.

| | | | |
|---|---|---|---|
| C | AMOUNT | $ | |
| O | FEE | $ | |
| D | TOTAL | $ | |

VALUATION

$50 or 50¢ lb ☐ $1.00 per lb. ☐

DECLARED VALUE OF SHIPMENT $

SHIPPER'S NO

SHIPPER

PER

Permanent Post Office Address of Shipper

AGENT

PER

1

ORIGINAL NOT NEGOTIABLE (This Bill of Lading – Express Shipping Contract is to be Signed by the Shipper and Carrier)

BEST COPY AVAILABLE

### COMPLETING PACKING SLIPS

Use the following information to complete a packing slip for a shipment that includes:

- 2 boxes of nails weighing 1 kg each, stock number 3421 (4 boxes were ordered)
- 2 carpentry tool sets, weighing 5 kg each, stock number CT55 (2 sets were ordered)
- 1 can of linseed oil weighing 1 kg, stock number L.0.5 (3 cans were ordered)
- 4 bottles of "Everlast Glue", each weighing .25 kg, stock number EG25 (4 bottles were ordered).

This order was sold to Master Wood Products, Box 4000, Red Deer, Alberta, T4P 2X9. The shipment will be sent by bus to Master Wood Carpentry Services, 16777 – 151 Street, Highwood, Alberta. The shipment was ordered on PO 67271 and Master Wood Products has account number 555-5 with your warehouse.

Wild Rose Warehouse
Box 100
Pleasantville, Alberta
T4J 2Z5

## PACKING SLIP

No. 76125

Sold to: ______________________
______________________

Ship to: ______________________
______________________

Acct. #

| | | |
|---|---|---|
| Customer's P.O. # | Date Shipped | Route Via |

| QUANTITY ORDERED | STOCK/PART NUMBER | DESCRIPTION | QUANTITY SHIPPED | QUANTITY BACK ORDERED |
|---|---|---|---|---|
| | | | | |
| | | | | |
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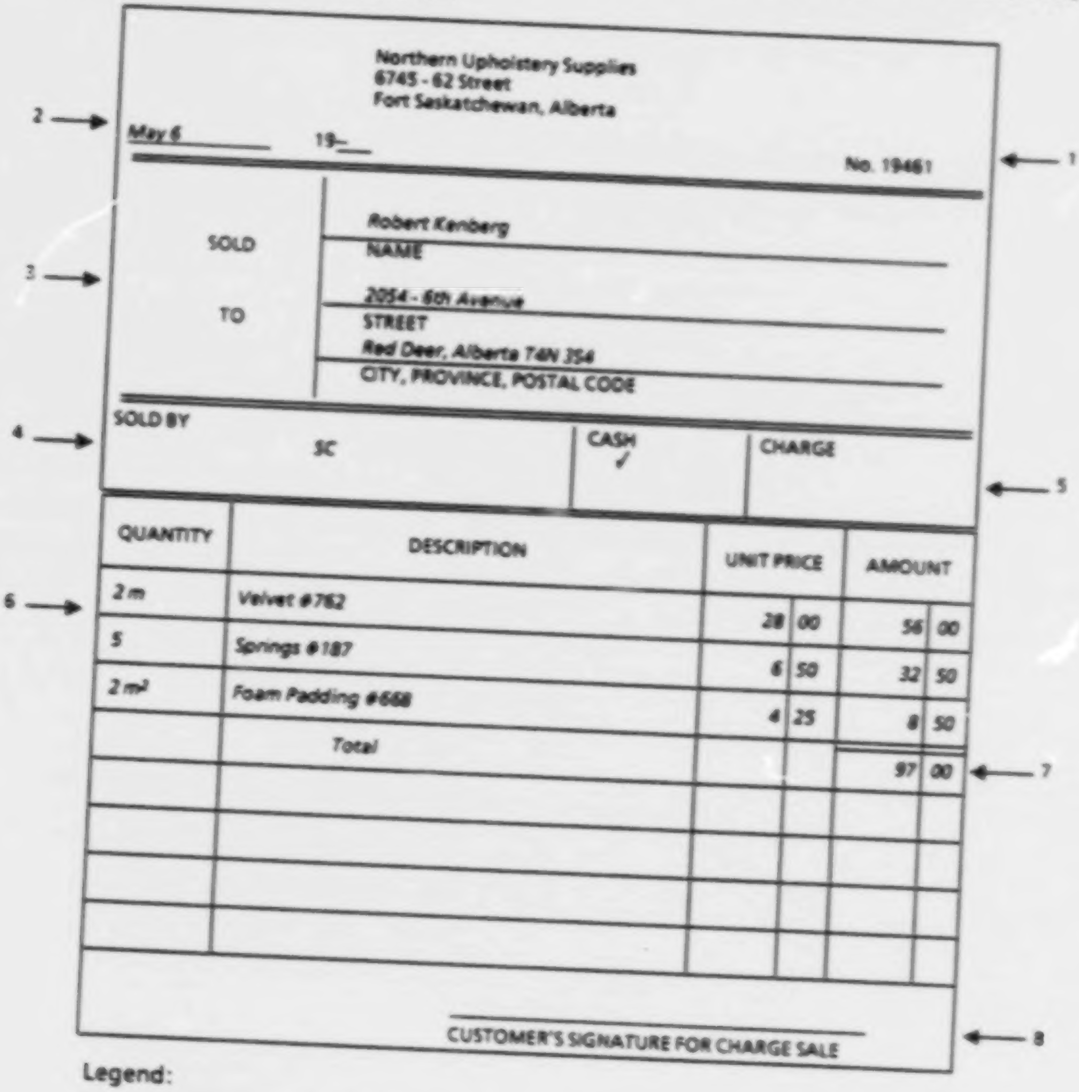
NOTE: All shipping damages must be reported to the carrier within 48 hours of receiving shipment.

# RESOURCE 18: COMPLETING SALES SLIPS

Some companies require that sales clerks prepare sales slips for both cash and credit transactions. The sales slip contains details of a sale, and provides the date, name and address of the customer, and a complete description of each item sold.



Northern Upholstery Supplies
6745 - 62 Street
Fort Saskatchewan, Alberta

*May 6* 19__

No. 19461

SOLD TO

*Robert Kenberg*
NAME

*2054 - 6th Avenue*
STREET

*Red Deer, Alberta T4N 3S4*
CITY, PROVINCE, POSTAL CODE

SOLD BY *SC*

CASH ✓

CHARGE

| QUANTITY | DESCRIPTION | UNIT PRICE | | AMOUNT | |
|---|---|---|---|---|---|
| *2 m* | *Velvet #762* | *28* | *00* | *56* | *00* |
| *5* | *Springs #187* | *6* | *50* | *32* | *50* |
| *2 m²* | *Foam Padding #668* | *4* | *25* | *8* | *50* |
| | *Total* | | | *97* | *00* |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

CUSTOMER'S SIGNATURE FOR CHARGE SALE

Legend:

1. Consecutively numbered sales slips.
2. Current date.
3. Customer's name and address.
4. Sales clerk's initials or employee number.
5. The cash or charge box is checked depending on the type of sale.
6. Details of the sale listed. The description number is the stock number of the item. Quantity × Unit Price = Amount.
7. Add the extensions in the Amount column to determine the sales slip total.
8. The customer signs the sales slip only for charge sales.

## RESOURCE 18: COMPLETING SALES SLIPS (continued)

Assume you are a sales clerk for Northern Upholstery Supplies of Fort Saskatchewan.

a. Prepare sales slip No. 19462 to record the following cash sale to Bill Berry, 8639 - 17 Avenue, Fort Saskatchewan, Alberta, T8L 2R9.

| | | | |
|---|---|---|---|
| 1 | Hammer | #126 | @ $9.86 |
| 6 | Springs | #186 | @ $6.45 |

b. Prepare sales slip No. 19463 to record the following charge sale to Alcide Brandon, 6895 - 26 Street, Fort Saskatchewan, Alberta, T8L 2R2.

| | | | |
|---|---|---|---|
| 2 m | Brocade | #345 | @ $15.00 |
| 1 box | Tacks | #466 | @ $ 1.22 |
| 4 m | Foam Padding | #667 | @ $ 3.25 |

**Northern Upholstery Supplies**
**6745 - 62 Street**
**Fort Saskatchewan, Alberta**

______________ 19___

No. 19462

SOLD TO

NAME ______________________________

STREET ______________________________

CITY, PROVINCE, POSTAL CODE ______________________________

| SOLD BY | CASH | CHARGE |
|---|---|---|
| | | |

| QUANTITY | DESCRIPTION | UNIT PRICE | | AMOUNT | |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
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| | | | | | |
| | | | | | |

______________________________
CUSTOMER'S SIGNATURE FOR CHARGE SALE

## RESOURCE 19: HANDLING CASH AND MAKING CHANGE

1. Complete the following chart. Assume you are making change when using a cash register that calculates and displays the amount of change due.

**MAKING CHANGE USING A CASH REGISTER THAT CALCULATES CHANGE DUE**

| Amount | $20 | $10 | $5 | $1 | 25¢ | 10¢ | 5¢ | 1¢ |
|---|---|---|---|---|---|---|---|---|
| $7.18 | | | | | | | | |
| $18.39 | | | | | | | | |
| $16.72 | | | | | | | | |
| $1.21 | | | | | | | | |
| $9.54 | | | | | | | | |
| $5.42 | | | | | | | | |
| $17.24 | | | | | | | | |
| $11.63 | | | | | | | | |
| $7.69 | | | | | | | | |
| $2.27 | | | | | | | | |
| $13.48 | | | | | | | | |
| $3.09 | | | | | | | | |
| $19.33 | | | | | | | | |
| $5.66 | | | | | | | | |
| $14.12 | | | | | | | | |
| $6.60 | | | | | | | | |
| $9.36 | | | | | | | | |
| $0.15 | | | | | | | | |
| $15.51 | | | | | | | | |
| $4.78 | | | | | | | | |

After completing the chart, practise with a partner how you would count and return change to the customer.

## RESOURCE 19: HANDLING CASH AND MAKING CHANGE (continued)

2. Complete the following chart. Assume you are making change when using a cash register that does not calculate and display the amount of change due.

**MAKING CHANGE USING A CASH REGISTER THAT DOES NOT CALCULATE CHANGE DUE**

| Amount of Purchase | Amount Tendered | 1¢ | 5¢ | 10¢ | 25¢ | $1 | $5 | $10 | $20 |
|---|---|---|---|---|---|---|---|---|---|
| $ 2.50 | $ 3.00 | | | | | | | | |
| $10.17 | $11.00 | | | | | | | | |
| $ 5.50 | $10.50 | | | | | | | | |
| $ 0.37 | $ 0.50 | | | | | | | | |
| $ 3.77 | $ 5.00 | | | | | | | | |
| $20.38 | $21.00 | | | | | | | | |
| $ 7.67 | $10.00 | | | | | | | | |
| $ 9.89 | $20.00 | | | | | | | | |
| $ 3.69 | $ 4.00 | | | | | | | | |
| $ 5.57 | $ 6.00 | | | | | | | | |
| $10.86 | $12.00 | | | | | | | | |
| $ 0.73 | $ 0.80 | | | | | | | | |
| $ 5.50 | $ 6.50 | | | | | | | | |
| $55.97 | $60.00 | | | | | | | | |
| $29.33 | $30.00 | | | | | | | | |
| $27.54 | $100.00 | | | | | | | | |
| $ 7.27 | $10.00 | | | | | | | | |
| $ 0.67 | $ 0.75 | | | | | | | | |
| $ 8.72 | $10.00 | | | | | | | | |
| $ 0.53 | $ 0.75 | | | | | | | | |

After completing the chart, practise with a partner how you would count and return change to the customer.

## RESOURCE 19: HANDLING CASH AND MAKING CHANGE (continued)

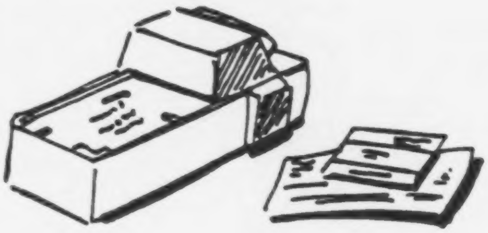
3. Why don't cashiers place the amount of money tendered for a purchase in the cash register compartments until after the customer has accepted change?

4. Why is it necessary to count change twice, once to yourself and once to the customer?

5. Why should a cashier use as few coins and bills as possible when giving change to a customer?

6. A cashier has just completed giving change from a $5.00 bill for a $3.50 purchase. The amount tendered has already been placed in the drawer. The customer insists that a $20.00 bill was tendered. What should the cashier do?

# RESOURCE 20: PROCESSING CREDIT CARD SALES



When accepting credit cards, company policies and procedures must be followed. The procedures listed below are representative of most companies that accept credit cards.

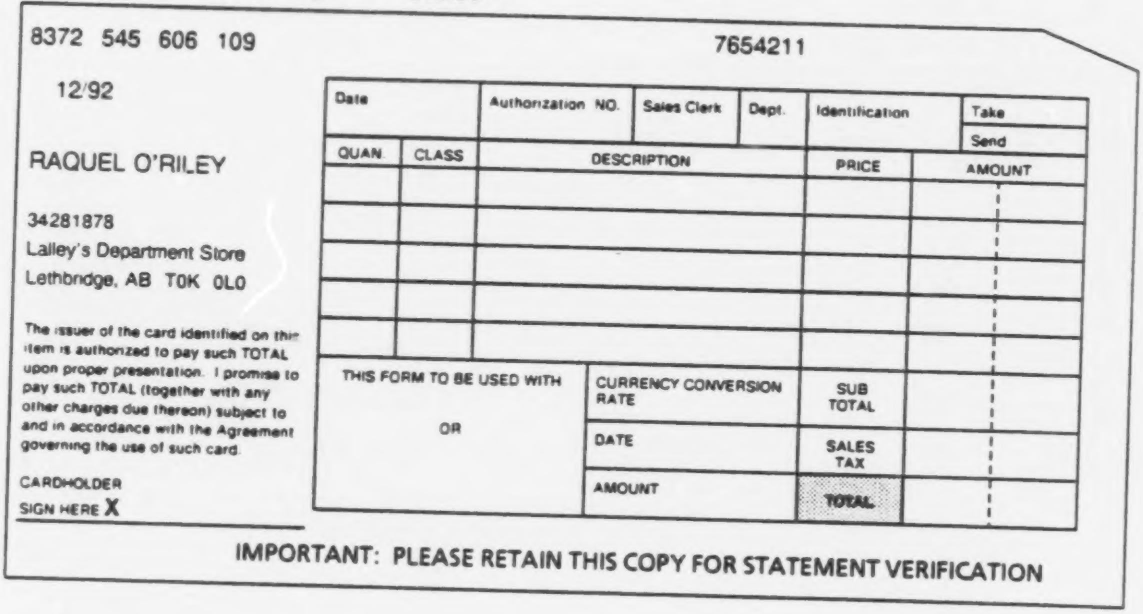
a. Run the credit card and form through the imprint machine.
b. Check that all figures and words are clearly printed.
c. Accurately complete the credit form.
d. Check the credit card's expiry date. Circle the date to indicate that it has been checked.
e. Check the card number against the current list of stolen, lost or bad-risk card numbers.
f. Call the credit centre for an authorization number if the amount of the purchase is greater than an established limit (often $50.00).
g. Obtain the customer's signature on the credit card form.
h. Compare the signature on the form with the signature on the credit card.
i. Return the credit card to the customer along with a copy of the credit form.
j. Place the merchant's copy of the form in its proper location.

**ACTIVITY**

Assume you are a sales clerk at Lalley's Department Store in Lethbridge. Prepare a credit card form, dated April 25, for each of the following sales. Your department number is 43.

a. Raquel O'Riley used her credit card to buy the following items. The authorization number for this sale is 304679.

| | | | | |
|---|---|---|---|---|
| 1 | Lamp | #2387 | @ | $59.78 |
| 1 | Shade | #2398 | @ | $19.99 |



8372 545 606 109

7654211

12/92

RAQUEL O'RILEY

34281878
Lalley's Department Store
Lethbridge, AB T0K 0L0

The issuer of the card identified on this item is authorized to pay such TOTAL upon proper presentation. I promise to pay such TOTAL (together with any other charges due thereon) subject to and in accordance with the Agreement governing the use of such card.

CARDHOLDER
SIGN HERE X

| Date | Authorization NO. | Sales Clerk | Dept. | Identification | Take<br>Send |
|---|---|---|---|---|---|
| | | | | | |

| QUAN. | CLASS | DESCRIPTION | PRICE | AMOUNT |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

| | | | |
|---|---|---|---|
| THIS FORM TO BE USED WITH<br>OR | CURRENCY CONVERSION RATE | SUB TOTAL | |
| | DATE | SALES TAX | |
| | AMOUNT | TOTAL | |

IMPORTANT: PLEASE RETAIN THIS COPY FOR STATEMENT VERIFICATION

## RESOURCE 20: PROCESSING CREDIT CARD SALES (continued)

b. Mrs. Salley Lloyd used her MasterCard to buy and take with her the following items. The authorization number for this sale is 304289.

| | | | | |
|---|---|---|---|---|
| 1 | Table Tennis Table | #2289 | @ | $150.00 |
| 2 | Table Tennis Sets | #2385 | @ | $14.95 |
| 1 | Dart Board | #1232 | @ | $15.98 |

8011 635 010 911 3762110

06/92

MRS. SALLEY LLOYD

34281878
Lalley's Department Store
Lethbridge, AB T0K 0L0

The issuer of the card identified on this item is authorized to pay such TOTAL upon proper presentation. I promise to pay such TOTAL (together with any other charges due thereon) subject to and in accordance with the Agreement governing the use of such card.

CARDHOLDER
SIGN HERE X

| Date | Authorization NO. | Sales Clerk | Dept. | Identification | Take<br>Send |
|---|---|---|---|---|---|

| QUAN. | CLASS | DESCRIPTION | PRICE | AMOUNT |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

| | | | |
|---|---|---|---|
| THIS FORM TO BE USED WITH<br><br>OR | CURRENCY CONVERSION RATE | SUB TOTAL | |
| | DATE | SALES TAX | |
| | AMOUNT | TOTAL | |

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c. Joseph Banker used his Visa card to buy over ten hardware items, which were listed on the cash register tape. The total of the items was $36.55.

2204 363 415 210 9013466

05/92

JOSEPH BANKER

34281878
Lalley's Department Store
Lethbridge, AB T0K 0L0

The issuer of the card identified on this item is authorized to pay such TOTAL upon proper presentation. I promise to pay such TOTAL (together with any other charges due thereon) subject to and in accordance with the Agreement governing the use of such card.

CARDHOLDER
SIGN HERE X

| Date | Authorization NO. | Sales Clerk | Dept. | Identification | Take<br>Send |
|---|---|---|---|---|---|

| QUAN. | CLASS | DESCRIPTION | PRICE | AMOUNT |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

| | | | |
|---|---|---|---|
| THIS FORM TO BE USED WITH<br><br>OR | CURRENCY CONVERSION RATE | SUB TOTAL | |
| | DATE | SALES TAX | |
| | AMOUNT | TOTAL | |

IMPORTANT: PLEASE RETAIN THIS COPY FOR STATEMENT VERIFICATION

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# RESOURCE 21: APPROVING PERSONAL CHEQUES

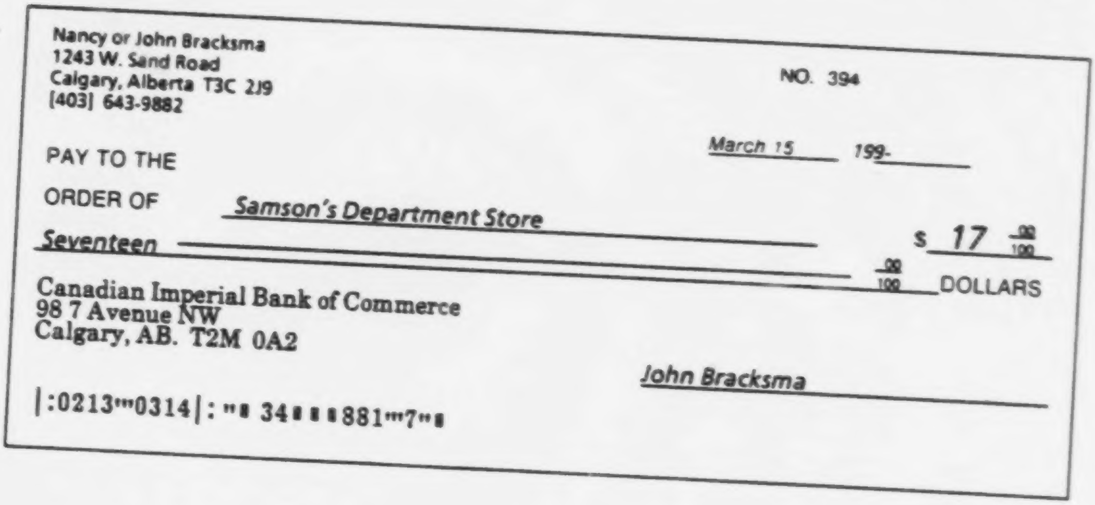
Every business has policies and procedures for cheque approval that must be followed when accepting cheques. Assume that your company has posted the following procedures for approving cheques.

a. Accept the cheque for only the amount of the purchase.
b. Examine the cheque for:
   - current date
   - correct spelling of the store's name
   - correct dollar amount in numeric and word form
   - signature of customer.
c. Ensure that the current address and telephone number of the individual is on the front of the cheque.
d. Request two pieces of identification (e.g., driver's license and credit card).
e. Record the name and number of the two pieces of identification on the back of the cheque.
f. Place your initials in the upper right corner of the cheque to indicate that you have completed the approval process.

**ACTIVITY**

Assumer you are a sales clerk at Samson's Department Store in Calgary, Alberta. On March 15, you received the following personal cheques as payment for sales. Examine each cheque carefully to determine if it has been properly prepared. Make note of missing information or errors by writing directly on the cheque. If the cheque has been properly prepared, place your initials on the cheque in the appropriate place.

1.



Nancy or John Bracksma
1243 W. Sand Road
Calgary, Alberta T3C 2J9
(403) 643-9882

NO. 394

*March 15* 199-

PAY TO THE

ORDER OF *Samson's Department Store*

$ *17* $\frac{00}{100}$

*Seventeen* $\frac{00}{100}$ DOLLARS

Canadian Imperial Bank of Commerce
98 7 Avenue NW
Calgary, AB. T2M 0A2

*John Bracksma*

|:0213'''0314|:'' 34'''881'''7''

## RESOURCE 21: APPROVING PERSONAL CHEQUES (continued)

2.

**ANTHONY MADSON**

NO. 715

March 15 199_

PAY TO THE

ORDER OF *Samson's*

*Thirty-nine* ________ $ *39.45*

[illegible]/100 DOLLARS

Bank of Nova Scotia
34 38 Street SW
Calgary, AB. T3C 1S8

*T. Madson*

|:0213‴0110|: ‴▪ 55▪▪▪115‴5‴▪

3.

***HAROLD BENNETT***
*14 River Street*
*Calgary, AB T2P 1M8*

NO.

March 15 199_

PAY TO THE

ORDER OF *Samson's Dept. Store*

*Thirty two* ________ $ *32.61*

[illegible]/100 DOLLARS

**Royal Bank**
**96 Wookpark Blvd. SW**
**Calgary, AB. T2W 3R5**

*Harold Bennett*

|:0213‴0110|: ‴▪ 55▪▪▪017‴8‴▪

4.

NO.

March 15 199_

PAY TO THE

ORDER OF *Samson's Department Store*

*eighty one* ________ $ *81. [illegible]/100*

[illegible]/100 DOLLARS

**National Bank**
**1502 12 Avenue SE**
**Calgary, AB T2M 0G3**

*David Lawton*

|:0213‴0314|: ‴▪ 34▪▪▪654‴2‴▪

# RESOURCE 22: BALANCING DAILY CASH RECEIPTS

Study the sample situation and balance sheet provided below. Then complete the balance sheet on the following page.

**SAMPLE SITUATION**

A cash register was opened with a change tray containing a $75 float. At the end of the cashier's shift, the change tray contained a total of $130 in cash and a number of customer receipts. The following procedure may be followed to balance the cash:

Step 1. Record the amount of the float.

Step 2. Record the cash on hand after sales by:

a. separating the cash into its various denominations and recording the amount of cash in each denomination

b. counting the total cash contained in the change tray.

Step 3. Total all the customer receipts and read the cash register sales tape. The total of the customer receipts should be equal to the total sales registered on the sales tape.

Step 4. Subtract the amount of the opening float from the cash on hand. Record the amounts.

Step 5. Add the amount of the opening float to the total of the customer receipts. Record the amounts.

The cash balances when the amounts recorded in step 4 and step 5 balance.

**SAMPLE BALANCE SHEET**

Step 1. Original change tray = $75.00

Step 2. Total cash on hand after sales = 130.00

| | | Coins | | | Bills | | |
|---|---|---|---|---|---|---|---|
| a. Half dollars | = | $15.00 | Ones | = | $28.00 | | |
| Quarters | = | 19.00 | Fives | = | 25.00 | | |
| Dimes | = | 8.30 | Tens | = | 10.00 | | |
| Nickels | = | 3.00 | Twenties | = | 20.00 | | |
| Pennies | = | 1.70 | | | | | |
| b. | | $47.00 | + | | $83.00 | = | $130.00 |

Step 3. Total of customer receipts = $55.00

Step 4. Total cash on hand = $130.00
Float = -75.00
Total (cash sales) = $55.00

Step 5. Float = $75.00
Total of customer receipts = +55.00
$130.00

# RESOURCE 22: BALANCING DAILY CASH RECEIPTS (continued)

## BALANCE SHEET

Original change tray = $75.00
Total cash on hand after sales = ________

Coins

Half dollars = $12.50
Quarters = 7.25
Dimes = 3.60
Nickels = 2.95
Pennies = .23

Bills

Ones = $21.00
Fives = 10.00
Tens = 20.00
Twenties = 60.00

$________
Total of customer receipts = $________ + $________ = $________
Total cash on hand $________
Float $ -75.00
Total (cash sales) $________
Float = $ 75.00
Total of customer receipts = $________
$________

## DISCUSSION TOPICS

- What would you do if the figures did not balance?

- Who is responsible if there is a shortage or surplus of money in the change tray? Why?

# RESOURCE 23: COMPLETING A CASH SUMMARY REPORT

Using the information provided below, complete the cash summary report.

You are a sales clerk at the Eastridge Gift Shop. You start the day with a $50 change tray. At the end of the day, the amount on the register tape is $1382.14 and your cash drawer contains the following items:

| | | | | | |
|---|---|---|---|---|---|
| 31 | Pennies | 189 | $1 Bills | Cheques: | $15.98 |
| 9 | Nickels | 28 | $5 Bills | | $9.55 |
| 21 | Dimes | 16 | $10 Bills | | |
| 4 | Quarters | 43 | $20 Bills | | |

**CASH SUMMARY REPORT**

**Cashier:** ______________________ **Date:** ______________________

| Quantity | Denomination | Amount |
|---|---|---|
| | Pennies | |
| | Nickels | |
| | Dimes | |
| | Quarters | |
| | Dollar Coins | |
| | $1 Bills | |
| | $2 Bills | |
| | $5 Bills | |
| | $10 Bills | |
| | $20 Bills | |
| | $50 Bills | |
| | $100 Bills | |
| | Cheques | |
| | Total Cash in Drawer | |
| | Less Change Fund | |
| | Cash Received | |
| | Less Cash Register Tape | |

Cash Proved: ______________ Cash Over: ____________ Cash Short: ______________

# PROBLEM SOLVING

Learning to solve problems is a major goal of the mathematics program. Today's rapidly changing technological society demands that students are able to apply the mathematical skills learned to new and unfamiliar problem situations in life. Practice in finding answers to routine word problems in mathematics will not provide students with the problem-solving skills they require. Instead, for solving problems, students must acquire strategies that foster the development of critical and creative thinking skills, and be given ample opportunity to apply the strategies and skills they acquire to a wide variety of problem situations in everyday life. The mathematics program must provide opportunities for students to apply knowledge, skill and experience in new and challenging situations where:

- no readily apparent solution or means to the solution is evident
- a person can be temporarily perplexed
- there may be no answer, a single answer, or many answers
- personal and societal factors are involved, as well as mathematical competencies.

Problem solving should not be viewed as an isolated activity. Strategies and skills appropriate to problem solving must be integrated and applied throughout all themes and strands of the curriculum. Teachers should remember that what may be a problem to some students may be only a routine exercise to others, and be prepared to adapt their presentations and expectations to reflect this. Teachers should also model problem-solving behaviour. Students will think problem solving is important if they are encouraged by their teachers to solve problems, and if they see their teachers solving problems.

The framework for problem solving outlined in the Mathematics 26 *Program of Studies/Curriculum Guide* parallels the framework used for solving problems in other subject areas within the Integrated Occupational Program. It suggests a general "plan for action" that can guide students through a broad range of investigative activities (e.g., problem solving, inquiry, decision making, library research). As students become familiar with the framework and discern similarities among the actions taken in different kinds of investigation, they will be more likely to transfer skills learned in specific contexts to other practical situations where methods and outcomes may be uncertain. Teachers should encourage students to recognize how the framework for solving problems in mathematics provides a general plan for action that is also effective in guiding investigations in English, science, social studies and the occupational courses, and how specific skills used within the framework will vary according to the nature of the problem or investigation.

A variety of suggestions and ideas for developing effective problem-solving techniques have been included in this section of the manual, and focus attention on:

- Developing a Desire to Solve Problems
- A Framework for Solving Problems
- Using Algebra in Problem-Solving Situations
- Cooperative Problem Solving
- Monitoring and Evaluating Progress.

Additional information on problem solving can be obtained by referring to Alberta Education's monographs ***Problem-Solving Challenge for Mathematics*** (1985) and ***Problem Solving in Mathematics: Focus for the Future*** (1987).

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## DEVELOPING A DESIRE TO SOLVE PROBLEMS

Critical to the development of skills in problem solving is the attitude with which students approach the task. The development of appropriate attitudes must be nurtured through an atmosphere that fosters flexibility and acceptance. Students must learn to accept and appreciate that being perplexed and unsure is often normal when first encountering a challenging situation, and to take risks in the development of particular problem-solving strategies. The guidelines that follow may assist teachers in planning effective problem-solving activities.

- Create a positive classroom atmosphere that facilitates the development of appropriate attitudes and a desire to solve problems by:
  - encouraging students to be creative, and to generate their own ideas and approaches to problem solving
  - providing adequate support and assistance to students, but not solving their problems for them
  - being willing to accept unconventional solutions, more than one solution, or no solution, where appropriate
  - challenging students to think critically and justify strategies and solutions
  - being enthusiastic and capable of recognizing the students' desire and perseverance to solve problems
  - providing appropriate questions and modelling for students.
- Select problem-solving activities that emerge from real life situations. Ensure that problems are relevant to the interests and experiences of students, and that cognitive demands of the problems match developmental levels of students.
- Modify and vary the approach used so as to ensure interest, participation, and some degree of success by all students. Most students have an inherent desire to accept the challenge provided by a problem.

Teachers can monitor the development of appropriate problem-solving attitudes and beliefs by asking students to respond to statements made about problem solving on an inventory. Student reaction to particular statements about problem solving will indicate to the teacher those attitudes that need further attention and development. Sample statements that might be used in constructing an inventory of problem-solving attitudes are provided on the following page.

Problem-solving attitudes can also be monitored by observing and questioning students as they work in problem-solving situations. Observations can be recorded on individual checklists and rating scales, thus indicating aspects of attitude and performance that require further attention. Sample checklists and rating scales that may be effective in monitoring attitudinal change and development through the school year are provided in the "Assessment/Evaluation" section of this document.

## CLARIFICATION/EXAMPLE

**Attitude Inventory Items**[1]

Mark true or false depending on how each statement describes you in problem-solving situations. There are no right or wrong answers.

_____ 1. I will put down any answer just to finish a problem.
_____ 2. It is no fun to try to solve problems.
_____ 3. I will try almost any problem.
_____ 4. When I do not get the correct answer right away I give up.
_____ 5. I like to try hard problems.
_____ 6. My ideas about how to solve problems are not as good as other students' ideas.
_____ 7. I can only do problems everyone else can do.
_____ 8. I will not stop working on a problem until I get an answer.
_____ 9. I am sure I can solve most problems.
_____ 10. I will work a long time on a problem.
_____ 11. I am better than many students at solving problems.
_____ 12. I need someone to help me work on problems.
_____ 13. I can solve most hard problems.
_____ 14. There are some problems I will just not try.
_____ 15. I do not like to try problems that are hard to understand.
_____ 16. I will keep working on a problem until I get it right.
_____ 17. I like to try to solve problems.
_____ 18. I give up on problems right away.
_____ 19. Most problems are too hard for me to solve.
_____ 20. I am a good problem solver.

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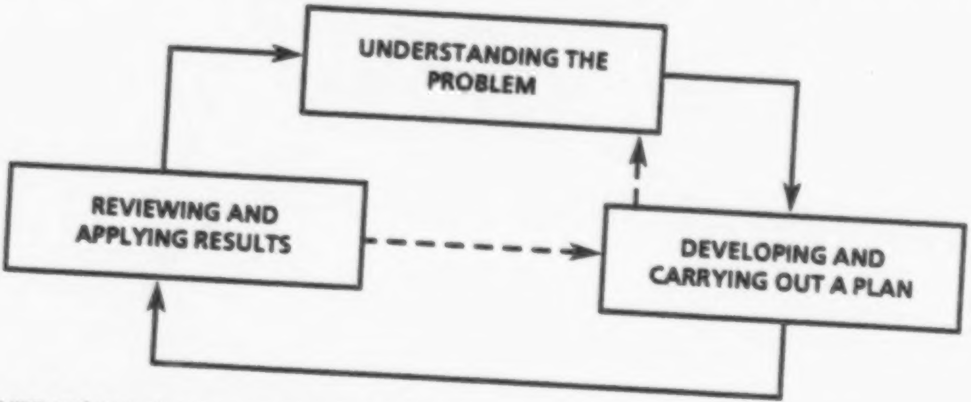
[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

# A FRAMEWORK FOR SOLVING PROBLEMS

A framework for problem solving is diagrammed below. Although the framework provides an overall structure that can help students to understand the processes used in solving problems, it should not be interpreted as a series of fixed and rigid stages and strategies. There are many ways to approach and solve problems, and the sequence of steps used will depend upon individual problems and individual students. Students may not always go through the steps in the order suggested, nor even use all steps in some problem situations. Nevertheless, an understanding of the steps, as well as individual skills that can be used at each stage of the process, will increase the students' repertoire of strategies that can be brought to bear on a problem.


UNDERSTANDING THE PROBLEM
REVIEWING AND APPLYING RESULTS
DEVELOPING AND CARRYING OUT A PLAN

Teachers can assist students to use the problem-solving process in practical situations by:

- sharing the problem-solving framework with all students. The framework gives structure to the overall process, and suggests specific strategies that might be used in solving problems
- relating the problem-solving framework used in mathematics to the frameworks used for solving problems in science, social studies, English and the occupational courses. Students should recognize how the frameworks are similar in the overall structure they provide for problem solving, and how they may differ in terms of individual strategies appropriate to each stage of problem solving
- keeping the framework and strategies for problem solving flexible and tentative. While useful in the support and structure they provide, students should be encouraged to be creative and experimental in their approach to problem solving.

Performance in problem solving can be enhanced by making students conscious of their thought process (i.e., metacognitive awareness). Students need to become aware of and discuss how they think in order to become more strategic in their learning repertoires, and to monitor their problem-solving efforts (e.g., check their answers when in doubt). Teachers can foster this awareness by thinking out loud as they solve problems, by asking questions, and by having students identify the strategies and processes used when solving problems. Teacher modelling of appropriate steps and strategies for solving problems will provide a structure that facilitates the development of effective problem-solving skills.

Various stages and steps in the problem-solving framework are clarified on the pages that follow. A sample lesson plan illustrating how specific strategies might be used in the problem-solving process is provided in Resource 1: Sample Lesson Plan for Problem Solving.

## UNDERSTANDING THE PROBLEM

During this stage of the problem-solving process, students must be encouraged to think about and interpret the problem situation. Teachers can assist students to focus their attention on information and conditions set in the problem by asking appropriate chains of questions. Teachers can also model and explicitly teach strategies that may be used by students in developing an understanding of a problem situation.

### CLARIFICATION/EXAMPLE

**Strategies for "Understanding the Problem":**

- reading the problem several times
- asking questions
- identifying key words and their meanings
- looking for patterns
- identifying wanted, given, and needed information
- identifying extraneous information
- internalizing the problem by restating in one's own words or by visualizing the problem
- drawing pictures/diagrams
- using concrete manipulatives
- interpreting pictures/charts/graphs
- relating the problem to other problems previously encountered
- simulating or modelling the problem situation
- considering alternative interpretations of the problem
- referring to other resources to clarify the meaning of the problem
- determining if there are hidden assumptions which contain information necessary to the solution of the problem.

Teachers can assist students to understand problem situations by:

- having discussions that focus on understanding the problem
  e.g.,
    - What is the question? What do we need to find?
    - What are the conditions/variables in the problem?
    - What data do we need?
- asking students to explain problems in their own words, or through the use of pictures and diagrams
- reminding students of similar problems
- using coloured markers to highlight important words, phrases or data in the problem
- recording data provided in the form of a list.

## DEVELOPING AND CARRYING OUT A PLAN

In this stage, students should plan strategies and then actually use these strategies to solve the problem. As students may lack the "strategic repertoire" required to develop a problem-solving plan, it may be necessary to teach various strategies appropriate to specific problem situations explicitly. Emphasize that there are often strategies other than computation that can be used effectively to solve the problem. Once appropriate strategies have been planned, the student simply "carries out the plan" to arrive at a solution.

## CLARIFICATION/EXAMPLE

**Strategies for "Developing and Carrying Out a Plan":**

- guessing and checking the result (thus improving the guess)
- using logic or reason
- choosing and sequencing the operations needed
- sorting and classifying information
- applying selected strategies
- presenting ideas clearly
- selecting appropriate calculating/measuring devices and methods
- visualizing the problem
- acting out or simulating the problem
- applying patterns
- estimating the answer
- documenting the process used
- working with care
- working in a group situation where ideas are shared
- visualizing the problem
- speaking to self with positive statements (e.g., "I can solve this")
- using a simpler problem (making an analogy)
- identifying factors relevant to the problem
- collecting and organizing data into diagrams,number lines, charts, tables, pictures, graphs or models
- experimenting through the use of manipulatives
- breaking the problem down into smaller parts
- formulating an equation
- recognizing limits and eliminating possibilities
- identifying and applying relationships
- constructing flow charts
- working through the problem backwards
- examining the problem from varying perspectives/points of view.

Teachers can assist students to develop and carry out their problem-solving plans by:

- suggesting a solution strategy
- giving the start of a solution or strategy, and then asking students to complete the solution in order to solve the problem
- giving direct instruction/practice with particular solution strategies
- discussing possible solution strategies
  e.g., – asking students to suggest reasons why they believe particular strategies might work
  – asking what action in the problem suggests a particular operation
- reminding students of similar problems
- providing a one-step or multiple-step problem without numbers, and asking students to identify the operations used in finding a solution
- providing a completed solution to a problem (e.g., a number sentence, organized list, or picture), and asking students to create a problem that would fit the solution.

## REVIEWING AND APPLYING RESULTS

This stage encourages students to assess the effectiveness of their solution, and to consider the accuracy of their results. Students should be encouraged to relate answers to the question in the problem in order to verify that the problem has indeed been solved. By evaluating strategies that have been used, students will become aware of their appropriateness and of other strategies that might also be used. Through discussion and reflection, encourage students to generalize and apply their results to related situations.

### CLARIFICATION/EXAMPLE

**Strategies for "Reviewing and Applying Results":**

- stating an answer to the problem
- restating the problem with the answer
- explaining the answer in oral/written form
- determining if the answer is reasonable
- discussing the process used with others
- suggesting other ways of solving the problem
- checking the answer
- considering the possibility of other answers/solutions
- making and solving similar problems
- generalizing the solution and applying the process used to other problems
- creating and writing other routine/non-routine problems that involve use of a similar process.

Teachers can assist students to review and assess the effectiveness of their problem-solving efforts by:

- asking students to explain why they chose particular solution strategies
- illustrating alternative strategies that might be used to solve the problem, and evaluating their usefulness
- discussing incorrect strategies and attempts, and explaining why these strategies were not appropriate
- using estimation to check the reasonableness of answers obtained
- ensuring that all relevant information in the problem has been used.

## USING ALGEBRA IN PROBLEM-SOLVING SITUATIONS

Students may be reluctant to use algebra in problem-solving situations because of limited experience with algebraic concepts and skills. Teachers should model the use of algebraic skills on a frequent basis in both formal and informal problem-solving situations. By encouraging students to use symbols to describe patterns and relationships in practical problem situations, they will gradually develop confidence in their ability to use algebra as a problem-solving tool.

## USING VARIABLES TO DESCRIBE NUMBER PATTERNS AND RELATIONSHIPS

Before developing formal equation-solving skills, students should establish an understanding of variables and their use in practical situations. Be aware that an understanding of the concept of variables develops slowly. Demonstrate the use of variables in a variety of concrete situations throughout each theme.

Teachers can assist students to use variables in describing arithmetical patterns and relationships by:

- playing number games
- drawing diagrams
- building tables
- using/modifying computer programs.

### CLARIFICATION/EXAMPLE

**A Number Game**

Ask each student to think of a number. Then instruct students to:
- double the number
- add 15
- subtract 10
- state the new number.

The teacher can determine each student's original number by reversing the process (i.e., subtract 5 from the answer and divide by 2). Ask students to suggest alternative methods of determining the original number. Describe the problem with an algebraic equation (e.g., $2N+15-10=2N+5$). Encourage students to recognize that in solving equations we frequently reverse operations and the order in which they are performed.

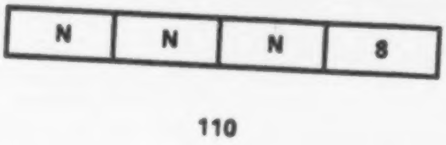
Modify the problem so as to involve the use of brackets and the distributive property:
- multiply the number by 2
- add 10
- divide this number in half
- subtract four.

The original number will be one less than the answer.

### CLARIFICATION/EXAMPLE

**Drawing a Diagram**

A piece of wood 8 centimetres long is cut off the end of a longer board that is 110 centimetres long. If the remaining board is divided into three equal pieces, how long will each piece be?



Equation: $8 + 3N = 110$

(Additional activities that involve the use of diagrams are provided in Resource 2: Using Diagrams to Solve Problems.)

### CLARIFICATION/EXAMPLE

**Building a Table**

If a car is travelling at 60 km/h, determine the distance it has travelled throughout each period of time indicated in the table.

| Hours travelled | 1 | 2 | 3 | 5 | 8 | 10 | 15 | h |
|---|---|---|---|---|---|---|---|---|
| Distance travelled | | | | | | | | |

Encourage students to write a formula that describes the relation illustrated in the table; e.g., $d = 60 \times h$

### CLARIFICATION/EXAMPLE

**Using a Computer Program**

The computer program illustrated below has been designed to generate a list of perfect squares. Ask students to run and possibly modify this program.

```
10   FOR X = 1 TO 15
20   PRINT X, X^2
30   NEXT X
40   END

RUN

1    1
2    4
3    9
4    16
5    25
•
•
•
```

## EVALUATING EXPRESSIONS AND SOLVING EQUATIONS

Initial equation-solving experiences should involve the use of "guess and check" strategies. Such activities will assist students to develop an understanding of variables and their function as place holders in an equation.

**CLARIFICATION/EXAMPLE**

**Using Guess and Check Strategies to Solve Equations**

Find a number that satisfies the equation "$2N+3=11$".

First guess:
$N=1$
$2(1)+3=5$
The number is too small.

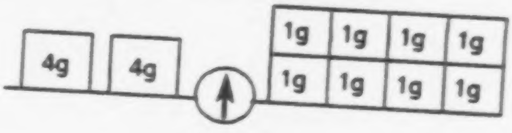
Second guess:
$N=5$
$2(5)+3=13$
The number is too large.

Third guess:
$N=4$
$2(4)+3=11$
The number satisfies the relationship described by the equation.

Following experience using guess and check strategies, students should develop an understanding of more formal strategies used in solving equations. Discuss the concept of "equality" as a statement of balance in number sentences where the left side equals the right side. Use the balance scale as a manipulative, and describe various states of balance on the scale using appropriate number sentences.

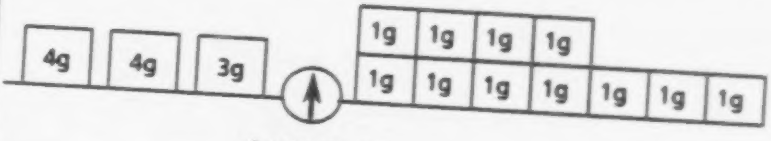
**CLARIFICATION/EXAMPLE**

**Relating Equality to the Balance Scale**



$4+4=8$

To maintain balance, whatever is added to/taken away from one side must be added to/taken away from the other side. Likewise, if one side is increased/decreased through multiplication or division, a similar operation must be performed on the other side of the equation to maintain balance.

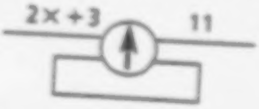


$4+4+3=11$

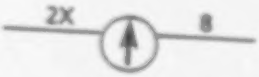
Consider the likeness of equation solving to balancing a scale, and the importance of maintaining balance (equality) in the equation-solving process.

**CLARIFICATION/EXAMPLE**

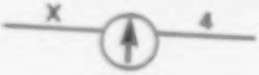
If $2x+3=11$, then the scale is balanced with $2x+3$ on one side and 11 on the other.



To solve for $x$, it must be isolated on one side of the scale. Subtract 3 from both sides. The scale is balanced with $2x$ on one side and 8 on the other side.



Divide each side of the scale by 2. The scale is balanced with $x$ on one side and 4 on the other side.



$X=4$

Another equation-solving strategy involves the analogy of packing/unpacking boxes. By making this comparison, students will develop an understanding of the need to reverse operations and the order in which they are performed when solving equations.

**CLARIFICATION/EXAMPLE**

**Packing/Unpacking Boxes**

Relate the process of writing an equation to "packing" a box. In solving the equation, the box must be "unpacked".

To solve $2x+3=11$, first determine how the box was packed. (How were the symbols in the equation combined?)

Packing

1. Start by placing an $x$ in the box. What was done to the $x$?
2. Multiply it by 2. What was done next?
3. Add 3. How many in total? $2x+3$ or 11.

Now unpack the box to solve for $x$. Remind students that when unpacking a box, actions taken to pack the box are performed in the reverse order.

Unpacking

Reverse the packing process as follows:

1. Subtract 3. This yields 8.
2. Divide by 2. The result is 4.

## A STRUCTURED APPROACH FOR APPLYING ALGEBRA SKILLS

A structured worksheet is often effective in helping students to cope with the cognitive demands of algebraic representation and equation solving. The structure and organization provided by a worksheet may clarify complexities of a problem, and prompt students to use appropriate strategies in correct sequence.

### CLARIFICATION/EXAMPLE

**Sample Format for a Problem-Solving Worksheet**

1. Problem-solving goal:
2. What I do not know:
3. What I know:
4. A description of the problem situation in my own words (or a picture of the problem):
5. Algebraic representation of the problem (i.e., an equation):
6. Solving the equation:
7. Solution:
8. Is my solution reasonable?
   (Estimate and check the answer.)
9. Does the solution achieve my problem-solving goal?

# COOPERATIVE PROBLEM SOLVING

Problem solving provides opportunities for students to develop strategies and skills by working in cooperative learning situations. Group work often provides students with a less threatening environment, where they may be more willing to take the risks associated with problem solving. Students participating in a group problem-solving activity can learn new strategies from others, and refine their own problem-solving skills.

Skills often best learned in group settings include the ability to:

- clarify one's own ideas
- consider alternative ideas and approaches
- compare and assess alternatives.

### CLARIFICATION/EXAMPLE

**A Paired Problem-Solving Strategy**[1]
Students are divided into pairs (problem solver, recorder) in order to work together in solving a problem. The use of a "thinking-aloud" procedure allows the student to see how their partner thinks and solves problems. Thinking steps are thus open to view and can be observed and communicated. The procedure used is as follows:

1. One member (the problem solver) "thinks aloud" while solving a given problem.
2. The other member (the recorder) listens carefully, noting the steps taken in the solving of the problem. At the end of this "think aloud" procedure, the recorder may ask clarification questions of the problem solver and/or may point out errors made in the problem-solving process.
3. Roles are reversed, repeating the same problem.
4. The modification/extension of both strategies are discussed with each person (or both if agreement is reached) documenting the best "modified" strategy.

This strategy could be expanded to include three people by subdividing the recorders' role into recorder and questioner. The three roles would rotate.

Interpersonal skills can also be enhanced through cooperative learning experiences. As students work in group settings, problems in social interaction may arise. A strategy for systematically analyzing a social problem is provided in Resource 3: Social Problem-Solving Strategy. This strategy helps students to identify:

- reasons for the difficulty/conflict
- strategies that may avoid the difficulty/conflict another time.

Teacher modelling and student use of this strategy may assist students to gain confidence in their ability to interact and communicate with one another effectively.

---

[1] Adapted from *SPELT: A Strategies Programme for Effective Learning/Thinking: Inservice Edition*, SPELT International Ltd., p. 197. Copyright 1987 by R. Mulcahy, K. Marfo, D. Peat and J. Andrews. Reprinted by permission.

Teachers can facilitate cooperative problem solving by establishing a "problem-solving corner" in their classrooms. This area of the classroom can be stocked with a variety of problems, puzzles and manipulative materials. Encourage students to contribute interesting problems and puzzles to the problem corner. By displaying a "daily puzzle" or "problem of the week", students can be motivated to use the corner on a regular basis as time permits. Additional ideas that may be useful in establishing a problem-solving corner are provided in an ensuing section of this manual (see "Situational and Concrete Approaches").

Sample problems provided in Resource 4: Problem-Solving Ideas have been selected on the basis of their cognitive demand as well as effectiveness in developing problem-solving strategies. While some of the problems are suited only to group settings, many can be solved by students working independently or in small groups. **Teachers are encouraged to develop their own collection of problems/puzzles that are appropriate to curriculum goals and student interest/ability.**

Additional problem-solving activities are suggested in:

- *Problem Solving Challenge for Mathematics* (Alberta Education, 1985)
- *Problem Solving in Mathematics: Focus for the Future* (Alberta Education, 1987)
- *The Arithmetic Teacher* (published by the National Council of Teachers of Mathematics)
- *The Mathematics Teacher* (published by the National Council of Teachers of Mathematics).

Books and kits providing suitable problems and puzzles can be obtained by contacting local library and resource centres.

## MONITORING AND EVALUATING PROGRESS

Checklists will enable the teacher to determine the extent to which students are using certain strategies in problem solving. By keeping a class checklist during work on a particular theme, teachers can diagnose both individual progress in the use of various strategies, as well as total group progress for each strategy that is developed.

**CLARIFICATION/EXAMPLE**

| | Identifies Key Words | Draws Pictures/ Diagrams | Uses Calculator | Applies Patterns | Applies Concepts | Asks Questions | Identifies Relationships | Makes Measurements | Estimates Answer | Explains Process | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Mary | I | II | IIII | I | I | | I | III | | II | 15 |
| Tom | I | | I | | II | | I | III | II | I | 11 |
| | 2 | 2 | 5 | 1 | 3 | 0 | 2 | 6 | 2 | 3 | |

In the example above, a tally is inserted in the appropriate column each time the teacher notes tangible evidence of the use of a particular strategy. In this instance, Mary is gaining experience in drawing sketches, making measurements, and in using several other strategies; she has shown no evidence of asking questions or estimating answers. The teacher can then plan special lessons to reinforce the strategies that have not been used.

Checklists can also be used to monitor problem solving attitudes and behaviours. By observing and questioning students as they work in problem-solving situations, teachers can make note of various aspects of attitude and behaviour and record these on a checklist. These records will be useful in assessing attitudinal change and development for individual students throughout the school year.

**CLARIFICATION/EXAMPLE**

**Observational Checklist of Problem-solving Attitudes and Behaviours**[1]

_____ 1. Likes to solve problems
_____ 2. Works cooperatively with others in the group.
_____ 3. Contributes ideas to group problem solving.
_____ 4. Perseveres – sticks with a problem.
_____ 5. Tries to understand what a problem is about.
_____ 6. Can deal with data in solving problems.
_____ 7. Thinks about which strategies might help.
_____ 8. Is flexible – tries different strategies if needed.
_____ 9. Checks solutions.
_____ 10. Can describe or analyze a solution.

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

Students need to recognize the importance of reflecting on the strategies they use, on what they have done, and on what they still need to do. Encourage students to monitor and evaluate their own thinking and progress in problem solving through the use of focus questions that ask them to think back and describe how they solved particular problems.

**CLARIFICATION/EXAMPLE**

**Focus Questions for Monitoring and Evaluating Progress**[1]

Use the following questions to help you look back and describe your thinking as you worked toward a solution to the problem.

1. What did you do when you first saw the problem? What were your thoughts?
2. Did you use any problem-solving strategies? Which ones? How did they work out? How did you happen to find a solution?
3. Did you try an approach that didn't work and have to stop and try another approach? How did you feel about this?
4. Did you find a solution to the problem? How did you feel about this?
5. Did you check your answer in any way? Did you feel sure it was correct?
6. How did you feel, in general, about this problem-solving experience?

Students can also be encouraged to reflect upon the thought processes they use by completing a "strategy inventory" that is based on a particular problem-solving experience.

**CLARIFICATION/EXAMPLE**

**Problem-Solving Strategy Inventory**[2]

Think about your use of strategies when solving the problem. Check the following that apply.

1. _____ I didn't think about using strategies at all.
2. _____ The idea of using strategies came to my mind, but I didn't think about it much more.
3. _____ I looked at a strategy list, but didn't try a strategy.
4. _____ I looked at a strategy list and picked a strategy, which I tried.
5. _____ I didn't look at a list, but just thought of a strategy to try.
6. _____ I used at least one strategy and it helped me find a solution.

7. I tried the following strategies:

_____ guess and check
_____ make a table
_____ look for a pattern
_____ make an organized list
_____ other ________________________________
_____ solve a simpler problem
_____ work backward
_____ draw a picture
_____ write an equation

---

[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

[2] Ibid.

Problem solving is often difficult to evaluate because it is process oriented. Ensure that evaluation strategies focus attention on the problem-solving process that is used, and not just the solution. A possible marking scale has been provided below. This scale assigns 0, 1 or 2 points for each stage of problem solving, according to the criteria stated. It should be noted that the marks assigned at any given stage should not be influenced by the marks assigned at other stages of the process.

**CLARIFICATION/EXAMPLE**

| **Marking Scale for Problem Solving**[1] | |
|---|---|
| **Understanding the Problem** | 0: Complete misunderstanding of the problem<br>1: Part of the problem misunderstood or misinterpreted<br>2: Complete understanding of the problem |
| **Developing and Carrying Out a Plan** | 0: No attempt, or totally inappropriate plan<br>1: Partially correct plan based on part of the problem being interpreted correctly<br>2: Plan could have led to a correct solution if implemented properly |
| **Reviewing and Applying Results** | 0: No answer or wrong answer based on an inappropriate plan<br>1: Copying error; computational error; partial answer for a problem with multiple answers<br>2: Correct answer and correct label for the answer |

Students should not be evaluated on their ability to solve problems simply by what they produce on paper. Attitudes and behaviours that may also be taken into account include:

- willingness to attempt a problem
- use of a systematic approach
- use of appropriate strategies
- willingness to try other strategies
- logical justification of strategies and solutions
- perseverance in the task
- confidence in ability to solve problems
- willingness to contribute to group problem-solving activities
- willingness to solicit/accept help from others.

Evaluation procedures should include a variety of techniques, including observation, questioning and interviewing. Checklists, inventories and rating scales provide a useful means of recording student attitudes and behaviours as they relate to problem solving. Assessment and evaluation techniques are further discussed in an ensuing section of this manual (see "Assessment/Evaluation").

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[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

# RESOURCE 1: SAMPLE LESSON PLAN FOR PROBLEM SOLVING

## PROBLEM SITUATION

Six people attend a meeting at 6:00 p.m. on June 6. If each person shakes hands with every other person, how many handshakes would take place?

### I. UNDERSTANDING THE PROBLEM

1. Have a class discussion about the problem.
   - Underline key words and important information.
   - Cross out extraneous information.
   - Determine if there are hidden assumptions that contain information necessary to the solution of the problem.
2. Ask questions to focus attention on key components of the problem.
   - How many people are in the room?
   - How many people shake hands at once?
   - If I shake your hand, is that the same as you shaking my hand?

### II. DEVELOPING AND CARRYING OUT A PLAN

1. Select an appropriate problem-solving strategy. Possibilities include:
   - Using a table or chart.
   - Drawing a diagram.
   - Looking for a pattern.
   - Acting out or simulating the problem.
2. If students experience difficulty, ask questions that will direct thought processes to appropriate strategies and considerations.
   - Once I shake everyone's hand, how many people will you still need to shake hands with?
   - How many handshakes would be needed if there were only two people at the meeting? Three people at the meeting?
3. Provide encouragement and support to students as they apply strategies and work toward a solution to the problem.

## III. REVIEWING AND APPLYING RESULTS

1. Restate the problem with the answer.
   - If six people are in a room, it would require 15 handshakes for everyone to shake hands with everyone else.

2. Discuss other ways to solve the problem.
   - Organize data into a table.

| Person No. | No. of Handshakes |
|---|---|
| 1 | 5 |
| 2 | 4 |
| 3 | 3 |
| 4 | 2 |
| 5 | 1 |
| 6 | 0 |

The solution can be obtained by systematically listing the number of handshakes made by each person in the group. The total number of handshakes will be the sum of the second column.

- Use logic and reason.

Each person must shake hands with five other people.

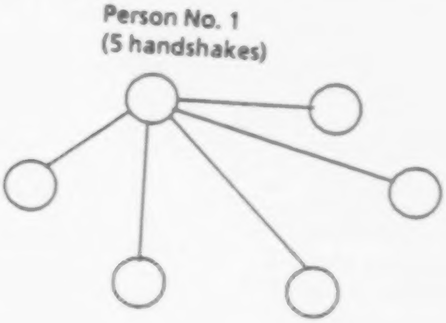
Since there are six people making five handshakes each, there would be a potential total of 6×5 or 30 handshakes.

However, each handshake has been counted twice (i.e., if I shake your hand, then you don't need to shake my hand). Therefore, the total number of handshakes will be 30 ÷ 2 or 15.

- Draw a Diagram.

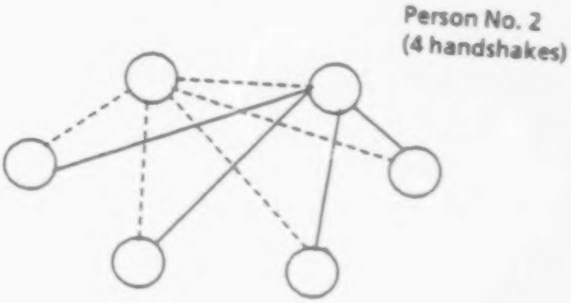
Step One

Draw the six people. Then draw a series of lines that represent the handshakes made by person No. 1.


Person No. 1
(5 handshakes)

Step Two

Draw additional lines in the diagram that represent the handshakes made by persons No. 2, No. 3, No. 4 and No. 5.

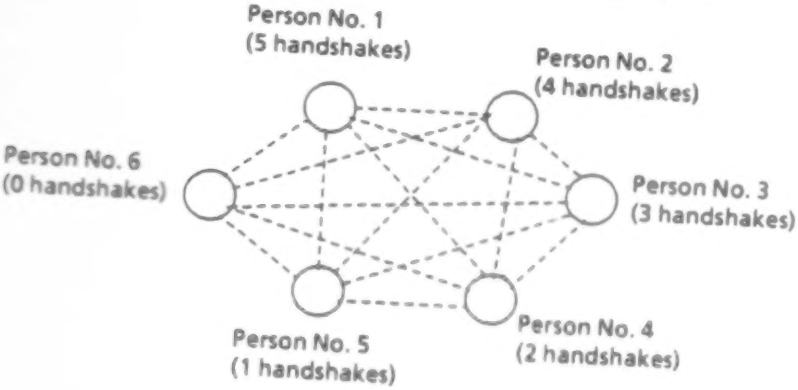

Person No. 2
(4 handshakes)

(This diagram can be extended to represent the handshakes made by persons No. 3, No. 4 and No. 5.)

## RESOURCE 1: SAMPLE LESSON PLAN FOR PROBLEM SOLVING (continued)

### Step Three

Notice, in the diagram below, that when person No. 5 has completed his or her handshakes, each person will have greeted every other person with a handshake.


Person No. 1
(5 handshakes)
Person No. 2
(4 handshakes)
Person No. 6
(0 handshakes)
Person No. 3
(3 handshakes)
Person No. 5
(1 handshakes)
Person No. 4
(2 handshakes)

The total number of handshakes, determined by counting the lines, must be 15.

3 Generalize the solution and apply the process used to other problems.

If ten teams enter a tournament and each team plays every other team, how many games must be scheduled?

## EVALUATION CRITERIA

### OBSERVATIONAL CHECKLIST OF PROBLEM-SOLVING ATTITUDES AND BEHAVIOURS

_____ 1. Enjoyed solving the problem
_____ 2. Worked cooperatively with others
_____ 3. Contributed ideas to group discussion and problem solving
_____ 4. Demonstrated perseverance
_____ 5. Considered more than one strategy or problem-solving approach
_____ 6. Demonstrated flexibility and openness to new ideas/approaches
_____ 7. Determined if the solution was reasonable
_____ 8. Generalized/transferred the solution to similar situations.

### INVENTORY OF PROBLEM-SOLVING STRATEGIES USED

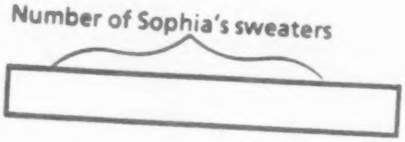
_____ 1. Guessed and checked
_____ 2. Made a table
_____ 3. Looked for a pattern
_____ 4. Simulated the problem
_____ 5. Worked backward
_____ 6. Drew a diagram
_____ 7. Wrote an equation
_____ 8. Other: ____________________

## RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS[1]

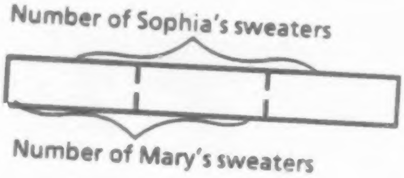
A. *Directions:* On this worksheet you will learn how to use diagrams to represent relationships between quantities in a given situation. The procedure uses a rectangular region to represent one amount; then that diagram is modified to represent another amount. Study the example given in problem 1, then complete the diagrams in problems 2 and 3.

1. Mary has 2/3 as many sweaters as Sophia.

   a. This region represents the number of sweaters that Sophia has.

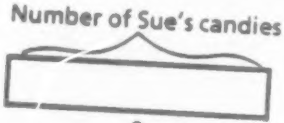

Number of Sophia's sweaters

   b. The initial diagram is then modified to include the number of sweaters that Mary has.

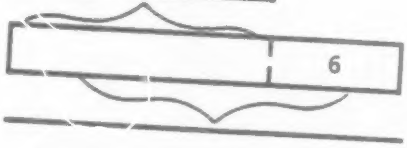

Number of Sophia's sweaters
Number of Mary's sweaters

2. Roosevelt has 6 more candies than Sue.

   a. Let the diagram represent the number of candies that Sue has.

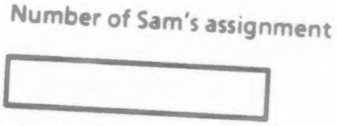

Number of Sue's candies

   b. Write the missing label for the modified diagram.



3. Cynthia has 4 more than twice as many mathematics problems to solve tonight as her younger brother Sam.

   a. Let the diagram represent the number of problems in Sam's assignment.


Number of Sam's assignment

   b. Modify the diagram to show the relationship between the number of problems in the two assignments.

## RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS (continued)

B. *Directions:* Draw and label diagrams to represent the relationship between quantities given in each of these problems.

1. Ronnie has 10 fewer baseball cards than Ritchie.

2. Elliot washed dishes 3 times as often as Bill.

3. Manuel saw 4 more than 3 times as many baseball games as Luiz.

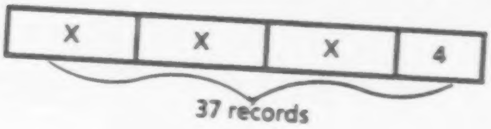
4. Vera practised 1 hour more than half as many hours as Tom.

5. Deanna worked 5 less than twice as many days as Tisha.

## RESOURCE 2: USING DIAGRAMS TO SOLVE PROBLEMS (continued)

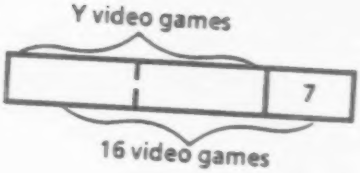
C. *Directions:* On this worksheet you are given diagrams that show relationships between certain numbers. The letters used in the diagrams represent unknown numbers. Dashed lines divide regions into congruent parts. Use the diagrams to find the values of the indicated quantities.

1.


X
X
X
4
37 records

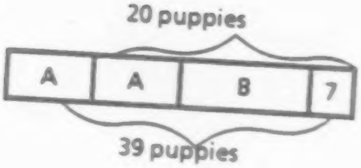
X= ______ records.

2.


Y video games
7
16 video games

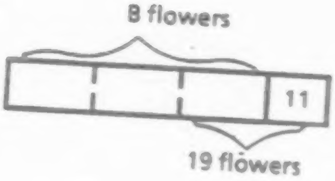
Y= ______ games.

3.


20 puppies
A
A
B
7
39 puppies

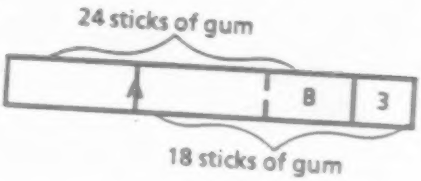
A= ______ puppies.

4.


B flowers
11
19 flowers

B= ______ flowers.

5.


24 sticks of gum
A
B
3
18 sticks of gum

B= ______ sticks.

(*Hint:* First, figure out the number represented by half of A plus B.)

---

D. *Directions:* On this worksheet, diagrams are used to solve word problems. Draw and label a diagram for each problem, then use the diagram to answer the question.

1. Nancy and Barbara are collecting craft sticks to send in for a prize. When they first started collecting, they had 30 sticks between them. Barbara has not collected any more, but Nancy has doubled the number of sticks she has. They now have 42 sticks. How many sticks did Nancy have in the beginning?

   Diagram:

   Answer: ______________

2. My two dogs, Mutt and Jeff, were eating 34 cans of dog food each week. The veterinarian put Mutt on a diet. Now he eats only 1/2 as much as before. As a result, the dogs are eating only 24 cans each week. How many cans of dog food has Jeff been eating all along?

   Diagram:

   Answer: ______________

3. Jennifer and Angeli share a locker at school. They have 14 textbooks between them. When classes change at the semester break, Jennifer will have half as many books, but Angeli will have 4 more. They'll still have 14 textbooks to cram in their locker. How many do they each have now?

   Diagram:

   Answer: ______________

4. Sam and Janet were bundling newspapers for the school's paper drive. Between the two of them, they had 20 bundles at the start of the afternoon. Sam gathered 8 more bundles and Janet gathered the same amount she already had. This effort gave them a total of 35 bundles. How many bundles did Sam have at the beginning of the afternoon?

   Diagram:

   Answer: ______________

5. Ryan and Clark decided to raise rabbits because they wanted to have lots of animals to sell. They started with 18 rabbits, some white and some brown. After three months, they now have 3 times as many white rabbits and 2 times as many brown rabbits, for a total of 46 rabbits. How many white rabbits do they have after the three months?

   Diagram:

   Answer: ______________

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[1] Reprinted with permission from *The Mathematics Teacher*, copyright 1989 by the National Council of Teachers of Mathematics.

## RESOURCE 3: SPOT (SOCIAL PROBLEM-SOLVING STRATEGY)[1]

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Teachers are encouraged to use the following social problem-solving strategy as a model when dealing with individual students or as a guide for students to use on their own.

**Description of Strategy**

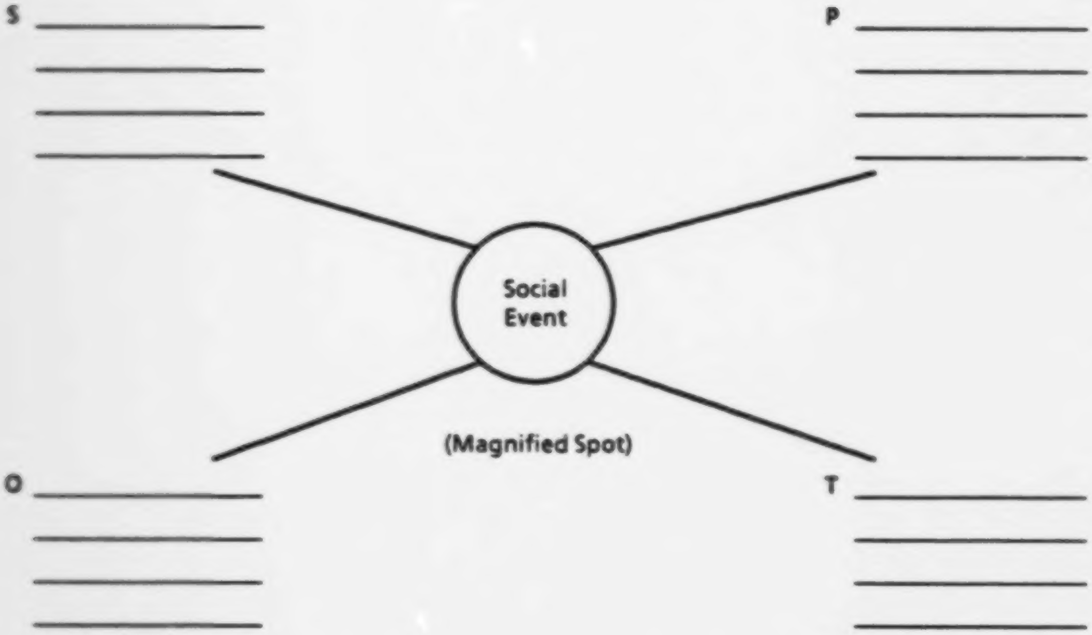
**S:** Setting: Who? What? Where? When?

**P:** Problem: What's the situation to be solved?

**O:** Order of action: What happened?

**T:** Tail End: What can be done next time?

**Problem-Solving Chart:**



(Magnified Spot)

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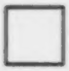
[1] From *Strategies for Teaching Students with Learning and Behaviour Problems* by Dr. C. Bos, and S. Baughn. Copyright 1988 by Boston, Allyn and Bacon. Reprinted by permission.

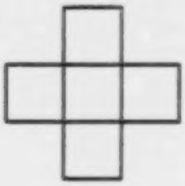
# RESOURCE 4: PROBLEM-SOLVING IDEAS

## 1. LOOK FOR A PATTERN

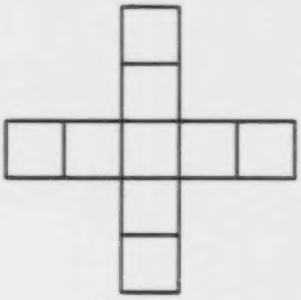
a. The 1st figure contains 1 square.
The 2nd figure contains 5 squares.
The 3rd figure contains 9 squares.



1st



2nd



3rd

If you made a drawing of the 4th figure, how many squares would it contain? How many squares would the 10th figure contain?

b. In each case, find the rule that will give the second number if you know the first. Then fill in the rest of the table according to the rule.

| | |
|---|---|
| 8 | 3 |
| 15 | 10 |
| 30 | 25 |
| 50 | ? |
| 25 | ? |
| ? | 100 |

| | |
|---|---|
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| ? | 64 |
| 6 | ? |
| ? | 1000 |

| | |
|---|---|
| 2 | 7 |
| 3 | 10 |
| 5 | 16 |
| 8 | ? |
| 22 | ? |
| ? | 22 |

c. Complete these patterns:

- 2, 5, 8, 11, ___, ___, ___, ___
- 1, 4, 9, 16, ___, ___, ___, ___
- 64, 32, 16, 8, ___, ___, ___, ___
- 1, 2, 4, 7, ___, ___, ___, ___

d. A school has 100 lockers and an enrolment of 100 students. All the doors are neatly closed for the summer holiday. On the first day of school, the first student to arrive opens all the locker doors. The second student to arrive closes every door with an even number. The third student reverses the position of all locker doors that have a number divisible by three (e.g., open becomes closed, closed becomes open). The fourth student reverses the position of all doors that have a number divisible by four. This pattern of events continues until all 100 students have entered the school.

Is locker number 100 opened or closed after the last student enters the school?

Teacher Note:
The number of lockers and students may be adjusted according to student ability. Start with 10 lockers and 10 students. Draw a diagram and use an "O" or "C" to indicate whether a particular locker is opened or closed.

Encourage students to use the number of factors of each locker number as a clue to the solution. Students should note that only numbers that are perfect squares will have an odd number of factors.

## 2. MAKE A SYSTEMATIC LIST

a. Two dice are thrown. How many different totals of the two dice are possible?

Complete this listing:

| 1st die | 2nd die | Sum |
|---|---|---|
| 1 | 1 | 2 |
| 1 | 2 | 3 |
| 1 | 3 | 4 |
| • | • | • |
| • | • | • |
| • | • | • |

b. The Canadian Football League has an interlocking schedule where each team plays every other team twice. How many games would have to be played? Make a season schedule for these games.

Western Conference

B.C. Lions
Edmonton Eskimos
Calgary Stampeders
Saskatchewan Roughriders

Eastern Conference

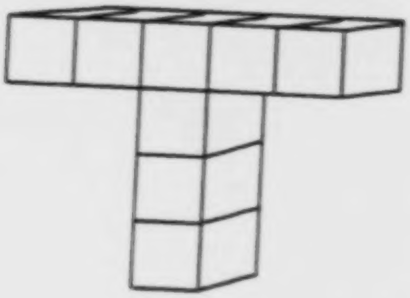
Winnipeg Blue Bombers
Hamilton Tiger Cats
Toronto Argonauts
Ottawa Roughriders

c. Kelvin has:
3 pairs of slacks – grey, black, blue
2 dress shirts – white, beige
3 sports jackets – plain, plaid, tweed.

How many different outfits can Kelvin make from this wardrobe?

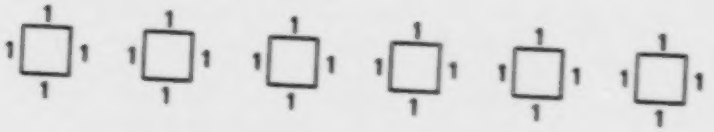
## 3. USE A MODEL OR DRAWING

a. Eight one-centimetre cubes are put together to form the T-shaped figure shown below. The outer faces of the T-shaped figure are painted red. The one-centimetre cubes are then separated. How many of the cubes have exactly four red faces?[1]

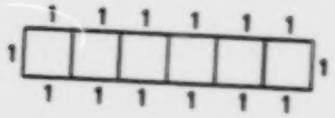


b. A restaurant has 6 square tables. Each table can accommodate one person on each side. How many different seating arrangements can be made using the 6 square tables?

Examples:



= 24 people



= 14 people

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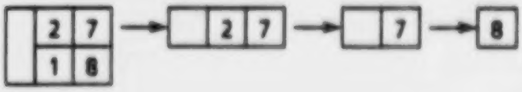
[1] Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 27.

c. Describe how to fold this "map" so that numbered sections lie on top of one another in order from 1 to 8.[1]

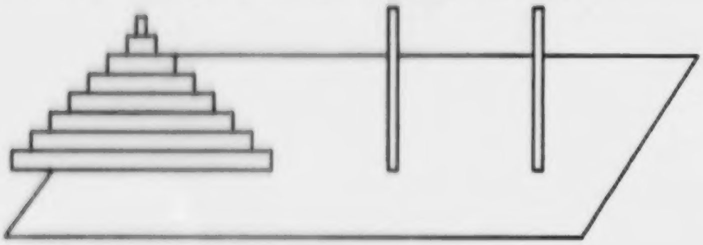
| | | | |
|---|---|---|---|
| 3 | 4 | 2 | 7 |
| 6 | 5 | 1 | 8 |

Solution:



d. TOWER OF HANOI

The "Tower of Hanoi" is a very old puzzle. The playing board consists of a flat surface with three vertical pegs. Seven rings with different radii are placed on one peg, in order of size with the largest ring on the bottom. The objective of the game is to transfer, using the least number of moves, all of the rings to another peg so they are arranged in the same order (i.e., with the largest ring on the bottom). At no time may a larger ring be placed on a smaller ring. The uppermost ring is the only ring in a pile that can be moved.



Teacher Note:
The puzzle can be easily made using paper tubes and cardboard rings. Adjust the difficulty of the game by increasing/decreasing the number of rings used. The least possible number of moves with seven rings is 127. If n rings are used, the least number of moves is $2^n - 1$.

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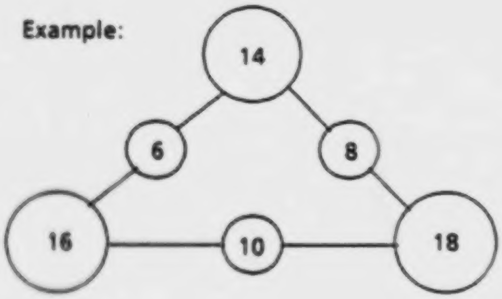
[1] Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 47.

### 4. GUESS AND CHECK

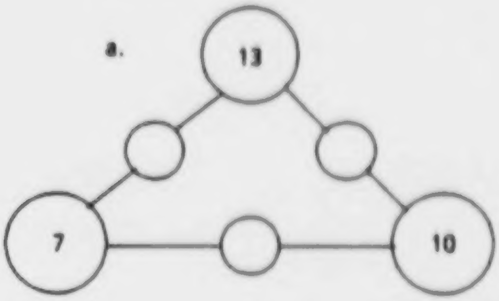
The numbers in the big circles are found by adding the numbers in the small circles.

Example:

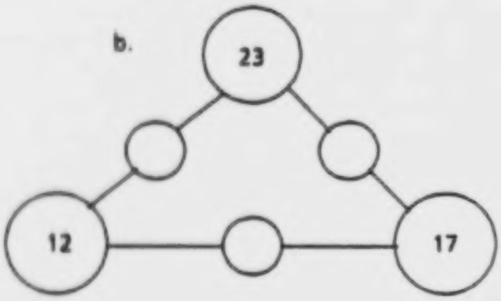


Find the numbers for the small circles in each problem.

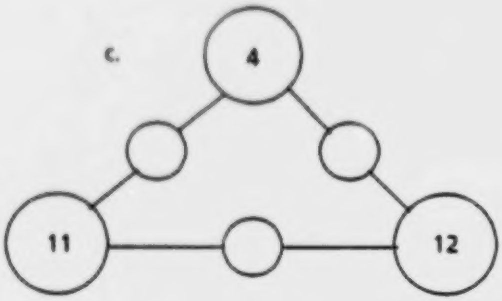
a.



b.



c.



### 5. A WINNING STRATEGY

Two players, A and B, have a pile of 6 toothpicks. Players alternately remove 1 or 2 toothpicks from the pile. The player who takes the last toothpick wins.

Teacher Note:
This game does not take long to play so students may be able to play it several times to discern the winning strategy. Students may discover several different strategies for winning. (e.g., The player who makes the second move always does the opposite to what the first player did. That is, if player A makes the first move and removes one matchstick, then player B will remove two, and so on.) The teacher may extend this game by changing the original number of toothpicks to 21, and instructing students to move one, two or three toothpicks at a time.

### 6. VISUALIZE THE PROBLEM

A tape is placed snugly around the earth at the equator. If another tape that is two metres longer is placed a uniform distance above the first tape, how far above the ground would it be? (Assume that the earth is a smooth sphere with a circumference of 40 000 km.)[1]

### 7. GUESSING AND CHECKING – ELIMINATING POSSIBILITIES

Choose a three- or four-digit number having no digits repeated. Ask someone to guess the number. Provide clues as to how close the guess is to the number chosen:
- say "Zilch" if no digits are correct
- say "Fermi" if there is a correct digit, but in the wrong position
- say "Pico" if there is a correct digit in the correct position.

For example, if the number chosen was 236, and someone guesses 532, the response is pico-fermi (i.e., one digit in the right position and another correct digit in the wrong position).

It may be advisable to choose two-digit numbers when introducing this activity. Students may play the game in pairs, keeping a record of the number of guesses made by each before determining the correct number. The winning student determines the number chosen with the least number of guesses.

### 8. LOGICAL THINKING

A used car dealer complained to his friend about having had a bad day. He told his friend that he sold two cars at $750 each. One of the sales yielded a profit of 25%. The other sale yielded a loss of 25%. "What are you worrying about?" asked his friend. "You had no loss whatsoever." "On the contrary, a substantial one", answered the car dealer. Who was right?[2]

### 9. IDENTIFY KEY WORDS

a. The average weight of four football players is 90 kg. The weights of three of the players are 82 kg, 92 kg, and 108 kg. What is the weight of the fourth player?[3]

b. Three friends ate lunch together. One person spent $3.60 for a glass of milk, two hamburgers and a donut. The second person bought two glasses of milk, one hamburger and two donuts for $3.00. If the third person bought one glass of milk, one hamburger and one donut, how much did this person pay?[4]

c. A freight train 500 m long passes through a 2000 m tunnel. If 60 seconds elapse from the time when the last car enters the tunnel to the time when the engine emerges from the other end, what is the speed of the train?[5]

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[1] Alberta Education. *Problem Solving in Mathematics: Focus for the Future*, 1987, p. 67.
[2] Alberta Education. *Problem Solving Challenge for Mathematics*, 1985, p. 40.
[3] Ibid. p. 42.
[4] Ibid. p. 22.
[5] Ibid. p. 39.

## 10. ALGEBRA PROBLEMS

Arthur found this equation in his brother's algebra book:

$15N+23=398$ (Remember, 15N means 15 times N.)

His brother explained that this equation is like a puzzle problem:
I'm thinking of a number. If you multiply it by 15 and add 23 you get 398. What is my number?

CLASS EXERCISES

a. Arthur's first guess was 12. Was his guess too large or too small?
b. Arthur's next guess was 30. Was the guess too large or too small?
c. What is the solution to $15N+23=398$?
d. Here are other equations Arthur found in the book. First read the equation as a puzzle problem. Then find the solution.

$2N-4=24$ $\frac{4A+6}{2}=7$

$\frac{M}{5}+1=3$ $\frac{2N}{5}-4=0$

# USE OF TECHNOLOGY

## THE CALCULATOR

In order to harness technology productively, students need to become proficient and discerning in their use of a calculator as an aid to computation and problem solving. All students shall have appropriate calculators available for use throughout the program.

Effective use of the calculator requires an understanding of place value and the ability to judge the reasonableness of the results of calculations. Skills in estimation and mental arithmetic become increasingly important in enabling students to anticipate and verify calculator results. These experiences will be more effective in developing number sense and cognitive process than long and tedious computations with paper and pencil. Competence in estimation and mental arithmetic can be expected to improve through frequent use of these skills while using a calculator. By using calculators appropriately, time previously spent on tedious computation can be used to develop cognitive process and problem-solving skills.

The guidelines that follow will assist in establishing learning environments where the calulator can be used as an effective instructional and computational tool.

- The calculator does not reduce the need for basic arithmetic skills. **Effective use of the calculator requires that an understanding of place value, basic number skills, and arithmetical operations is emphasized throughout the instructional program.**
- There will always be a need to possess limited paper-and-pencil computational skills. **For most students, the calculator should be used after the mathematical concepts and algorithms are understood.** Students who simply cannot master their basic number skills or who have persistent problems with computation should use calculators more extensively.
- The calculator should always be used in situations where long and/or extended computations are required.
- **When using the calculator in problem situations, place emphasis on estimation and mental arithmetic,** on proper documentation of numbers and operations used, and on the reasonableness of answers.
- Do not assume that students understand how to use a calculator. **Be prepared to teach students how and when to use a calculator properly.** Remember, however, that creating a situation just for the sake of using calculators will not develop an understanding of their appropriate use.

A variety of suggestions and ideas for developing effective calculator skills have been included in this section of the manual, and focus attention on:

- Entry Procedures
- Order of Operations
- Auditory Cueing
- Rounding Results
- Finding Remainders
- Using the Percent Key
- Using the Memory Keys
- Checking the Reasonableness of Results
- Developing Confidence and Speed.

## ENTRY PROCEDURES

Provide opportunities for the development of basic entry skills on the calculator. Many errors made by students in using their calculators are due to incorrect entries that have been made. Students often enter numbers for division/subtraction/exponentiation in the wrong sequence, and frequently experience difficulty with questions that involve multiple entries.

Ensure that students are familiar with the basic operations of their calculators. Keys operated include:

| | | | | | |
|---|---|---|---|---|---|
| $\boxed{+}$ | addition | $\boxed{X}$ | multiplication | $\boxed{\sqrt{x}}$ | square root |
| $\boxed{-}$ | subtraction | $\boxed{C}$ | clear | | |
| $\boxed{\div}$ | division | $\boxed{=}$ | equals | $\boxed{y^x}$ | exponentiation. |

Use an overhead transparency to demonstrate appropriate procedures for entering numbers and performing operations on the calculator. Assist students to develop confidence in using these procedures by providing both oral and written exercises.

### CLARIFICATION/EXAMPLE

**Procedure for Decimal Operation**
**(e.g., Find the mean of 8.1, 9.2, 7.6.)**

| Entry | | Display |
|---|---|---|
| Turn on | - - - - - - - - - - - - - | $\boxed{0.}$ |
| Enter | $\boxed{8}\ \boxed{.}\ \boxed{1}$ | $\boxed{8.1}$ |
| Enter | $\boxed{+}$ | $\boxed{8.1}$ |
| Enter | $\boxed{9}\ \boxed{.}\ \boxed{2}$ | $\boxed{9.2}$ |
| Enter | $\boxed{+}$ | $\boxed{17.3}$ |
| Enter | $\boxed{7}\ \boxed{.}\ \boxed{6}$ | $\boxed{7.6}$ |
| Enter | $\boxed{+}$ | $\boxed{24.9}$ |
| Enter | $\boxed{\div}\ \boxed{3}$ | $\boxed{3.}$ |
| Enter | $\boxed{=}$ | $\boxed{8.3}$ |

**CLARIFICATION/EXAMPLE**

**Procedure for Calculating Exponent Values and Square Roots:**
**(e.g., $4^3$)**

| Entry | | Display |
|---|---|---|
| Enter | 4 | 4. |
| Press | $y^x$ | 4. |
| Enter | 3 | 3. |
| Press | = | 64. |

(e.g., $\sqrt{6.25}$ )

| Entry | | Display |
|---|---|---|
| Enter | 6 . 2 5 | 6.25 |
| Enter | $\sqrt{\ }$ | 2.5 |

## ORDER OF OPERATIONS

Students need to perform operations in correct sequence when working with multiple step calculations and problems. Illustrate the order of operations on a poster in the classroom, and provide students with a mnemonic device that will facilitate recall of the skills they develop. Remind students that exponentiation is a form of multiplication and should be performed prior to any addition or subtraction.

**CLARIFICATION/EXAMPLE**

**The "BODMAS" Mnemonic for Order of Operations**

| B | O | D | M | A | S |
|---|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| Brackets | Of | Divide | Multiply | Add | Subtract |

The problems that follow illustrate the importance of performing operations in the correct sequence. Although each problem may be solved through one set of key strokes on some calculators, students should be encouraged to document the process used and record intermediate solutions.

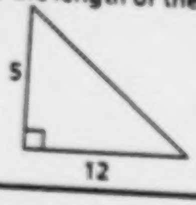
**CLARIFICATION/EXAMPLE**

1. Jane used her calculator to compute the number of calories in the snack she ate after school. She had a coke with 145 calories, 18 pretzel sticks with 4 calories each, and 7 potato chips with 12 calories each. She entered the following numbers on her calculator:

   $145 + 18 \times 4 + 7 \times 12 =$

   To her horror, the calculator showed 7908 calories! Jane knew that her allowance of calories for an entire day was only 2280 calories. What went wrong?

2. Calculate the length of the third side.



$$C^2 = 5^2 + 12^2$$
$$C = \sqrt{5^2 + 12^2}$$

Activities that may assist students to perform sequential operations on their calculators are provided in Resource 1: Using Brackets and Parentheses.

## AUDITORY CUEING

The wording of directions and problems that involve arithmetical operations is often a source of confusion for students. Variations in wording may cause students to enter numbers into the calculator in the wrong sequence. Be consistent with the wording/auditory cues you provide for students. Students should develop the ability to handle alternative auditory cues, but only after basic skills and procedures are understood.

**CLARIFICATION/EXAMPLE**

**Alternative Auditory Cues for Subtraction**

- subtract 3 from 6
- 6 minus 3
- take away 3 from 6
- 6 take away 3
- What is the difference between 6 and 3, or 3 and 6?

**Alternative Auditory Cues for Division**

- divide 6 by 3
- 6 divided by 3
- 3 divided by 6
- What is 6 over 3?

**Alternative Auditory Cues for Exponentiation**

- 4 squared
- 4 to the power of 2
- 4 to the second power
- the second power of 4

## ROUNDING RESULTS

Students need to learn how to interpret results of the computations they perform on the calculator. This is particularly true in division, where digits are displayed to the right of the decimal point when the answer is not a whole number. Encourage students to consider the context in which the computation was performed, and to determine appropriate procedures for rounding. (e.g., Should the answer be rounded to the correct tenth or hundredth?)

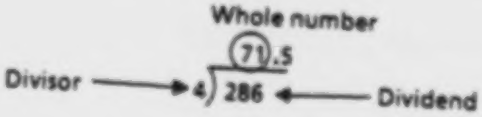
A strategy that may be effective in helping students to understand the process of rounding numbers is provided in Computational Facility and Estimation, "Developing Strategies for Estimation".

## FINDING REMAINDERS

Division may be a difficult operation when performed by hand or on the calculator. If the divisor is a factor of the dividend, the process is relatively simple on the calculator. If the divisor is not a factor of the dividend, however, students must recognize that the answer can be expressed as a decimal (which may require rounding) or as a whole number with remainder. As some problem situations require exact remainders to be known, students should be familiar with procedures for finding whole number remainders, using numbers displayed on the calculator.

### CLARIFICATION/EXAMPLE

**What is the whole number remainder for 286 ÷ 4?**



Step 1: Multiply the divisor by the whole number in the answer.

$4 \times 71 = 284$

Step 2: Subtract the result of step 1 from the dividend to obtain the remainder.

$286 - 284 = 2$ (remainder)

Students may benefit by initially documenting the process they use for finding whole number remainders in a table or chart. The table that follows provides a structure for the process, and enables students to record intermediate steps and solutions in an effective way.

CLARIFICATION/EXAMPLE

| Question | Calculate Answer | Step 1 Multiply Divisor by Whole Number | Step 2 Subtract Answer in Step 1 from Dividend | Remainder |
|---|---|---|---|---|
| $8\overline{)47}$ | | | | |
| $4\overline{)550}$ | | | | |
| $2\overline{)961}$ | | | | |
| $62\overline{)4217}$ | | | | |

## USING THE PERCENT KEY

There are two main types of calculators, each using a different logic system as its base. One is "algebra-based", and the other is "arithmetic-based". The calculators do not differ in appearance, but in how calculations are executed. While this duality of logic systems does not present a problem in most instances, it does affect how the percent function operates. Students should recognize that:

- *if* the calculator is algebra-based, pressing the percent key causes the number in the display to be divided by 100
- *if* the calculator is arithmetic-based, the percent key can only be used as a multiplier.

Ask students to determine which logic system their calculator uses by entering the number 25 and then pressing the percent key. If the display reads 0.25, the calculator is algebra-based. If the display does not change, reads 0 or E, the calculator is arithmetic-based.

CLARIFICATION/EXAMPLE

**Find 25% of 36.**

Key strokes on algebra-based calculator:

$\boxed{25}\ \boxed{\%}\ \boxed{\times}\ \boxed{36}$ or $\boxed{25}\ \boxed{\times}\ \boxed{36}\ \boxed{\%}$

Key strokes on arithmetic-based calculator:

$\boxed{36}\ \boxed{\times}\ \boxed{25}\ \boxed{\%}$

## USING THE MEMORY KEYS

Most hand-held calculators have a memory storage system. Although the steps to follow in using memory will vary according to the type of calculator being used, most memory systems are quite similar. Memory systems usually consist of four keys:

$\boxed{M+}$ $\boxed{M-}$ $\boxed{CM}$ $\boxed{RM}$

The M+ key is used to place the contents of the display into memory, or to add the contents of the memory to the display. The M− key is used to subtract the contents of the display from memory, or to subtract the contents of memory from the display. The CM key (or MC key on some calculators) is used to clear the memory. Remember always to clear the memory before starting a new set of calculations. The RM key (or MR key on some calculators) denotes memory recall and will display the contents of the memory.

The following examples illustrate use of the four memory keys.

**CLARIFICATION/EXAMPLE**

$(12\times 10) + (2\times 8) =$

| Entry | Display |
|---|---|
| CM | 0. |
| 12 | 12. |
| × | 12. |
| 10 | 10. |
| = | 120. |
| $M^+$ | 120. |
| 2 | 2. |
| × | 2. |
| 8 | 8. |
| = | 16. |
| $M^+$ | 16. |
| RM | 136. |

**CLARIFICATION/EXAMPLE**

$(12\times 10) - (2\times 8) =$

| Entry | Display |
|---|---|
| CM | 0. |
| 12 | 12. |
| × | 12. |
| 10 | 10. |
| = | 120. |
| $M^+$ | 120. |
| 2 | 2. |
| × | 2. |
| 8 | 8. |
| = | 16. |
| $M^-$ | 16. |
| RM | 104. |

Model appropriate strategies for using the memory key when solving problems that involve several steps and operations. Frequent practice will enable students to develop confidence in using the calculator's memory.

## CHECKING THE REASONABLENESS OF RESULTS

Encourage students to anticipate and verify the results they obtain on their calculators, and to monitor their own work through sub-vocal rehearsal and questioning.

e.g.,
- Is my answer reasonable?
- Within what range of numbers must my answer lie?

Model strategies that will enable students to estimate the results of computations they are about to perform on the calculator. A variety of effective estimation strategies are provided in the "Computational Facility and Estimation" section of this document.

### CLARIFICATION/EXAMPLE

| Question | Estimated Answer | Calculated Answer |
|---|---|---|
| 48 + 50 =<br>\$10.00 − \$1.96 =<br>$7^3 =$<br>37 ÷ 7 =<br>26 × 9 =<br>$\sqrt{30} =$<br>47 + 80 =<br>\$3.01 ÷ \$0.98 = | | |

**CALCULATOR GAME OF 'ESTIMATION' (2 players)**

Two players each have 30 seconds to estimate a product or quotient of two-, three- or four-digit numbers. The player with the closer estimate wins a point. The first player with ten points wins the game.

## DEVELOPING CONFIDENCE AND SPEED

Confidence and skill in performing computational procedures on the calculator will increase as students gain experience in using the calculator in a variety of contexts. Provide opportunities for students to develop and refine calculator skills in situations that involve:

- drill and practice
- timed challenges
- problem solving
- puzzles and games.

Sample puzzles and games that may be useful in developing calculator skills are provided in Resource 2: Calculator Puzzles and Games.

**CLARIFICATION/EXAMPLE**

### CALCULATOR GAME OF '21' (2 players)

On each turn a player can push only one of the three keys $\boxed{1}$, $\boxed{2}$ or $\boxed{3}$ along with the $\boxed{+}$ key. The first player to reach 21 is the winner. Ask students to determine the winning strategy.

### CALCULATOR GAME OF '50' (2 players)

This game is similar to the game of '21'. Players may use the $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$ and $\boxed{6}$ keys. The first player to reach 50 is the winner.

# THE COMPUTER

Current demands of society place emphasis not only on the ability to communicate through the written word, but also on the skills of interacting with computer technology. The mathematics program must provide opportunities for students to demonstrate an understanding of:

- the basic operation of computers, by
  - identifying major parts of a computer
  - distinguishing between hardware and software
  - recognizing that computers get their instructions from a program written to accomplish a specific task.

- the basic capabilities, applications and limitations of computers, by
  - recognizing that computers are best suited to tasks requiring speed, accuracy, repeated operations and the processing of large amounts of data
  - identifying major areas in society where computers are used, and the tasks performed by computers in these areas
  - recognizing tasks that computers cannot accomplish.

To the extent that facilities and equipment are available, the mathematics program should also provide opportunities for students to:

- work independently with prepared software
- enter, run and modify simple programs that have been written for particular purposes.

This section of the manual describes two "user friendly" programming languages that will enable both student and teacher to interact with the computer without relying solely upon commercially prepared software. Commands and procedures provided on the pages that follow will enable students to enter, run and/or modify simple programs in:

- BASIC computer language
- LOGO computer language.

BASIC computer language has been selected because most microcomputers use this language. The use of BASIC may vary slightly in response to the type of computer being used. The programs described throughout this manual refer to the BASIC programming language used on an Apple computer.

LOGO computer language has been selected because of its simplicity and graphic capabilities. Different versions of LOGO are also available. The version described in this manual is Terrapin LOGO.

Although neither BASIC or LOGO require extensive programming knowledge, the degree to which they are used within the mathematics program will depend upon student interests/abilities and available resources. Teachers are encouraged to adapt and modify simple programs as required in meeting the needs of individual students and/or classes.

## BASIC COMPUTER LANGUAGE

Most microcomputers are capable of interpreting BASIC language commands. A disc is not needed to boot the computer for BASIC as the computer will automatically interpret the commands. A disc is only needed if the program is to be saved. When using the computer without a disc in the drive, the drive will "whir" for a time before the cursor appears. To stop the whirring on your own, press "control reset". The cursor should appear in the lower left hand corner.

A list of common commands used in BASIC programming is provided below.

| Key Word | Description |
|---|---|
| DATA | DATA 45, 67, 35<br>Holds the data for use by READ statement. |
| END | Terminates the program. |
| FOR | FOR X = 1 to 10<br>Substitutes the numbers from 1 to 10 into the program. Causes a LOOP to occur. |
| GOTO | GOTO 50<br>Tells the computer to go to line 50 of the program. |
| HOME | Clears the screen and sends the cursor to the upper left-hand side of the screen. |
| IF . . . THEN | IF X > 1 THEN 90<br>The computer makes a decision as to the truth of a statement and if true, in this example, will proceed to line 90. If the condition is not true it will go to the next line. |
| INPUT | INPUT :N$<br>Calls for input from the keyboard during program execution. |
| INT | PRINT INT (23.6)<br>Will give the largest integer less than or equal to the value given. In this case the computer would print 23. |
| LET | LET X = 10<br>Assigns a value to a variable. |
| NEXT | NEXT X<br>Closes a FOR . . . NEXT loop. |
| PRINT | PRINT "HELLO"<br>The computer will print the expression in the quotation marks. |

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| Key Word | Description |
|---|---|
| READ | READ X, Y, Z<br>Reads values from a DATA statement. |
| REM | REM - - "THIS IS A COMPUTER PROGRAM"<br>The REM command allows the programmer to insert remarks for explanation into the program. |
| STEP | FOR X = 0 to 100 STEP 10<br>The computer will substitute the numbers from 0 to 100, counting in increments of 10. |

The arithmetic commands in BASIC include:

$\boxed{+}$ – addition
$\boxed{-}$ – subtraction
$\boxed{*}$ – multiplication
$\boxed{/}$ – division
$\boxed{\wedge}$ – exponentiation

Additional commands that may be needed to program in BASIC include:

| Command | Execution |
|---|---|
| LIST | Displays the entire program contained in the computer's memory. If the program is long it may be necessary to display only part of it at a time. This can be accomplished by listing the lines to be displayed. For example, LIST 10 – 100 would display all lines from 10 to 100. |
| LOAD | To retrieve a program from a disc, type LOAD and the name of the program. |
| NEW | Before entering a program, type NEW to clear the computer memory. |
| RUN | After a program has been entered type RUN to have the computer execute the program. |
| SAVE | If the program is to be saved on a disc, type SAVE and the program name. |

When programming in BASIC, one must realize that the computer executes commands in order of their line number. It is important that all lines be numbered, and that they are numbered sequentially according to the order of the commands.

## CLARIFICATION/EXAMPLE

**A BASIC program for finding the factors of a number:**

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE: "
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END
```

Notice that the line numbers are listed in increments of ten. This allows the programmer to insert a new line without having to re-type the rest of the program. For example, if the programmer wished to insert a new line between 10 and 20 it could be entered as line 15 and the computer will automatically execute the new line in its numerical order.

Here is an explanation of the program.

| Line Number | Execution |
|---|---|
| 10 | The REM command has allowed the programmer to state the purpose of the program. Note that the REM statement has no effect on the running of the program. It is only there for explanatory purposes. |
| 20 | When the program is run the statement "WHAT IS THE NUMBER" will be printed. |
| 30 | INPUT N calls for the operator to input the number to be factored. |

**CLARIFICATION/EXAMPLE**

| Line Number (continued) | Execution (continued) |
|---|---|
| 40 | After the operator inputs the number, the computer will print the statement: "THE FACTORS ARE". Note that the statement to be printed is enclosed in quotation marks. |
| 50 | FOR K = 1 to N tells the computer to test the numbers from 1 to N (the number specified in INPUT) in increments of one. Notice that the STEP statement does not need to be included if the step is one. |
| 60 | The LET statement defines the variable X as the quotient of N ÷ K. |
| 70 | The LET statement here defines the variable Y as the integer value of N ÷ K. The integer value is the greatest integer not larger than N ÷ K. |
| 80 | The conditional statement IF . . . THEN tests a condition and begins a loop. In this case the computer is determining if X = Y. That is, when N is divided by K, does it result in an integer (no remainder)? If the statement is true then K will be printed. |
| 90 | NEXT K tells the computer to test the next value of K. Remember, the computer will start testing at 1 and end at N. |
| 100 | When the computer reaches this line number it will cease executing the program. |

Run the previous program to be sure that it works. A sample run would be:

Type RUN

The screen should read:

```
WHAT IS THE NUMBER
```

Enter a number (e.g., 36). The computer screen will now look like this:

```
THE FACTORS ARE:
  1
  2
  3
  4
  6
  9
 12
 18
 36
```

Once a program has been run, the teacher may wish to make revisions to the program. Revisions may often involve adding additional lines and commands to the original program.

**CLARIFICATION/EXAMPLE**

**A revised BASIC program for finding the factors of a number:**

```
 10  REM FIND ALL FACTORS OF A NUMBER
 20  PRINT "WHAT IS THE NUMBER"
 30  INPUT N
 40  PRINT "THE FACTORS ARE: "
 50  FOR K = 1 TO N
 60  LET X = N/K
 70  LET Y = INT (N/K)
 80  IF X = Y THEN PRINT K
 90  NEXT K
100  END

 25  PRINT "YOU WISH TO FACTOR"
 95  PRINT "DO YOU HAVE ANOTHER NUMBER"
 96  INPUT R$
 97  IF R$ = "YES" THEN 20
 98  PRINT "OK! GOOD BYE FOR NOW."
```

Line 25 will be inserted between lines 20 and 30. It will cause the computer to print YOU WISH TO FACTOR. This insertion only serves to clarify the statement WHAT IS THE NUMBER. The entire statement could not be placed on line 20 because there are only 27 character spaces left on the line after the print statement.

Line 95 is the first of three lines that allow the program to be repeated. When the program is run after the initial list of factors, the computer will ask if another number is wanted.

Line 96 calls for input from the operator. Note the form of the input, R$. This is called a string variable. It prepares the computer for input which may be alphabetic or numeric, and which may be several characters in length. If the input is to be a word or letter, a string variable must be used.

Line 97 is the conditional statement. If the input from the operator is YES (not just Y), the computer will go back to line 20 and begin the factoring process again. If the input is anything other than YES the computer will go on to the next line.

Line 98 causes the computer to print "OK! GOOD BYE FOR NOW.".

The examples and explanations provided on BASIC language are fairly simple and include only those commands that a novice may use. Teachers may wish to consult other programming texts for additional commands and strategies that are used when programming in BASIC.

## LOGO COMPUTER LANGUAGE

The LOGO computer language is an interpretive language in that commands may be executed immediately, or can be stored for use in "procedures". A procedure is a group of one or more instructions and commands that the computer stores for future use. Once a procedure has been defined, it may be used as a part of other procedures.

There are many facets to the LOGO language, but perhaps the most appealing characteristic of LOGO is its graphic capability. Very little knowledge of the language is required prior to using the graphics mode of LOGO. This section will highlight some of the graphics commands to be used in Terrapin LOGO. Be prepared to make adjustments to these commands if using another version of LOGO (e.g., LOGO writer).

To enter the graphics mode, type DRAW. The LOGO turtle, Δ, should now appear in the middle of the screen. This is the turtle's HOME position. Once you are in the draw mode you can command the turtle to perform various moves. Some of the more commonly used graphics commands are provided below.

| Graphics Command | Execution |
|---|---|
| HOME | This command places the turtle in his home position in the centre of the screen. |
| FD ______* | This command followed by a number will move the turtle forward that number of turtle steps. For example, FD 100 would move the turtle forward 100 steps. |
| BK ______* | This command followed by a number will move the turtle backwards by that number of turtle steps. |
| LT ______* | This command followed by a number will designate a left turn by that number of degrees. For example, LT 45 will turn the turtle to the left 45 degrees. |
| RT ______* | This command will turn the turtle by the desired number of degrees to the right. |
| PU | Means "pen-up", and ensures that the turtle's path that is executed after the PU command will not show. |
| PD | Means "pen-down". If a PU command has been used, it is necessary to type PD prior to any further commands to make the turtle's path visible. |
| CS | Means clear screen. This command erases all previous paths from the screen. |

* Do not type a blank. The blank lines represent a need for numerical input.

To access the text mode of LOGO for programming, simply type TO and the name of your procedure. LOGO will automatically switch into the text mode.

The edit commands listed below will be useful in defining procedures and writing simple programs.

| Edit Command | Result |
|---|---|
| CONTROL D (CTRL D) | Deletes character under the cursor. |
| CTRL P | Moves the cursor up one line. |
| CTRL N | Moves the cursor down one line. |
| CTRL E | Moves cursor to the end of the line. |
| CTRL G | Stops the procedure . |
| CTRL C | Defines the procedure. |

CLARIFICATION/EXAMPLE

**A sample procedure to be executed in LOGO:**

```
TO   SQUARE  :S
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
FD   :S
RT   90
END
```

Explanation:

TO SQUARE  :S

The TO tells the computer that you are going to define a procedure which you are naming SQUARE. Procedure names can only be one word. If it is necessary to have more than a one word description, the words may be separated with periods (e.g., TO SQUARE.MAZE). The :S indicates to the computer that there will be a variable in the procedure. In this case the variable is the length of the side.

The directions that follow instruct the turtle to move forward a specified number of steps, followed each time by a 90° right turn. Note that the final RT 90 is not necessary for drawing the square, but it is good programming practice to have the turtle end in the home position facing forward.

To run the procedure simply type SQUARE and a length of a side (e.g., SQUARE 10). The computer should now draw a square whose sides are 10 turtle steps in length.

The procedure for square could also be written as:

```
TO SQUARE :S
REPEAT 4 [FD :S  RT 90]
END
```

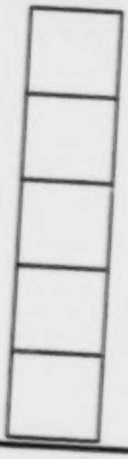
Defining the procedure this way makes use of the REPEAT command and shortens the program.

Once a procedure has been defined in LOGO, it may be used in other procedures.

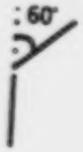
**CLARIFICATION/EXAMPLE**

```
TO STACK.SQUARE :N :S
REPEAT :N [SQUARE :S FD :S]
END
```

If STACK.SQUARE 5 20 is entered, the computer will draw five squares whose side length is 20 turtle steps.



When drawing polygons with LOGO, encourage students to use the "total trip theorem". This theorem states that the turtle must turn through a total of 360 degrees in order to produce a closed figure. Instructions that cause the turtle to turn by a desired number of degrees (i.e., draw an angle) must therefore be stated in terms of the supplement of each angle in the polygon. The command "FD 10 RT 60 FD 10" would cause the turtle to draw a 120 degree angle in the manner illustrated below:



The "total trip theorem" will enable students to construct both acute-angled and obtuse-angled polygons. Right-angled polygons are more easily constructed as the command "RT 90" will produce an angle of 90 degrees (i.e., the supplement of 90 degrees is 90 degrees).

## CLARIFICATION/EXAMPLES

**A LOGO Program for Producing a Regular Pentagon**

A pentagon has five angles and the turtle must turn through 360°. Therefore, each turn must be $\frac{360°}{5} = 72°$.

```
TO PENTAGON  :S
REPEAT 5[FD :S RT 72]
END
```

After entering a procedure be sure to press CTRL C to have the computer put the definition into its memory. If you wish to save a procedure on a disc, type SAVE "FILE NAME". To read from the disc, type READ"FILE NAME".

This brief description of LOGO is intended to enable teacher and student to begin using the graphics and text modes of LOGO. Only the graphic capabilities of LOGO have been discussed. The arithmetic capabilities of the language could be investigated through the use of a LOGO manual.

## RESOURCE 1: USING BRACKETS AND PARENTHESES

A. Calculate 165 + (168 × 337)

First multiply: (168 × 337) = ______________

Then add: 165 + 56 616 = ______________

Here is the sentence to use. 168 × 337 + 165 = ______________

B. Rewrite the following so you can use your calculator. Then do the question.

| | Calculator Sentence | Answer |
|---|---|---|
| 1. 2411 + (3768 × 72) | ______________ | ______________ |
| 2. 180 164 × (165 + 28) | ______________ | ______________ |
| 3. 273 × (8927 − 7373) | ______________ | ______________ |

If there is more than one set of parentheses, sometimes square brackets are used.

$$[41 + \underset{\substack{\uparrow\\ \text{Next}}}{} (43 \underset{\substack{\uparrow\\ \text{Do first}}}{\times} 44)] \div 40$$

Then do the rest of the work.

C. Complete the following:

| | Calculator Sentence | Answer |
|---|---|---|
| 1. [36 × (71 + 32)] − 5 = | ______________ | ______________ |
| 2. [57 + (472 − 150)] ÷ 8 = | ______________ | ______________ |
| 3. 586 + [35 + (483 ÷ 7)] = | ______________ | ______________ |

# RESOURCE 2: CALCULATOR GAMES AND PUZZLES

A. Your calculator can spell for you. Certain numbers on your calculator, when turned upside down, look like letters.

1. Find out which numbers produce the following letters:

O_ I_ E_ S_ L_ B_ G_ Z_

2. What word does each of the following numbers spell?

77 345 _____ 0.7734 _____

3504 _____ 3080 _____

3. Solve each of the following to find the word:

- a telephone greeting: (1 − .30394) ÷ 9 = ____ (hello)
- what a snake does: 471 × 265 + 410 699 = ____ (hisses)
- what water does at 100°C: 305 644 ÷ 43 = ____ (boil)
- where a turtle lives: 11 345 = ____ (shell)
- the name of a book: 4202 × .5 × 18 = ____ (bible)
- how math is for you: 16 × 3 × 5 × 83 × 277 = _____ (oh bliss)

4. Solve these expressions in order to answer the questions:

- What did Amelia Earhart's father say the first time he saw her fly an airplane?

  (.023 × 3 + 10 141) × 5 = ____ (she solos)

- How do fish breathe?

  5787 ÷ 3 × 4 = ____ (gill)

- What is a pit?

  59 × 59 + 223 = ____ (hole)

5. Create other words and phrases that are the results of pre-planned calculations.

B. Guess the Number.

1. Ask a friend to enter any number (not more than six digits) on the calculator and then multiply by 2; add 4; multiply by 5; add 12; multiply by 10; press the "=" key.

2. Take the calculator back from your friend and subtract 320. The result will end in one or more zeros. Drop the zeros, and you'll have the number your friend started with.

Example: Choose 65219

- 65 219 × 2 + 4 × 5 + 12 × 10 = 6 522 220.
- 6 522 220 − 320 = 6 521 900; discard the two trailing zeros, and you have 65 219, the number your friend started with.

C. Sum of 1089.

1. Ask a friend to select any 3-digit number. (The first and third digits cannot be the same.)

2. Ask your friend to:
   - reverse the digits
   - find the difference between the two numbers (larger minus smaller).

3. Reverse the digits of the difference. Then find the sum of these two numbers. The answer will always be 1089.

   Example: Choose 155

   - Reverse the digits : 551
   - Find the difference : 551 − 155 = 396
   - Reverse the digits : 693
   - Find the sum : 396 + 693 = 1089

D. Your Birthday and Age.

1. Hand someone your calculator and ask them to enter the number representing the month of their birthday; multiply it by 100; add their date of birth; multiply by 2; add 9; multiply by 5; add 8; multiply by 10; subtract 422; add their age; then press the "=" key.

2. Take the calculator back and subtract 108. The result will be a five- or six-digit number. The first digit(s) tells the month of birth; the next two the date of birth; and the last two, the age of your friend. Your friend will be impressed.

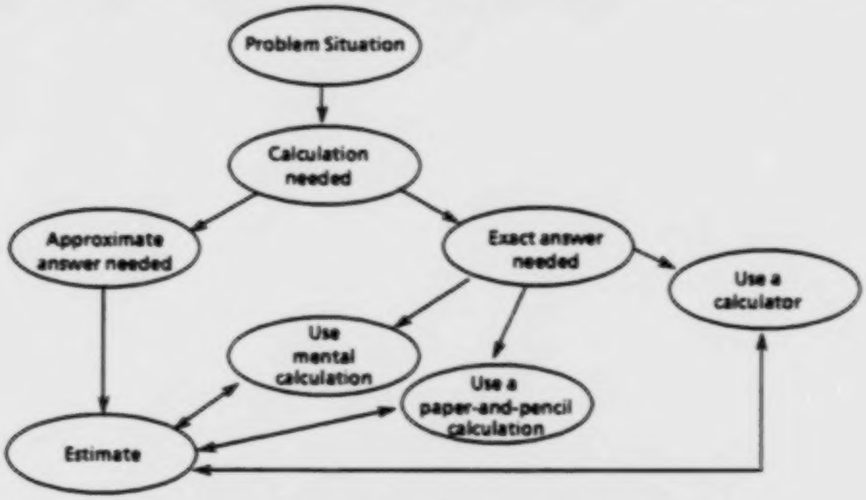
Example: Your friend is 17 years old and was born on March 13.

- 3 × 100 + 13 × 2 + 9 × 5 + 8 × 10 − 422 + 17 = 31 425.
- Take 31 425 − 108 = 31 317 or 3/13/15.

Your friend was born in the third month (March), on the 13th day, and is 17 years old.

# COMPUTATIONAL FACILITY AND ESTIMATION

The availability and use of technology has caused emphasis in the skills required for computational competence to change over the last decade. Computational facility includes more than the knowledge and skills required to perform paper-and-pencil computation with standard algorithms. While these skills are important, responsible participation at home and work also requires facility in performing **mental arithmetic**, in the application of **calculator skills**, and in applying strategies of **estimation**. Surveys show that mental computation and estimation are used in more than 80 percent of all practical problem-solving situations encountered outside the classroom. As important as each method of computation is in itself, even more important is the **understanding of when each strategy is most appropriately used** in everyday problem-solving situations.



Making Decisions About Computational Procedures
in Problem-Solving Situations[1]

Suggestions and ideas for developing student ability to compute by a variety of methods have been included in this section of the manual, and focus attention on:

- Developing Computational Process
    - Understanding Place Value
    - Using Manipulatives.
- Developing Mental Arithmetic Skills
    - Applying Basic Number Skills
    - Using Formal Strategies.
- Developing Strategies for Estimation
    - Front-End Estimation
    - Rounding
    - Compatible Numbers
    - Clustering (or Averaging).

Suggestions for developing calculator skills are included in another section of this manual (see "Use of Technology").

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[1] Adapted with permission from *Curriculum and Evaluation Standards for School Mathematics*. Copyright 1989 by the National Council of Teachers of Mathematics.

## DEVELOPING COMPUTATIONAL PROCESS

Instruction in paper-and-pencil computation should emphasize place value and basic number skills prior to the application of algorithms, and should de-emphasize calculation with large numbers. (Algorithmic procedures for addition and subtraction should generally involve numbers with no more than four digits. Algorithmic procedures for multiplication and division should involve numbers containing up to four digits, using multipliers/divisors of no more than two digits.) Concrete and visual materials will assist in concept development. Students who have not experienced past success with standard computational procedures may react more favourably and experience more success with less sophisticated forms of algorithms.

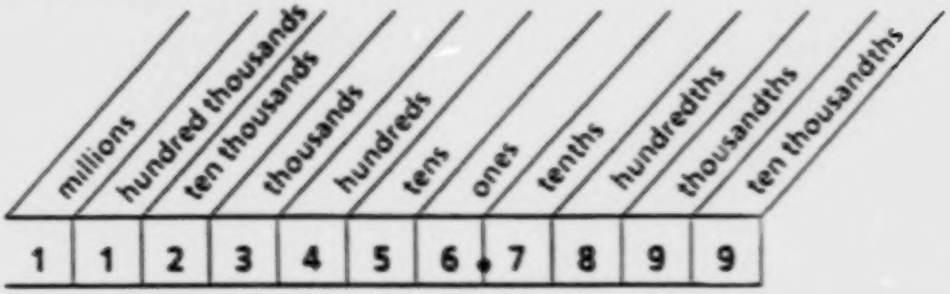
### UNDERSTANDING PLACE VALUE

An understanding of place value is critical to the development of computational process. Develop and reinforce whole number and decimal place value through discussion, questioning and a variety of concrete/visual experiences. Teachers may find it beneficial to:

- display a large place value chart in the classroom that students can refer to as necessary

**CLARIFICATION /EXAMPLE**

Place Value Chart



- design activities that require students to apply their knowledge of place value to computations performed on the calculator.

**CLARIFICATION/EXAMPLE**

- Enter the number 21.455 into a calculator.
- Suggest an amount that can be added/subtracted to this number in order to obtain the following results:

  26.455 (+5)
  21.405 (−0.05)
  61.455 (+40)
  21.055 (−0.4).

- Verify each idea by performing the necessary operation on the calculator.

## USING MANIPULATIVES

Manipulative materials provide students with a concrete base upon which to build concepts and skills. Students who experience difficulty with arithmetical process should be given opportunities to develop an understanding of number operations and relationships through tactile and visual learning activities.

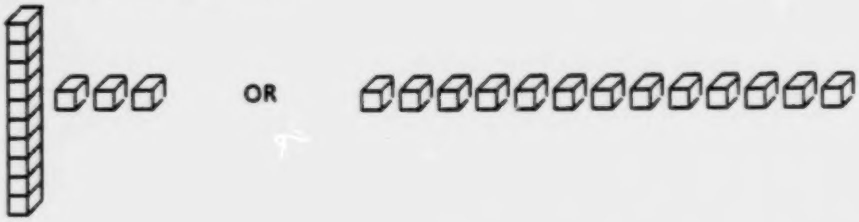
Manipulatives are particularly useful in clarifying the division process. Encourage students to recognize division as a "sharing process", and guide students in the use of appropriate thought processes by asking relevant chains of questions.

e.g.,
- How many are to be shared?
- How many are doing the sharing?
- Can we share so that everyone gets a ten? a hundred?
- What is the greatest number each will have when the sharing is done?
- Are there any left over?
- How can we determine the greatest number that can be shared?

### CLARIFICATION/EXAMPLE

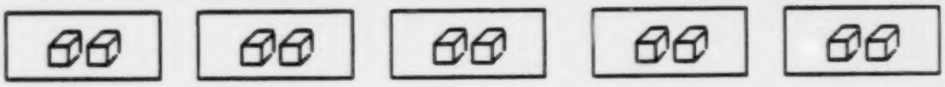
13 ÷ 5 = ?

- How many are to be shared?

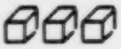


- How many are doing the sharing?

- What is the greatest number that each will have when the sharing is done?



- Are there any left over?

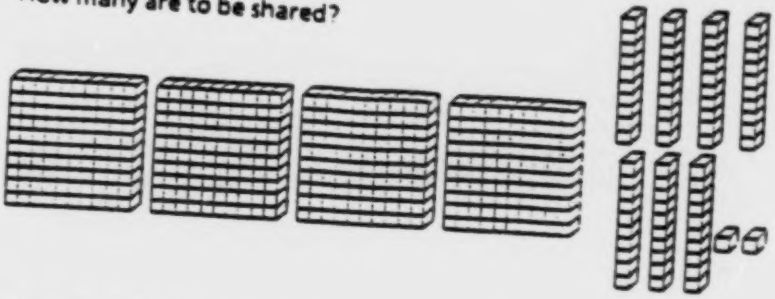


Therefore, 13 ÷ 5 = 2 R 3.

## CLARIFICATION/EXAMPLE

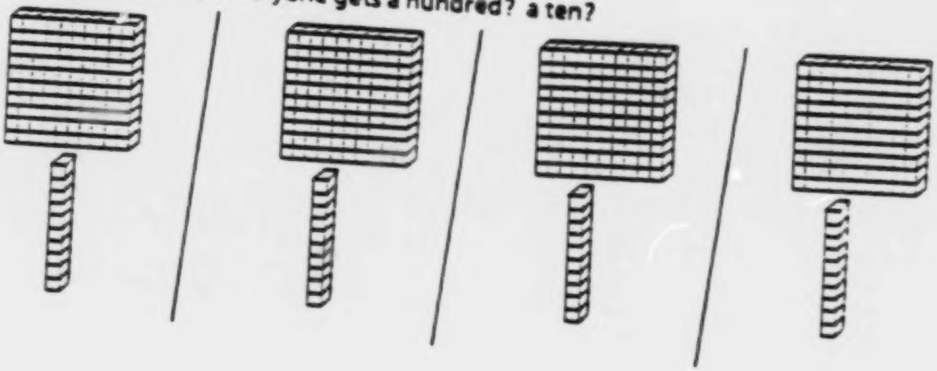
**472 ÷ 4 = ?**

- How many are to be shared?

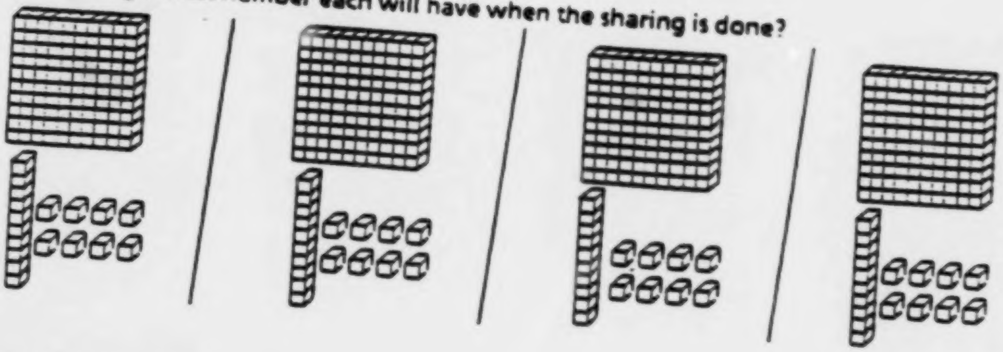


- How many are doing the sharing?

- Can we share so that everyone gets a hundred? a ten?



- What is the greatest number each will have when the sharing is done?



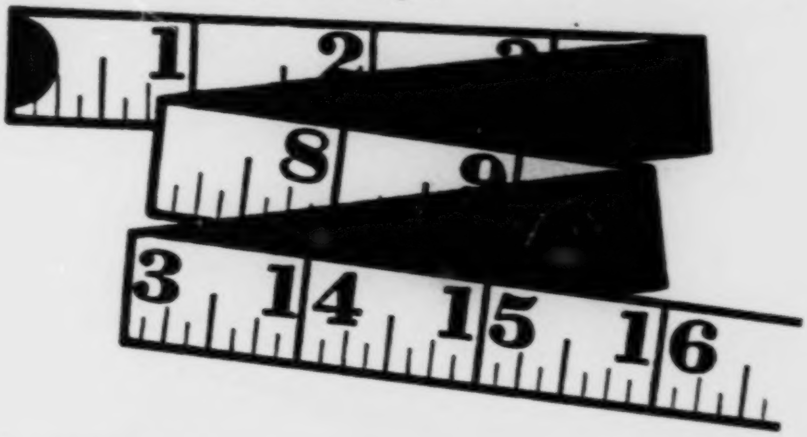
**Therefore, 472 ÷ 4 = 118 R 0.**

Students may benefit from representation of the division process in a variety of forms. The tape measure provides an alternative method of demonstrating the division process.

**CLARIFICATION/EXAMPLE**

**16 ÷ 3 = ?**

Locate the first 16 units on the tape measure. Starting at 16, fold this portion of the tape measure into sections that are 3 units in length.



The number of folded sections will represent the quotient. The number of units left beyond the last fold will represent the remainder.

Therefore, 16 ÷ 3 = 5 R 1.

Some students may experience difficulty in understanding the traditional algorithm for division. If students find other algorithms more meaningful, encourage their use.

**CLARIFICATION/EXAMPLE**

**Alternative Division Algorithms:**

```
   ______
4 )  978
     400     100
     ---
     578
     400     100
     ---
     178
     160      40
     ---
      18
      12       3
      --
       6
       4       1
       -     ---
       2     244
```

```
   ______
4 )  978
     800     200
     ---
     178
     160      40
     ---
      18
      16       4
      --     ---
       2     244
```

```
           4
          40
         200
   _________
4 )      978
         800
         ---
         178
         160
         ---
          18
          16
          --
           2
```

```
     244
   ______
4 )978
   8
   -
   17
   16
   --
    18
    16
    --
     2
```

## DEVELOPING MENTAL ARITHMETIC SKILLS

Mental arithmetic and estimation have a reciprocal relationship in that the ability to perform simple mental calculations is basic to refining one's estimation skills, and some methods of estimation provide a framework for performing mental arithmetic. Nevertheless, two distinguishing characteristics of mental arithmetic are that it:

- produces an exact answer
- is performed mentally, without the aid of external devices such as paper and pencil.

Teachers can assist students to develop effective mental arithmetic skills throughout the mathematics program by:

- devoting time each day to the development of strategies for mental arithmetic. Plan to teach specific strategies and offer opportunities for practice on a regular basis
- making a list of mental arithmetic strategies that you consider are most appropriate for your students. Encourage discussion and sharing of these strategies
- asking students to "think aloud" and to share the strategies that they already use. In addition to helping students to focus on their own strategies, this process may provide other students with new strategies for performing mental computation
- including the use of whole numbers, decimals, fractions and percent in mental arithmetic activities. Develop strategies for performing various operations with these numbers
- providing frequent opportunities for students to practise mental arithmetic through oral activities. Emphasis on written computational work often tends to discourage the application of skills in mental computation
- developing an evaluation plan that reflects instructional commitment to mental arithmetic. Regular assessment of mental arithmetic skills will focus students' attention on the importance of these skills.

## APPLYING BASIC NUMBER SKILLS

Teachers can provide opportunities at the beginning or end of each class for students to apply basic number skills. Questions should be answered without the use of paper and pencil, and may involve:

- single digit products and dividends (e.g., $7 \times 9$; $63 \div 7$)
- "extended" products and dividends (e.g., $7 \times 90$; $630 \div 7$)
- sequences of operations (e.g., $5 \times 6 + 5 - 7$)
- multiplication and division by powers of ten.

Activities that may assist students to apply basic number skills include:

- the number game "Krypto". This game can be played for short periods of time, and reinforces single digit number skills and sequences of operations.
- "mad-minute" drills. Timed challenges that become part of the daily routine will encourage recall of single digit products and quotients, as well as foster a habit for their use.
- the use of "graph-paper arrays" or "algebra tiles". Students may benefit from constructing arrays that illustrate the factors for "hard to remember" numbers.
- the use of computer programs. A variety of computer programs that reinforce basic number skills are available (e.g., Fast Facts).

Encourage students to "extend" application of their basic number skills to related situations. This approach lends variation to practice sessions, and further develops number sense.

**CLARIFICATION/EXAMPLE**

- Extended Number Facts for "2+3":
  - 20+30
  - 20 000+30 000
  - 0.2+0.3.
- Extended Number Facts for "6×7":
  - 6×70
  - 60×70
  - 0.6×700.

Develop strategies for "doubling" and "halving" and demonstrate the application of these strategies in a variety of practical situations involving mental computation.

**CLARIFICATION/EXAMPLE**

Use a doubling strategy to recall a forgotten multiplication fact, such as 12×7.

Step One: Take half of 12 and multiply by 7.
e.g., 6×7=42

Step Two: Double the result.
e.g., 42×2=84

Therefore, 12×7=84.

Factoring is often an effective method of simplifying multiplication, and can assist students to find answers through the process of mental arithmetic.

**CLARIFICATION/EXAMPLE**

24×15=?

Step One: Factor the numbers.

24 X 15
= (4 X 6) X (5 X 3)

Step Two: Rearrange the factors.

(4 X 6) X (5 X 3)
= (4 X 5) X (6 X 3)

Step Three: Apply basic facts.

(4 X 5) X (6 X 3)
= 20 X 18
360

Model and discuss the use of appropriate shortcuts for multiplying by 5, 25 and 50.

e.g.,
- to multiply a number by 5, multiply by 10 and divide by 2
- to multiply a number by 25, multiply by 100 and divide by 4
- to multiply a number by 50, multiply by 100 and divide by 2.

Encourage students to identify and share other shortcuts they presently use when performing mental arithmetic.

## USING FORMAL STRATEGIES

Two strategies that are frequently used in performing calculations with mental arithmetic include:

- computing from left to right
- compensation.

Model and discuss the use of these strategies on a regular basis. Prompt students to use these and other effective strategies in their daily work.

### CLARIFICATION/EXAMPLE

**STRATEGIES FOR ADDING WITH MENTAL ARITHMETIC**

**LEFT TO RIGHT**

④8 + ③5

1. Add the tens. — 40 and 30 is 70
2. Add the ones. — 8 and 5 is 13
3. Add the two sums. — 70 + 13 = 83

㊽ + ③5

1. Look at the whole first number. Add to it the tens in the second number. — 48 and 30 is 78
2. Add the ones in the second number to the sum. — 78 + 5 = 83

**COMPENSATION**

㊽$^{+2}$ + ㉟$^{-2}$

1. Add a number to one addend to make it a multiple of ten. — 48 and 2 is 50
2. Subtract the same number from the other addend. — 35 − 2 is 33
3. Add the two numbers. — 50 + 33 = 83

㊽ + 19 (for use in adding 19, 29, etc.)

1. To the first number add 20 (the next higher multiple of ten). — 48 and 20 is 68
2. Subtract 1 from the sum. — 68 − 1 = 67

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**CLARIFICATION/EXAMPLE**

**STRATEGIES FOR SUBTRACTING WITH MENTAL ARITHMETIC**

**LEFT TO RIGHT**

$\boxed{62} - \boxed{2}5$

1. From the first number subtract the tens in the second number.

   62 minus 20 is 42

2. Now subtract the ones in the second number.

   42 – 5 = 37

**COMPENSATION**

$\overset{+5}{\boxed{62}} - \overset{+5}{\boxed{25}}$

1. Add a number to the minuend to make it a multiple of ten.

   25 and 5 is 30

2. Add the same number to the subtrahend.

   62 + 5 = 67

3. Subtract the two numbers.

   67 – 30 = 37

**CLARIFICATION/EXAMPLE**

**STRATEGIES FOR MULTIPLYING WITH MENTAL ARITHMETIC**

**LEFT TO RIGHT**

$\boxed{6 \times 72}$

1. Multiply the tens in the second factor by the first factor. 6 × 70 = 420
2. Multiply the ones in the second factor by the first factor. 6 × 2 = 12
3. Add the two products. 6 × 72 = 432

**COMPENSATION**

$\boxed{6 \times 39}$

1. Round the second factor to the next higher ten. 39 → 40
2. Multiply the tens by the first factor. 6 × 40 = 240
3. Multiply the difference by the first factor. 40 – 39 = 1<br>6 × 1 = 6
4. Subtract the second product from the first one. 240 – 6 = 234

Additional strategies and classroom activities that will assist students to compute using mental arithmetic are provided in *Mental Math in Junior High*.

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# DEVELOPING STRATEGIES FOR ESTIMATION

Skill in estimation is necessary for effective problem solving and calculator use. Students need to be able to determine if a particular result is precise enough for the purpose at hand, and be alert to the reasonableness of computational results. More often than not, practical problems involve estimations rather than exact numbers. Estimation, not computation, gives answers to everyday questions like "Do I have enough cash to buy groceries?" and "How many hot dogs should I order for the party?".

Thinking skills and problem solving assume important roles in developing estimation skills. It should be recognized that the process of estimation:

- is performed mentally, usually without paper and pencil
- is done quickly
- produces answers that are not exact, but adequate for making necessary decisions
- reflects individual strategies and produces a variety of estimates as answers.

Students should practise estimation in a variety of ongoing activities involving whole numbers, decimals, fractions and percentages. Evaluation should include assessment of students' ability to estimate in a variety of practical situations. The guidelines that follow will assist in creating a classroom climate that establishes the usefulness of estimation and builds student confidence in making estimates.

- Introduce estimation by referring to situations where estimated or rounded counts are used in everyday life. Extend this activity by having students identify numbers used in the newspaper that are exact amounts and those that are estimates:
  - "Over 1500 Fans View Final Home Game"
  - "$200 000 Spent on Recreational Centre"
  - "2/3 of the Athletes Use Brand X".
- Identify other situations where only an estimate is required. Estimation is useful both as a check on the reasonableness of a computed answer and as an end in itself. Many daily situations require only an estimate.
- Use easy examples in early stages of instruction, and avoid emphasis on a high degree of precision in estimates. Students need to be convinced that estimation is easy and develop a desire to use it.
- Focus attention on the language of estimation. By using appropriate phrases, teachers can communicate the spirit of estimation and help students to understand it.
  e.g., – about 12 1/2
  - close to 9
  - just about 15
  - a little less than 3.5
  - between 8 and 9, but probably closer to 8
  - somewhere between 30 and 40.
- Accept a variety of answers. Students need to understand that there is no one "correct" estimate. Any estimate that is reasonably close to the exact answer is valid. If multiple responses are encouraged, students will learn more about the process of estimation.
- Use oral work and group discussion. In the early stages of instruction, students will be tempted to compute the exact answer and adjust it by rounding. By encouraging oral response, anxiety levels will be reduced and students will be more likely to apply estimation strategies. Oral work promotes "doing estimation in your head" rather than with paper and pencil.

- Present situations where students must determine the degree of precision that is required. Estimates can range from rough to precise. Sometimes only a ball-park estimate is required, but at other times it is important to get closer to the exact answer. Through discussion, encourage students to recognize that both ball-park estimates and more precise ones are appropriate, and the choice of which one to use depends on the situation.
- Ask students to identify the number of digits that an answer to a specific computational question should contain. Focusing on the size of the answer provides a quick and useful check for reasonableness.
- Provide activities that require students to:
  - identify estimates that are unreasonable
  - explain why an estimate is an overestimate or an underestimate
  - adjust an initial estimate by changing it to a closer estimate.
- Assess student progress in estimation. A possible evaluation technique involves using an overhead to project a variety of problems. Display each problem individually for a short period of time (fifteen to twenty seconds). Establish scoring intervals for each problem in advance. Take time to discuss selected problems and the strategies that students have used to solve them.

A computer program in BASIC that will provide students with an opportunity to practise their skills in estimation has been included in this section of the manual (see Resource 1: Estimation Game).

## FRONT-END ESTIMATION

This method of estimation is useful in addition and subtraction with numbers of three or more digits. The left-hand digits are added or subtracted. An estimate for the remaining part of the problem is then determined by "eyeballing" the remaining digits.

$$\begin{array}{r} 426 \\ 275 \\ +126 \\ \hline \end{array}$$

a. Add the hundreds ⟶ 400 + 200 + 100 = 700
b. Estimate the sum of tens and ones ⟶ greater than 100 but less than 200
c. Estimate the total ⟶ between 800 and 900

$$\begin{array}{r} 626 \\ -147 \\ \hline \end{array}$$

a. Subtract the hundreds ⟶ 600 − 100 = 500
b. Compare 26 and 47 ⟶ 26 is less than 47
c. Estimate the difference ⟶ The answer is between 400 and 500.

The front-end method can also be used for some multiplication questions.

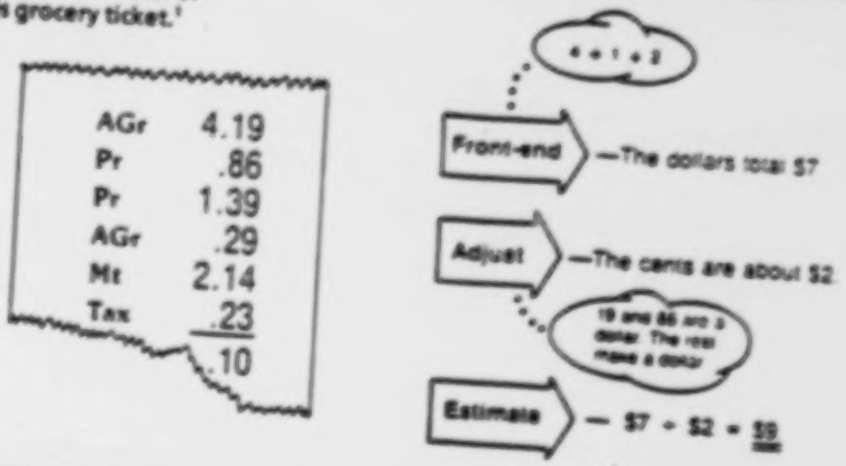
4 X 648 = □

a. 4 X 600 = 2400
b. 48 is about 50
c. 4 X 50 = 200
d. the product is about 2600

One advantage of the front-end method of estimating is that it is less likely to require the use of paper and pencil than other methods.

### CLARIFICATION/EXAMPLE

**Estimate** the total of this grocery ticket.[1]


AGr 4.19
Pr .86
Pr 1.39
AGr .29
Mt 2.14
Tax .23
.10
4 + 1 + 2
Front-end
—The dollars total $7
Adjust
—The cents are about $2
19 and 86 are a dollar. The rest make a dollar
Estimate
— $7 + $2 = $9

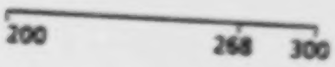
## ROUNDING

The concept of rounding numbers needs careful development before students can use it effectively to estimate answers. Students need to understand the thought processes and steps employed in rounding a number. The illustration below uses a number line in developing the concept of a rounded number. Students can apply this thought process to all rounding situations.

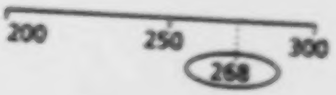
1. Round 268 to the nearest hundred.

268

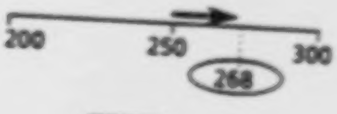
2. Determine which two hundreds the number is between.



3. Find the "midpoint" number.


200
250
300
268

4. Determine which hundred the number is closer to by noting whether it is greater or less than the midpoint number.


200
250
300
268

5. Write the rounded number.

300

Although rounding is the most familiar approach to estimating, it should not be used to the exclusion of other approaches.

---

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1985 by the National Council of Teachers of Mathematics.

CLARIFICATION/EXAMPLE

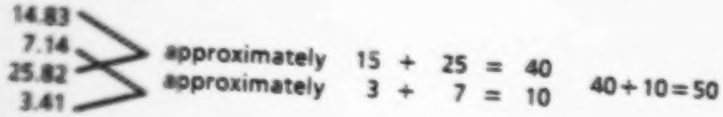
| *Example* | | *Estimate* |
|---|---|---|
| 362<br>+825 | → | 400<br>+800 |
| 49<br>x6 | → | 6 x 50 |
| 68<br>x32 | → | 70 x 30 |
| 1782 ÷ 3 | → | 1800 ÷ 3 |

## COMPATIBLE NUMBERS

Rounding and the use of front-end numbers do not always result in easy mental computation. By rounding, $17.6\overline{)338}$ becomes $18\overline{)340}$. By using front-end numbers, $17.6\overline{)338}$ becomes $17\overline{)330}$. Neither method results in an easy division. Consider, instead, the possibilities of $17\overline{)340}$ or $18\overline{)360}$. Each of these division questions has a quotient that is easy to calculate mentally.

This strategy of estimating with numbers that are easy to calculate mentally is called "estimating with compatible numbers". What constitutes compatible numbers depends upon the operation as well as the estimator's choice.

The compatible numbers strategy can also be used with addition, subtraction and multiplication. As illustrated below, the student looks for pairs of numbers that "fit together" or are easy to calculate mentally.



Model this estimation strategy, and prompt students to use the strategy in their daily work.

**CLARIFICATION/EXAMPLE**

**Estimate the monthly payments.**[1]

**Financed cost: $15 629**

**Payments: 48 months**

The problem is easier to think of as

$$50\overline{)15\,000}.$$

Compatible number estimate: $300

## CLUSTERING (OR AVERAGING)

The clustering or averaging strategy is suited to situations where a group of numbers cluster around a common value. To successfully use this strategy, students should understand the concept of "average", and recognize the link between repeated addition and multiplication. Although clustering is limited to particular kinds of problems, the strategy is useful. Many students will discover and use this strategy on their own.

**CLARIFICATION/EXAMPLE**

**Estimate the total attendance.**[2]

| | |
|---|---|
| Monday | 72 250 |
| Tuesday | 63 891 |
| Wednesday | 67 490 |
| Thursday | 73 180 |
| Friday | 74 918 |
| Saturday | 68 490 |

The figures all cluster around 70 000, so about 70 000 people attended each day.

$$6 \times 70\,000 = 420\,000$$

Averaging estimate: 420 000.

---

[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1985 by the National Council of Teachers of Mathematics.

[2] Ibid.

## RESOURCE 1: ESTIMATION GAME[1]

The BASIC program which follows provides students with an opportunity to practise their skills in estimation. The program involves addition, but can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals.

Players take turns entering a value of their choice. Each player must then make an estimate of the results of adding the numbers that have been entered. The estimate closest to the actual answer is declared the winning estimate.

```
  4  PRINT "ESTIMATION GAME"
 10  LET OP$ = "SUM"
 15  PRINT "IN TURN, EACH PLAYER WILL ENTER"
 20  PRINT "A NUMBER OF THEIR CHOICE"
 30  PRINT "THEN EACH PLAYER WILL ENTER AN"
 35  PRINT "ESTIMATE THE OP$ OF THE NUMBERS"
 45  PRINT "THE CLOSEST ESTIMATE WINS"
 55  PRINT "FIRST PLAYER'S NUMBER": INPUT A
 60  PRINT "SECOND PLAYER'S NUMBER": INPUT B
 70  PRINT "FIRST PLAYER'S ESTIMATE. . .": INPUT A1
 80  PRINT "SECOND PLAYER'S ESTIMATE. . ." : INPUT B1
 85  LET C = A + B:  REM FINDS THE SUM
 90  IF ABS (C - A1) < ABS (C - B1) THEN 115
 96  IF ABS (C - A1) = ABS (C - B1) THEN 117
100  PRINT "PLAYER #2 WINS!"
105  GOTO 130
115  PRINT "PLAYER #1 WINS!"
116  GOTO 130
117  PRINT "IT'S A TIE!"
130  PRINT "PLAY AGAIN?  (Y OR N)"
131  INPUT AN$$
132  IF AN$$ = "Y" THEN GOTO 55
133  IF AN$$ < > "N" THEN GOTO 130
140  END
```

Line 10 can be changed to indicate a different operation. By changing line 10 to LET OP$ = "PRODUCT", the instructions in line 35 will tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also be changed. If OP$ = "PRODUCT", then line 85 must say LET C = A*B.

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[1] Alberta Education, Curriculum Support, *Junior High Mathematics Teacher Resource Manual*, 1988.

# SITUATIONAL AND CONCRETE APPROACHES

Student developmental levels and learning styles suggest an experiential approach to teaching and learning, where students are actively involved in:

- concrete, transitional and formal levels of cognition
- deliberate observation and questioning that promote thinking.

Students need to have direct access to the use of manipulative and visual materials while developing a new concept. Tactile experiences are particularly important in assisting students to deal with cognitive demands of fractions, ratio and percent, as well as two- and three-dimensional geometry. Manipulative materials need not be expensive, and in many instances can be borrowed or improvised. Teachers are encouraged to establish a mathematics lab by borrowing, collecting and/or purchasing a variety of manipulative materials appropriate to the mathematics program. Materials useful in providing instruction at concrete and transitional levels throughout the mathematics program may include:

- assorted measuring devices (e.g., tape measures/rulers/metre sticks, carpenter's squares, measuring cups, graduated cylinders, scales for measuring mass/weight, protractors, compasses, calipers, micrometers)
- assorted mathematical puzzles and games
- base-ten blocks
- cash register tapes and cash summary sheets
- cuisenaire rods
- fraction circles
- geoblocks
- geoboards
- graph paper
- LOGO software
- nails and screws of assorted size
- nuts, bolts and wrenches of assorted size
- paper money and plastic coinage
- specimen bank forms and income tax forms
- tangram pieces
- three-dimensional rectangular solids, cubes, cylinders and cones.

Students also need to observe, question, verbalize and discuss the relationships they investigate at the concrete level, and eventually to translate relationships into the abstract symbols of mathematics. While the purpose of manipulatives is to help students understand and remember, there comes a time when each student should become efficient in making application of concepts in their abstract form. It is important, therefore, that teachers supplement and alternate the use of manipulative materials with other more abstract learning strategies and activities.

Suggestions for planning situational and concrete learning activities in mathematics have been included in this section of the manual and focus attention on:

- Number Systems and Operations
- Ratio, Proportion and Percent
- Geometry and Measurement
- Project Work.

# NUMBER SYSTEMS AND OPERATIONS

## WHOLE NUMBERS AND DECIMALS

Puzzles and games provide a worthwhile method of reinforcing basic number skills and arithmetical process. Several mathematical puzzles and games have been provided below as examples.

### 1. 100 GAME

Two players, starting from 0 and taking turns, add any number from 1 to 10. The player who reaches 100 is declared the winner. As students play this game, encourage them to think about a strategy that will ensure their success (i.e., the first player to reach 89 wins the game). Are there other boundary numbers that will also ensure their success?

**CLARIFICATION/EXAMPLE**

| PLAYER 1 | PLAYER 2 |
|---|---|
| 0 | (+9) 9 |
| (+8) 17 | (+7) 24 |
| (+9) 33 | (+2) 35 |
| (+9) 44 | (+5) 49 |
| (+8) 57 | (+6) 63 |
| (+7) 70 | (+9) 79 |
| (+1) 80 | (+9) 89 |
| (+3) 92 | (+8) 100 |
| | Player 2 is the winner. |

### 2. LETTER PUZZLES[1]

Students may benefit from designing/solving letter puzzles in addition or subtraction.

In the following puzzle, each letter represents a digit:

$$\begin{array}{r} \text{SEND} \\ \underline{+\text{MORE}} \\ \text{MONEY} \end{array}$$

The solution is:

$$\begin{array}{r} 9\,5\,6\,7 \\ \underline{+\ 1\,0\,8\,5} \\ 1\,0\,6\,5\,2 \end{array}$$

Letter puzzles may also be used to develop and reinforce problem-solving strategies and number properties. In the example that follows, each letter again represents a digit:

$$\begin{array}{r} \text{YZ} \\ \underline{\times\ 2} \\ \text{XZZ} \end{array}$$

Ask students to identify the number, which when doubled, yields itself. Students will recognize the multiplicative property of zero, and should then be able to solve the rest of the puzzle.

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[1] Alberta Education. *Let Problem Solving Be The Focus For The 1980's*, 1983.

The solution is:

$$\begin{array}{r} 50 \\ \underline{\times\ 2} \\ 100 \end{array}$$

Another problem:

$$\begin{array}{r} \text{VZVZ} \\ \underline{\times \quad\ 2} \\ \text{X2X2Z} \end{array}$$

The solution is:

$$\begin{array}{r} 6060 \\ \underline{\times \quad\ 2} \\ 12120 \end{array}$$

Relate the use of letters in these puzzles to the use of variables in algebra.

### 3. PERCENT-DECIMAL DOMINOES (2 PLAYERS)

Shuffle the deck (20 cards) and place cards face down on the table. Each player takes 4 cards from the table.

Play alternates.

Player 1 begins play by placing a domino face up on the table.

Player 2 plays a domino on any edge of the first domino, making sure that the touching edges name the same number. If unable to play, the player draws one card from the face-down cards and plays it if possible. If it is not possible to play, the other person takes a turn.

The first person to play all of their cards wins the game.

Note:
The game may be extended to more than one dealing. Score one point for each card left at the end of each game. The first person to reach 10 points loses the game.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.07 | 13% | 0.2 | 80% | 0.02 | 4% | 0.9 | 60% |
| 0.13 | 8% | 0.8 | 70% | 0.04 | 50% | 0.6 | 31% |
| 0.08 | 40% | 0.7 | 18% | 0.5 | 6% | 0.31 | 9% |
| 0.4 | 0.05 | 0.18 | 3% | 0.06 | 10% | 0.09 | 1% |
| 5% | 20% | 0.03 | 2% | 0.1 | 90% | 0.01 | 7% |

### 4. PERCENT – DECIMAL PUZZLE

Distribute one copy of this grid to each student in the class. Instruct students to cut out the 16 pieces and assemble the puzzle so that touching edges name the same number (i.e., decimal-percent equivalents).

The finished puzzle will be a 4×4 square.

Note:
Other variations of this puzzle are easily made using a 3×3, 4×4 or 5×5 grid.

| | | | |
|---|---|---|---|
| 83%<br>0.61 0.1<br>40% | 0.04<br>1% 52%<br>0.72 | 0.03<br>5% 0.8<br>0.13 | 23%<br>0.05 20%<br>0.79 |
| 70%<br>80% 0.29<br>0.38 | 9%<br>10% 16%<br>3% | 0.61<br>39% 5%<br>0.32 | 0.6<br>29% 33%<br>4% |
| 0.06<br>0.18 8%<br>61% | 72%<br>17% 0.52<br>0.31 | 49%<br>0.02 81%<br>60% | 13%<br>0.08 25%<br>0.23 |
| 0.4<br>0.39 0.05<br>6% | 38%<br>0.25 0.01<br>9% | 0.09<br>0.2 0.17<br>30% | 27%<br>0.16 2%<br>0.7 |

## 5. MAGIC SQUARE

Place the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 and 1.6 in a square like the one below so that you may add the four numerals in any line and the sum will always be 3.4.

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

## 6. SEVEN/FIVE/THREE (2 players)

Lay out fifteen matches as shown.

¶ ¶ ¶ ¶ ¶ ¶ ¶

¶ ¶ ¶ ¶ ¶

¶ ¶ ¶

Players take turns removing matches. Any number of matches can be removed at each turn, providing they are all taken from the same row. The player who takes the last match loses the game.

Students should be encouraged to discover a winning strategy.

## INTEGERS

An understanding of integer concepts should be developed by investigating their application in everyday situations. Provide opportunities for students to demonstrate addition of pairs of integers through concrete manipulation and diagramatic representation. An instructional sequence for developing integer concepts is suggested below.

1. Positive and negative integers can be represented to students through the analogy of having money (positive integers) and owing money (negative integers).

**CLARIFICATION/EXAMPLE**

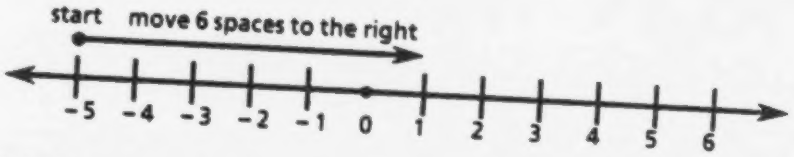
Frank makes $7.00 cutting the lawn. He uses this money to pay back a debt of $10.00 which he owes his father. How much money does Frank owe his father after repaying him $7.00?

$-10+7=-3$

2. Integer addition may be represented as moves on the number line. Moves to the right are positive, while moves to the left are negative.

**CLARIFICATION/EXAMPLE**

$-5+6=?$



Start at −5. Because 6 is positive, move 6 spaces to the right.

$-5+6=1$

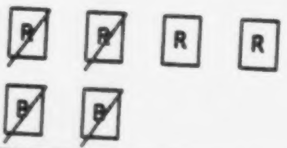
3. Use the thermometer to demonstrate integer addition in the same way that the number line is used. Positive integers represent a rise in temperature, while negative integers will represent a drop in temperature.

4. Coloured cards may be used to demonstrate the addition of integers. Represent positive integers with cards of one colour (e.g., black), and negative integers with cards of another colour (e.g., red).

**CLARIFICATION/EXAMPLE**

Black cards are positive.
Red cards are negative.

$-4+2=?$



Pairs of red and black cards cancel each other.

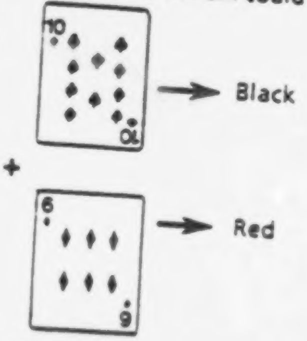
The answer is two red or − 2.

5. Integer addition can also be demonstrated using a regular deck of playing cards. Represent positive integers with black cards and negative integers with red cards. Pairs of red and black cards with the same number will cancel each other.

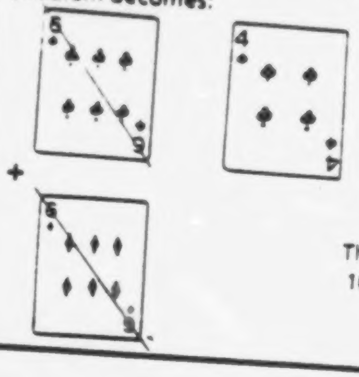
**CLARIFICATION/EXAMPLE**

$10 + -6 = ?$

Initially, the problem could be represented as:



These cards do not cancel each other. Replace the 10 with a six and a four. The problem becomes:



The answer is a black 4.
$10 + -6 = 4$

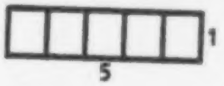
## PRIME NUMBERS

The concept of "prime" is abstract, and should be developed using concrete ideas. Suggestions for developing an understanding of prime numbers are provided below.

1. The following strategy may be effective in developing an understanding of prime numbers:
   (a) Provide students with a set of square blocks or square tiles. (Pieces of square paper will be sufficient.)
   (b) Select a number of blocks (square tiles) equal to a number which is to be factored.
   (c) Form as many rectangles as possible with this number of blocks (square tiles). Record your results.
   (d) The number being factored is prime if only one rectangle can be formed with the given number of blocks (square tiles).

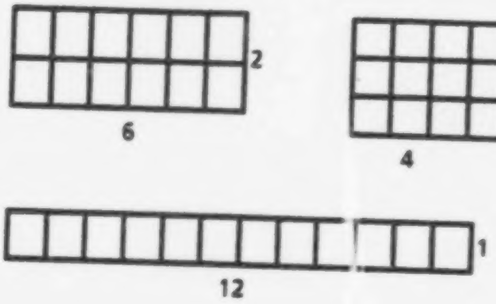
**CLARIFICATION/EXAMPLE**

- Factor the number "5":



The factors are 1 and 5. Since only one rectangle was formed, the number "5" is prime.

- Factor the number "12":



The factors are 1, 2, 3, 4, 6, and 12. Since more than one rectangle was formed, the number "12" is not prime.

2. Students may not recognize the importance of being able to identify factors and prime numbers. Encourage students to practise these skills by designing games that require their use.

**CLARIFICATION/EXAMPLE**

List the numbers from 1 to 50.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~1~~ | ~~2~~ | ~~3~~ | 4 | 5 | ~~6~~ | 7 | 8 | ~~9~~ | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | ~~18~~ | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

The first player selects a number (e.g., 18) and crosses it out. The second player crosses out all the factors of 18 and scores their total (e.g., 1+2+3+6+9=21). This process is then repeated, with the second player selecting a number and the first player crossing out its factors. Once a number has been crossed out it cannot be used again.

Exchange turns until all numbers have been crossed out. The player with the highest point total is declared the winner.

3. Computer programs can be used by students to generate sets of factors for numbers they are given.

**CLARIFICATION/EXAMPLE**

**BASIC Program For Generating a Set of Factors**

```
 10  PRINT "WHAT IS THE NUMBER"
 20  PRINT "YOU WISH TO FACTOR?"
 30  INPUT N
 40  PRINT "THE FACTORS ARE: "
 50  FOR K=1 TO N STEP 1
 60  LET X=N/K
 70  LET Y=INT (N/K)
 80  IF X=Y THEN PRINT K
 90  NEXT K
100  END
```

**CLARIFICATION/EXAMPLE**

**BASIC Program For Generating Prime Factors**

```
 10  REM LISTS THE PRIME FACTORS
 20  PRINT "ENTER A WHOLE NUMBER"
 30  INPUT N
 40  PRINT "PRIME FACTORS ARE: "
 50  READ P
 60  IF N / P < P THEN 160
 70  IF N / P < P = INT (N / P) THEN 90
 80  GOTO 30
 90  PRINT P
100  LET N = N / P
110  IF N / P < P THEN 160
120  GOTO 70
130  DATA    2, 3, 5, 7, 11, 17, 19
140  DATA    23, 29, 31, 37, 41, 43, 53
150  DATA    59, 61,67, 71, 73, 79, 83, 89, 97
160  PRINT N
170  END
```

4. Prime factors can be used to find the square roots of numbers that are perfect squares. These activities will assist students to develop concepts of "prime" and "square root".

**CLARIFICATION/EXAMPLE**

**Find the square root of 576.**

1. Write 576 as the product of prime factors:
   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
2. Divide the prime factors into two identical groups:
   $(2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)$
3. Find the product of each group of factors $24 \times 24$
4. If $576 = 24 \times 24$, then $\sqrt{576} = 24$.

## FRACTIONS

The cognitive demands of fraction concepts suggest that instruction include extensive use of manipulative and visual materials. Select activities that will encourage students to:

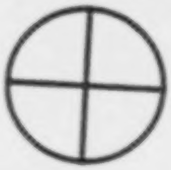
- work with real models, pictures and diagrams
- experiment and explore
- ask questions
- verbalize ideas and discoveries
- make associations and draw conclusions.

Sample activities that provide opportunities for students to manipulate materials, discover relationships and generalize the results of investigation are described below.

1. Fraction circles provide a useful method of developing the concept of a fraction and an understanding of equivalence among fractions. A variety of blackline masters for producing fraction circles have been provided in Resource 1: Fraction Circles.

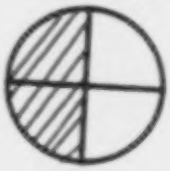
**CLARIFICATION/EXAMPLE**

- Draw a diagram that illustrates the relationship between 4/4 and 1.



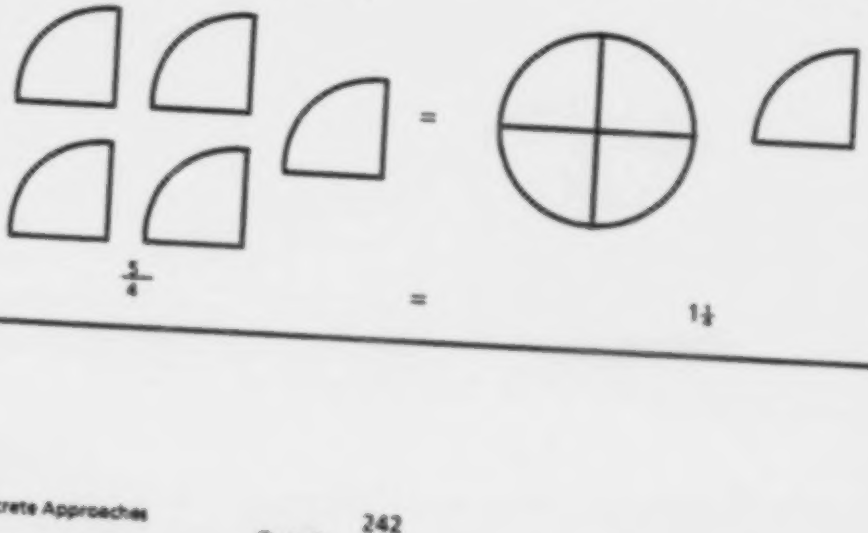
$\frac{4}{4} = 1$

- Draw the diagram that illustrates the relationship between 2/4 and 1/2.



$\frac{2}{4} = \frac{1}{2}$

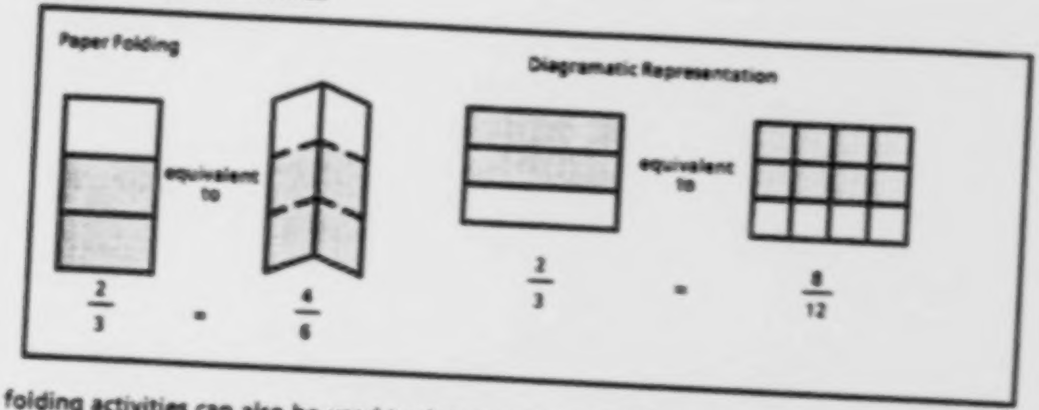
- Using pieces of a fraction circle, investigate the relationship between improper fractions and mixed numbers.



$\frac{5}{4} = 1\frac{1}{4}$

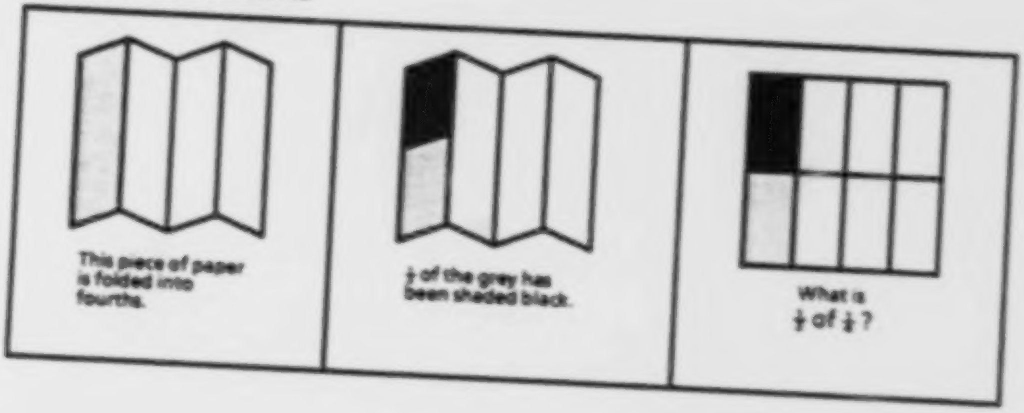
2. The concept of equivalent fractions (ratios) can be developed through paper folding activities, or through diagramatic representation with graph paper/geoboards. Encourage students to translate the relationships discovered into number sentences, and to look for patterns in the ratios obtained.

**CLARIFICATION/EXAMPLE**


Paper Folding
Diagramatic Representation
equivalent to
$\frac{2}{3}$ = $\frac{4}{6}$
equivalent to
$\frac{2}{3}$ = $\frac{8}{12}$

Paper folding activities can also be used to demonstrate multiplication of fractions . Encourage students to discover their own strategies for multiplying fractions that are based on the patterns and relationships discovered.

**CLARIFICATION/EXAMPLE**



This piece of paper is folded into fourths.

$\frac{1}{2}$ of the grey has been shaded black.

What is $\frac{1}{2}$ of $\frac{1}{4}$ ?

3. The multiple board is a worthwhile tool for investigating equivalent fractions.

**CLARIFICATION/EXAMPLE**

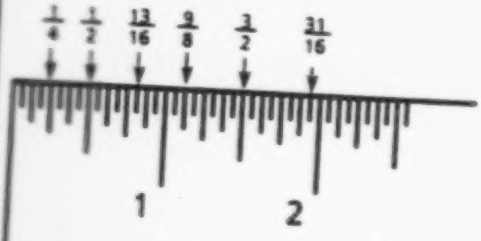
To generate equivalent fractions using the multiple board, locate the original fraction in the first column. Numbers that are in the same position throughout the nine remaining columns will represent fractions that are equivalent to the original fraction.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ② | ④ | ⑥ | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| ④ | ⑧ | ⑫ | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

4. The "inch" unit provides a useful vehicle for developing/reinforcing fraction concepts. Design activities involving fractional parts of an inch that will develop the concepts illustrated below. Black line masters of the inch unit are provided in Resource 2: Using the Inch Unit.

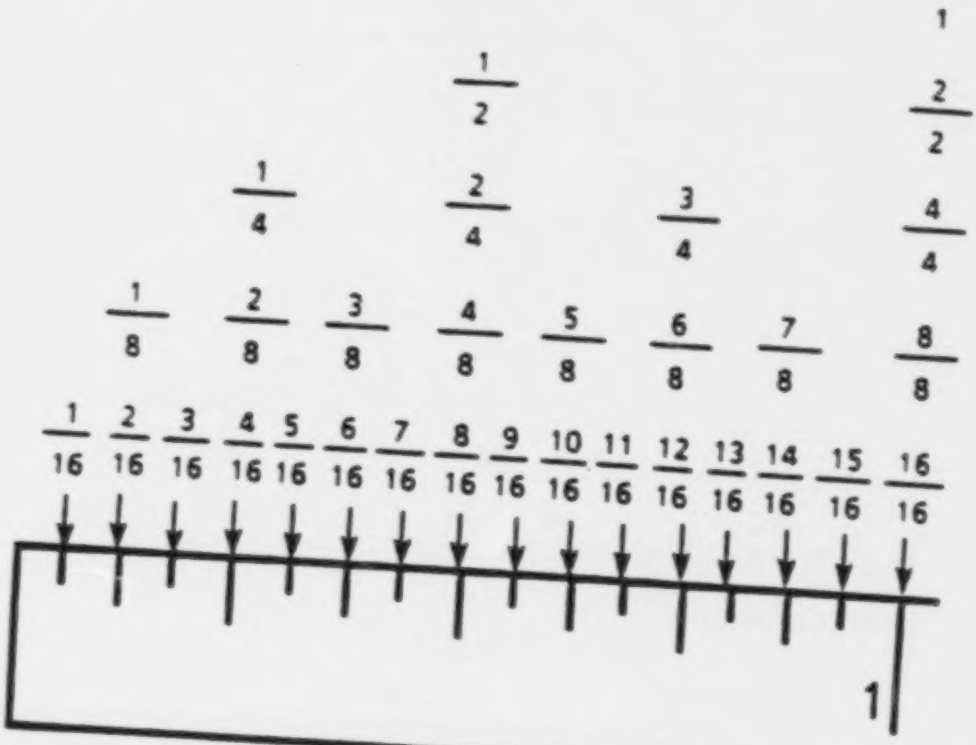
**CLARIFICATION/EXAMPLE**

Order of Fractions on the Number Line

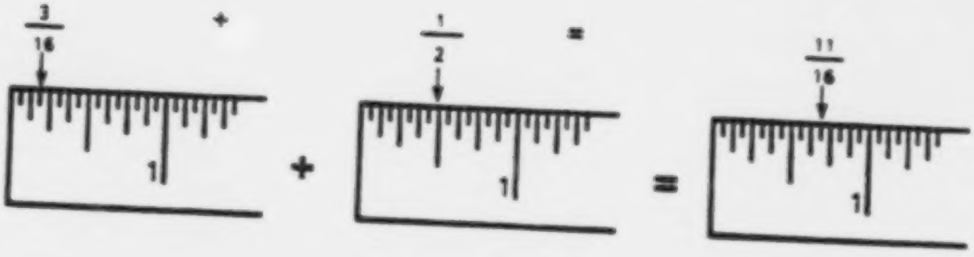


CLARIFICATION/EXAMPLE

**Equivalent Fractions**


1
1/2
2/2
1/4
2/4
3/4
4/4
1/8
2/8
3/8
4/8
5/8
6/8
7/8
8/8
1/16
2/16
3/16
4/16
5/16
6/16
7/16
8/16
9/16
10/16
11/16
12/16
13/16
14/16
15/16
16/16
1

**Addition and Subtraction of Fractions**


3/16
+
1/2
=
11/16
1
+
1
=
1

5. An understanding of greatest common factors (GCF's) and least common multiples (LCM's) will assist students to simplify and add/subtract fractions. The following examples illustrate alternative methods for finding the greatest common factor and least common multiple of a pair of numbers.

**CLARIFICATION/EXAMPLE**

**What is the GCF of 8 and 12?**

Write each number as the product of its prime factors.

8 = 2 × 2 × 2

12 = 2 × 2 × 3

The GCF is the product of all factors common to 8 and 12. Therefore, the GCF is 2 × 2 or 4.

**What is the LCM of 8 and 12?**

Write the first few multiples of 8 and 12:

8, 16, 24, 32, . . .

12, 24, 36, 48, . . .

The LCM is the smallest multiple that is common to 8 and 12. Therefore, the LCM is 24.

**CLARIFICATION/EXAMPLE**

**An Algorithm for Finding GCF's and LCM's**

Divide the numbers by a common factor. The common factor need not be the greatest common factor. Repeat the process until the numbers are relatively prime (i.e., have one as their only common divisor).

2 ) 8 12

2 ) 4 6

2 3

The greatest common factor is found by multiplying the common factors (i.e., numbers on the side).

② ) 8 12

② ) 4 6

2 3

The GCF is 2×2 or 4.

The least common multiple is found by multiplying the common factors (i.e., numbers on the side) by the numbers across the bottom.

② ) 8 12

② ) 4 6

② ③

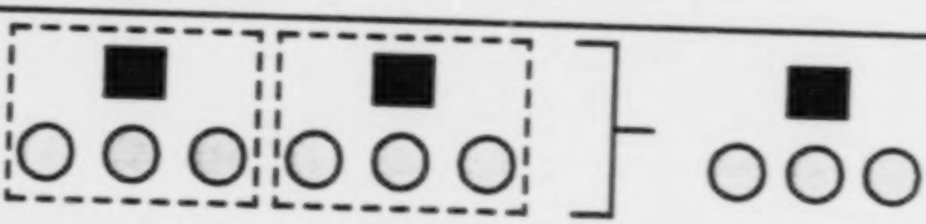
The LCM is 2×2×2×3 or 24.

# RATIO, PROPORTION AND PERCENT

It is sometimes assumed that high school students understand the concepts of ratio, proportion and percent because they have been exposed to these ideas throughout their school experience. This assumption may be false, and care should be taken to develop and/or reinforce these concepts through meaningful activities. Diagnose areas of student strength/weakness, and plan remedial activities that provide concrete and visual experiences for students. Suggestions for developing related concepts and skills are provided below.

1. Encourage students to discover number relationships that are present in equivalent ratios by mapping a correspondence between objects in one collection and objects present in another collection. Through discussion, develop an understanding of how the equivalence of two ratios can be verified using common factors or multiples.

**CLARIFICATION/EXAMPLE**



$$\begin{tikzcd} \frac{2}{6} \arrow[rr, bend left, "\div 2"] \arrow[rr, bend right, "\div 2"'] & = & \frac{1}{3} \end{tikzcd}$$

2. Students frequently fail to understand that the relationship between the two parts of a ratio is multiplicative rather than additive. The multiplicative relationship in ratios must be frequently illustrated and explained to students.

**CLARIFICATION/EXAMPLE**

$\frac{2}{3} = \frac{?}{6}$

Students may incorrectly reason that:

$$\xcancel{\begin{tikzcd} \frac{2}{3} \arrow[rr, bend left, "+3"] \arrow[rr, bend right, "+3"'] & = & \frac{5}{6} \end{tikzcd}}$$

Emphasize the multiplicative relationship:

$$\begin{tikzcd} \frac{2}{3} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{4}{6} \end{tikzcd}$$

Students who continue to experience difficulty with this concept should be given additional experience in working with collections of real objects, and in mapping a correspondence between objects in one collection with objects in another.

3. Keep number relationships simple when working with ratio and proportion. When applying ratio concepts to practical problems, select problem situations that involve both integral and non-integral ratios:

   - $\frac{1}{3} = \frac{4}{N}$ (integral)
   - $\frac{4}{5} = \frac{N}{15}$ (non-integral).

4. Discuss the use of certain words in making comparisons that can be described with ratios:
   - six books for each student
   - six books per student
   - six for a dollar.

5. When finding the missing component in a proportion, encourage students to use the common factor/multiple method whenever possible.

**CLARIFICATION/EXAMPLE**

**Common Multiple Method**

$$\frac{2}{3} \overset{\times 3}{\underset{\times 3}{=}} \frac{N}{9}$$

Numerator and denominator are multiplied by 3.

**Common Factor Method**

$$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{N}{3}$$

Numerator and denominator are divided by 4.

6. Develop formal equation-solving strategies that can be used to find the missing component in proportions not easily solved using the common factor/multiple method. Encourage students to recognize the "cross-product" method of solving proportions as a shortened form of these more formal equation-solving strategies.

**CLARIFICATION/EXAMPLE**

**Find the missing component in** $\frac{N}{3} = \frac{1}{4}$.

Formal Strategy

$$\frac{N}{3} = \frac{1}{4}$$

$$12 \times \frac{N}{3} = 12 \times \frac{1}{4}$$

$$\overset{4}{\cancel{12}} \times \frac{N}{\underset{1}{\cancel{3}}} = \overset{3}{\cancel{12}} \times \frac{1}{\underset{1}{\cancel{4}}}$$

$$4 \times N = 3 \times 1$$

$$N = \frac{3}{4}$$

Cross-Product Method

$$\frac{N}{3} = \frac{1}{4}$$

$$\frac{N}{3} \times \frac{1}{4}$$

$$4 \times N = 3 \times 1$$

$$N = \frac{3}{4}$$

7. Assist students to design or modify a simple computer program that will calculate the missing component in a proportion (see Use of Technology, "The Computer").

**CLARIFICATION/EXAMPLE**

**A BASIC Program for Solving Proportions**

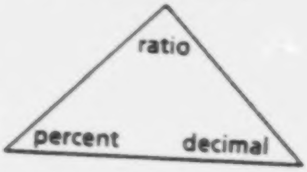
```
 10  REM GENERATE EQUIVALENT
 20  REM RATIOS OF THE FORM
 30  REM A / B = C / D
 40  PRINT "WHAT ARE THE VALUES OF A AND B?"
 50  PRINT "ENTER AS A,B"
 60  INPUT A,B
 70  PRINT "IF C IS GIVEN ENTER C"
 80  PRINT "IF D IS GIVEN ENTER D"
 90  INPUT 0$
100  IF 0$ = "C" THEN 170
110  PRINT "ENTER THE VALUE OF D"
120  INPUT D
130  PRINT "THE VALUE OF C IS"
140  LET C = A * D / B
150  PRINT C
160  GOTO 220
170  PRINT "ENTER THE VALUE OF C"
180  INPUT C
190  PRINT "THE VALUE OF D IS"
200  LET D = C * B / A
210  PRINT D
220  END
```

8. Use a variety of procedures to reinforce understanding of common fraction, decimal and percent equivalents:
   - number line activities
   - fraction-decimal-percent dominoes.

Triangular drill cards similar to the one illustrated below will be useful in developing student ability to recall equivalents that are commonly used.

**CLARIFICATION/EXAMPLE**

Construct a set of triangular cards. The vertices of each card will name a ratio, its equivalent decimal, and its equivalent percent.



Encourage students to make their own set of triangular cards for fractions, decimals and percents that they frequently use. Students can cover one or two vertices on each triangle, and practise naming the missing equivalents.

9. Students may benefit from producing a table of ratio/decimal/percent equivalents frequently encountered in work situations and everyday life. Encourage students to make such a table and refer to it as necessary.

The following computer program will generate decimal and percent equivalents for ratio "families". A sample run has been illustrated.

**CLARIFICATION/EXAMPLE**

**A BASIC Program for Generating Ratio/Decimal/Percent Equivalents**

```
 10  REM THIS PROGRAM WILL GENERATE
 20  REM DECIMAL AND PERCENT EQUIVALENTS
 30  REM FOR RATIO FAMILIES
 40  PRINT "WHAT IS THE DENOMINATOR"
 50  PRINT "OF YOUR RATIO FAMILY?"
 60  INPUT D
 70  FOR K = 1 TO D
 80  PRINT K "/"D, K/D, K/D * 100 "%"
 90  NEXT K
100  END
RUN
WHAT IS THE DENOMINATOR OF YOUR RATIO FAMILY?

 ?16

 1/16              .0625            6.25%
 2/16              .125             12.5%
 3/16              .1875           18.75%
 4/16              .25                25%
 5/16              .3125           31.25%
 6/16              .375             37.5%
 7/16              .4375           43.75%
 8/16              .5                 50%
 9/16              .5625           56.25%
10/16              .625             62.5%
11/16              .6875           68.75%
12/16              .75                75%
13/16              .8125           81.25%
14/16              .875             87.5%
15/16              .9375           93.75%
16/16              1                 100%
```

# GEOMETRY AND MEASUREMENT

## GEOMETRY

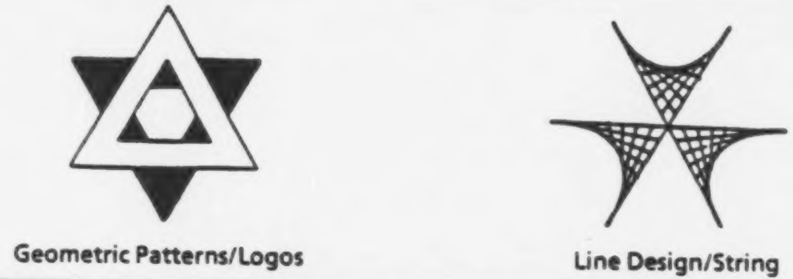
Concepts of one-dimensional space are usually well understood. Students find two-dimensional and three-dimensional concepts more difficult, and may require concrete support for successful experience in these areas. Manipulation and construction will assist students to recognize spatial relationships inherent in these figures. A variety of instructional activities that may assist students to develop concepts in geometry and skills in visual perception are provided below.

1. Develop and reinforce an understanding of the properties of two- and three-dimensional shapes by:
   - identifying/discussing the properties of two- and three-dimensional shapes present in the classroom
   - making a list of the two- and three-dimensional shapes observed in a photograph/filmstrip/movie, or after taking a short walk
   - making a collage, using various two- and three-dimensional shapes.

2. Provide opportunities for students to draw and construct the one-, two- and three-dimensional figures being studied, using compass/protractor/straightedge/ruler.

   Many geometric concepts and skills can be developed/reinforced as students construct the geometric logos or patterns described in Resource 3: Geometric Patterns. The line design and string art projects provided in Resource 4: Line Design are also useful vehicles for delivering skills in geometry.

   **CLARIFICATION/EXAMPLE**



Geometric Patterns/Logos

Line Design/String

   Encourage students to be creative in the designs and patterns they produce. Displays of student work in the classroom and hallway will enhance student motivation and effort.

   Additional ideas for geometric patterns, line design and string art projects can be obtained from a variety of resources available on these topics.

3. Use a LOGO computer program to produce geometric figures and designs. A list of LOGO commands is provided in the Use of Technology: The Computer section of this manual. Encourage students to be creative in the designs they generate. Provide direction to students as to which line relationships/geometric figures to incorporate into the designs that are produced.

4. Ask students to create a wallpaper pattern, fabric pattern or linoleum design based on repetition of a line design/geometric pattern that has been constructed.

5. Ask students to construct the 60 attribute shapes that are required to play "Attribute Dominoes". Directions for making the shapes and playing the game are provided in Resource 5: Attribute Dominoes.

6. Provide frequent opportunities for students to solve puzzles that involve recognition of spatial patterns and relationships. Puzzles might involve experimentation with:

- tangrams
- tessellations
- mazes
- paper folding.

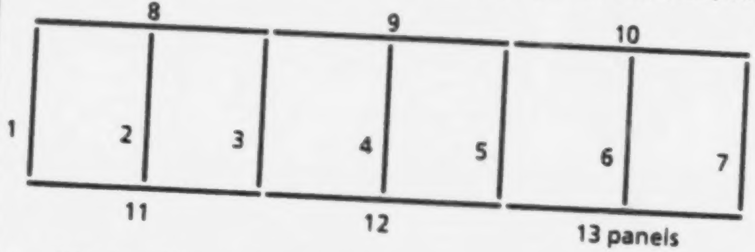
Sample activities involving tangrams and tessellations are provided in Resource 6: The Tangram and Resource 7: Tessellations.

A variety of books containing recreational problems and activities in mathematics can be obtained from local media centres and libraries.

**CLARIFICATION/EXAMPLE**

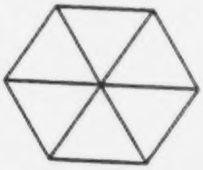
**PEN PROBLEM[1]**

Six turkey pens were made for a 4H Club display with 13 panels, all of equal length.



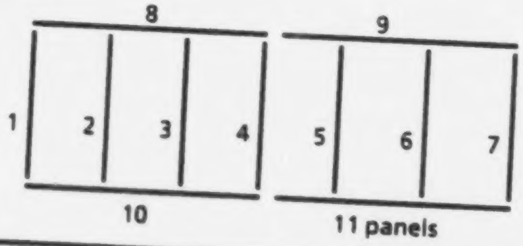
a. One panel was broken when a truck backed into it. The club members then arranged the 12 remaining panels to form 6 pens of equal size and shape. How did they do it?

**ANSWER:**



b. The manager needed one of the panels in another part of the display. The club members then made 6 pens of equal size and shape with 11 panels. How did they do it?

**ANSWER:**



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[1] Alberta Education, *Problem-Solving Challenge for Mathematics*, 1985, p. 47.

7. Introduce students to historical problems and achievements in geometry. Examples might include:

- The Möbius Strip
- The Königsberg Bridge Problem
- Great Pyramids of Egypt
- The Leaning Tower of Pisa
- The Theorem of Pythagoras
- Paintings by M.C. Escher.

**CLARIFICATION/EXAMPLE**

### THE MÖBIUS STRIP

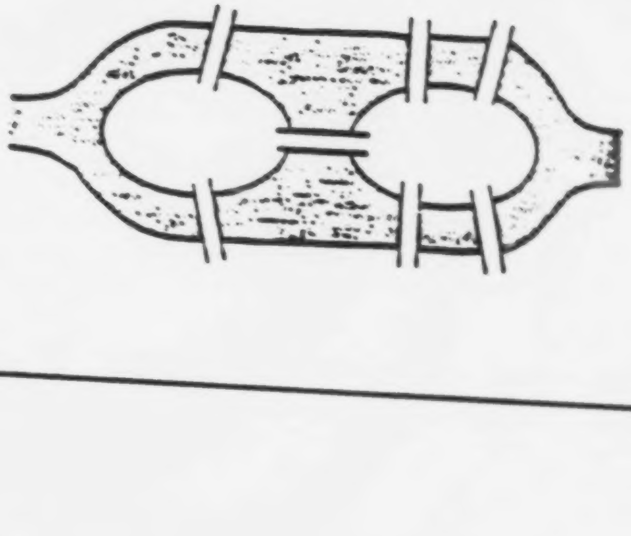
A 19th century mathematician, August F. Möbius, invented what was to become known as the Möbius Strip. The Möbius Strip is a strip of paper that has been given a twist and the ends joined. Some strange results are caused by the twist:

- The strip has no inside or outside. If you draw a line on the strip without raising your pen you will eventually end up where you started.
- If an ordinary strip of paper is cut along the centre you will end up with two identical strips of paper. If you cut the Möbius strip along the centre you will have only one strip twice as long as the original.
- If you cut the strip 1/3 of the width from the edge and continue until it meets up with itself, you will not make the strip longer, but rather will have two interlocking rings.

**CLARIFICATION/EXAMPLE**

### THE SEVEN BRIDGES OF KÖNIGSBERG

This problem originated at Königsberg, a Prussian town on the Preger River, which has seven bridges. The problem was whether it was possible to take a walk and cross each of the bridges exactly once. Try to solve the problem by tracing your path with a pencil.



## MEASUREMENT

Students need to develop a "feel" for standard units of measure through the use of referents and visual imagery. Develop skill in making reasonable estimates of measure before engaging in activities that involve actual measurement and precision. Consider the following guidelines when planning activities in estimation and measurement.

- Develop skill in estimation by having students make an estimate and then check their estimate through direct measurement.
- Although some experience in direct measurement may immediately follow the introduction of a particular unit, only a few measurements of this nature should be taken before students are asked to make estimates prior to performing direct measurement.
- Encourage good estimates, but do not provide penalties for inaccurate ones.
- The ability to estimate is based on previous experience and must be practised on a regular basis throughout the program.

Students may require concrete support in developing an understanding of the two-dimensional concept of area and the three-dimensional concept of volume. Instruction should include the use of concrete and visual materials (e.g., tiles, grid paper, cubes and models).

Additional suggestions for developing and refining measurement skills are provided below.

1. Provide abundant opportunities for hands-on experience in estimating and measuring the length, mass and capacity of familiar objects.

   One method of developing familiarity with frequently used units is through the use of "mind pictures" or referents. Some examples include:
   - millimetre – thickness of a dime
   - centimetre – width of little finger
   - kilometre – distance from school to __?__
   - gram – weight of a small paper clip
   - kilogram – weight of (subject) textbook.

   Encourage students to use these referents in making estimates, and to check their estimates through actual measurement. Coach students to select units that are appropriate to each item or object being measured. Students can record their work in tables similar to those illustrated.

| ITEM | ESTIMATED LENGTH | ACTUAL LENGTH |
|---|---|---|
| | | |
| | | |
| | | |

| ITEM | ESTIMATED MASS | ACTUAL MASS |
|---|---|---|
| | | |
| | | |
| | | |

| ITEM | ESTIMATED CAPACITY | ACTUAL CAPACITY |
|---|---|---|
| | | |
| | | |
| | | |

2. Another method of estimating measurements is through the use of a strategy called "chunking". The object being measured is divided into portions or chunks whose measure the student is familiar with. The measures of individual portions are then added in order to obtain an estimate of the total measurement. This strategy is particularly effective when the object being measured is large.

   Example: If you wish to measure the height of a room, divide the height into two parts:

   - the height of the doorway
   - the distance between the doorway and the ceiling.

3. Students may also find the strategy of "unitizing" to be an effective method of estimating the measure of a large object. Ask students to divide the object being measured into equal portions. After estimating the measure of one of the equal portions, multiply the estimate by the number of parts into which the object was divided.

4. Encourage students to recognize the importance of measurement in everyday life. Make a display of articles/pictures collected from newspapers and magazines that depict the use of:

   - length
   - perimeter
   - area
   - volume
   - capacity
   - mass
   - time
   - temperature
   - angle.

5. Plan a "scavenger hunt" that will require students to use their skills of estimation and measurement. Divide the class into small groups, and provide each group with the same list of measurements. Each group must go on a scavenger hunt and find objects that have approximately the same measurements as those on the list. No measuring devices may be used. At the conclusion of the hunt, ask each group of students to measure the objects collected. Determine the difference between the estimated and actual measurements. The team with the lowest cumulative total error is declared the winning team.

6. Provide opportunities for students to check the accuracy of their estimates of length/mass/capacity using the activities provided in Resource 8: Accuracy in Estimation.

7. Develop and reinforce concepts of perimeter, area and volume through the use of tactile and manipulative materials:

   - geoboards
   - dot paper
   - tiles/tessellations
   - boxes/building blocks.

   Activities with the geoboard are provided in Resource 9: Understanding Area.

   Compare the concept of perimeter to "distance around", the concept of area to "surface covered", and the concept of volume to "space occupied".

   **CLARIFICATION/EXAMPLE**

   **Distinguishing Between Perimeter, Area and Volume**

   Provide students with a small box. Ask the students to determine:
   - the length of string required to go around the box
   - the amount of paper required to cover the box
   - the number of $cm^3$ blocks required to fill the box.

   Encourage students to distinguish between perimeter, area and volume in each of these situations. Discuss familiar applications of perimeter, area and volume.

8. Encourage students to develop "mind pictures" for the square centimetre and square metre. Ask students to draw on paper and cut out each of these units.

   Provide opportunities for students initially to verify estimates of the area they make by filling rectangular figures with tiles or square cutouts and counting the number of squares needed to fill the region. Once the concept of area is understood, encourage students to develop their own formulas for determining the area of familiar geometric figures.

9. Assist students to develop "mind pictures" for the cubic centimetre and cubic metre by making three-dimensional models of these units of measure.

   Provide opportunities for students to make three-dimensional models of rectangular solids and cylinders using the patterns provided in Resource 10: Three-Dimensional Patterns. Ask students to determine the surface area and volume of the three-dimensional models they make.

10. Students need to develop strategies for using the meters, scales and gauges that they encounter in occupational courses and in everyday life. Mathematics teachers are encouraged to confer with occupational teachers in planning activities that will develop measurement skills that are required by students.

    Students are given practice in interpreting the calibrations on a variety of meters, scales and gauges in Resource 11: Meters, Scales and Gauges.

## PROJECT WORK

Project work in mathematics class often increases student motivation and provides opportunity for the application and maintenance of skills and processes. When effectively planned, a project will integrate mathematical skills with skills learned in other subject areas. Opportunity for group work will foster the development of social skills and responsibility.

Topics around which projects might be developed include:

- Planning a Vacation
- Buying a Car
- Preparing a Personal Time Schedule
- Developing a Budget
- Finding a Place to Live
- Orienteering in the Outdoors
- Operating a Simple Retail Business
- Decorating a Home.

## SAMPLE PROJECT: BUYING A CAR

Assume that you are going to buy a car. After considering your monetary resources, decide whether you will buy a new car or used car. Obtain information on buying, financing and maintaining a car from a variety of sources:

- the newspaper
- local financial institutions
- the insurance bureau.

### OUTLINE

A. **Assessing Personal Finances**

- How much money do you have at present?
- How much money do you earn?
- How much money can you save before you buy the car? (Prepare a personal budget.)
- How much money can you afford to spend on a car?

B. **Considering Additional Expenses**

- Insurance:
  - What type of insurance do you require?
  - How much coverage do you need?
  - What deductible will you need?
- Registration and licencing fees.
- Maintenance and operating costs.

C. **Shopping for a Car**

- Consumer reports
- Newspaper ads

- Bartering:
  - What does "book value" mean?
  - How do prices compare among competing businesses?
  - What constitutes a reasonable offer?

- Questions to ask:
  - What is the odometer reading?
  - What options does the car have?
  - Was the car in previous accidents?
  - Is the car in good condition? (e.g., Is the car mechanically safe? What is the condition of the car's exterior?)
  - Are there any liens against the car?

### D. Financing

- Amount of down payment.

- Value of trade-in.

- Amount to be borrowed.

- Choice of lending institution.

- Type of loan:
  - interest rate
  - payment schedule
  - penalty for late or missed payment
  - amortization period.

### E. Driver Responsibilities

- Safe driving practices:
  - driving lessons
  - safe stopping distances
  - safe following distances
  - use of seat belts.

- Routine safety checks:
  - brakes
  - tires
  - lights and turning signals
  - steering and alignment.

## INTEGRATED SKILLS

Skills from mathematics and other subject areas that might be integrated with this project are identified below.

### Mathematics

- problem solving
- estimation
- calculator skills
- reading and interpreting tables and charts
- personal budgeting skills
- calculation of interest
- calendar skills
- ratio, proportion and percent skills.

### English

- gathering information
- reading and interpreting information presented in the newspaper
- considering the opinions of others
- interpreting the presenter's intent
- writing business letters.

### Science

- recognizing the subsystems in an automobile
- describing simple maintenance procedures that will contribute to vehicle efficiency and safety
- identifying the impact of technological developments on the automobile industry
- using problem-solving and decision-making skills.

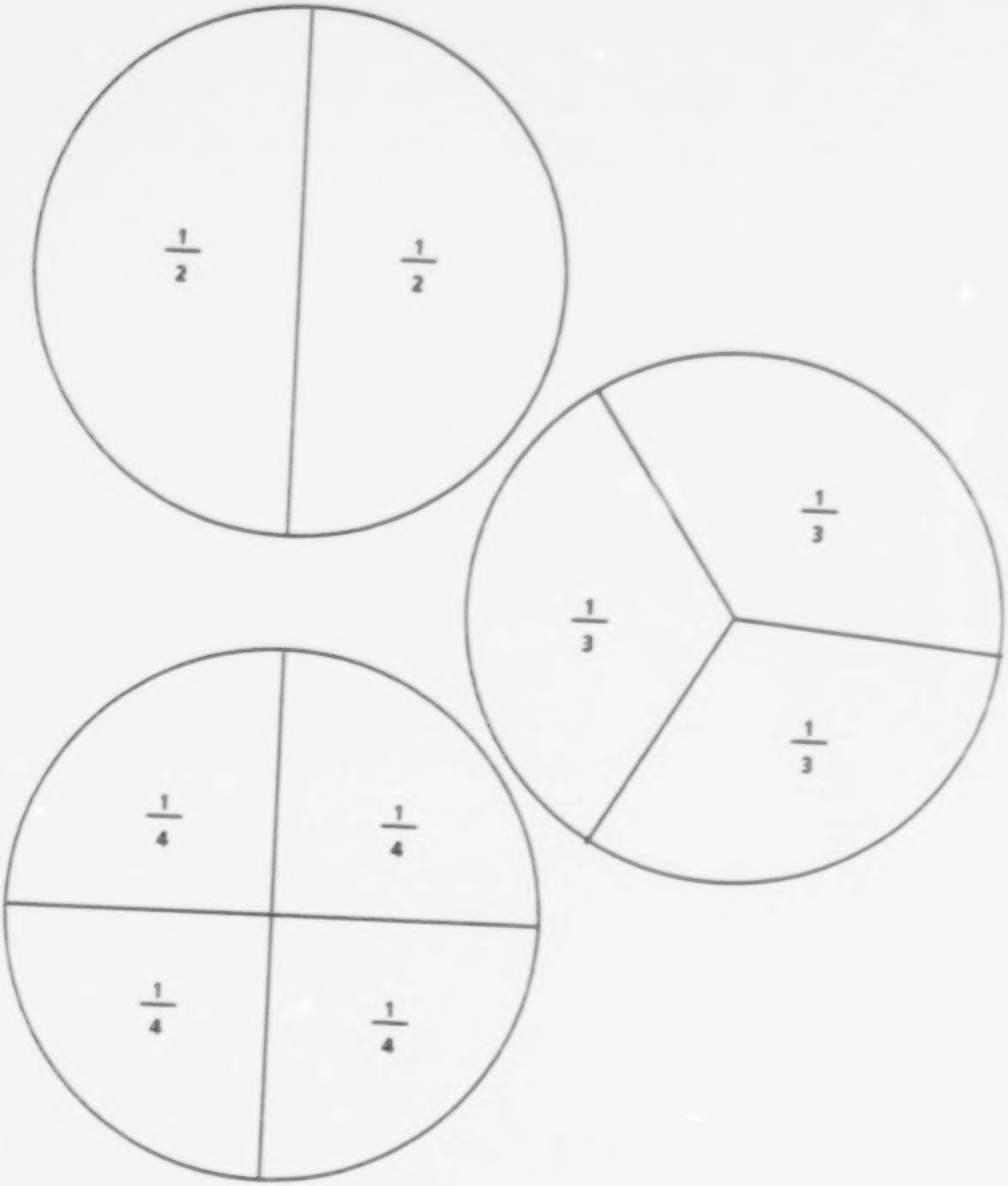
### Social Studies

- decision making
- personal development and personal organization
- interpretation of laws and bylaws
- personal economics
- knowledge of resources in the local community.

### Occupational Courses

- budgeting
- consumer awareness and skills
- personal and interpersonal skills
- business communication skills
- entrepreneurial awareness.

## RESOURCE 1: FRACTION CIRCLES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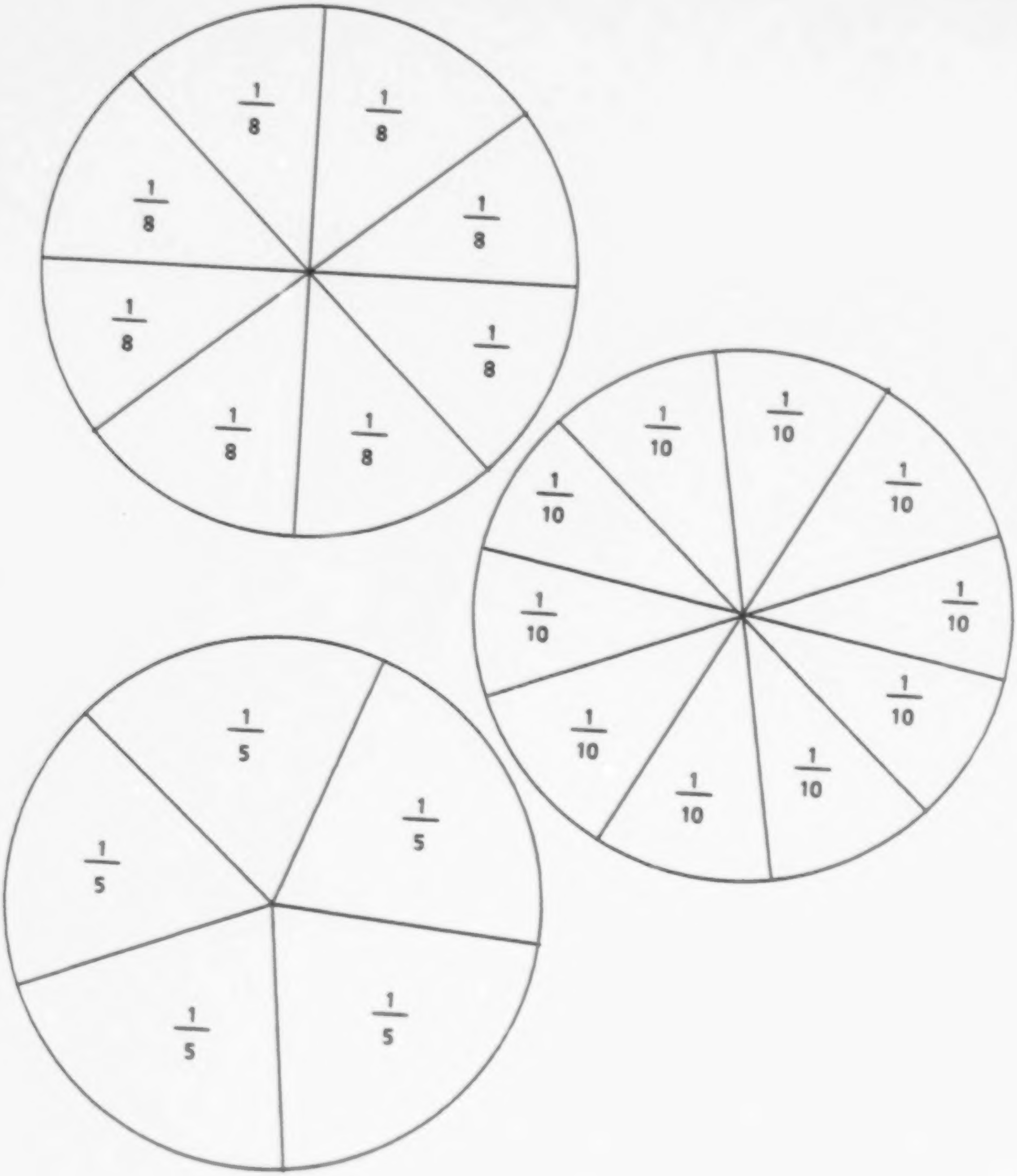
## RESOURCE 1: FRACTION CIRCLES (continued)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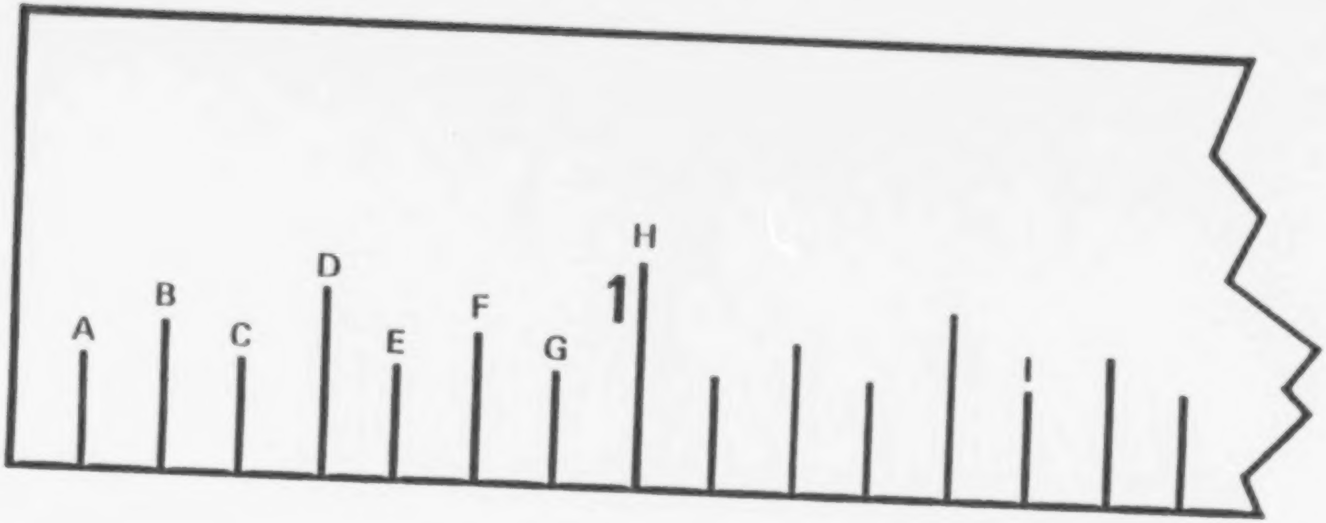
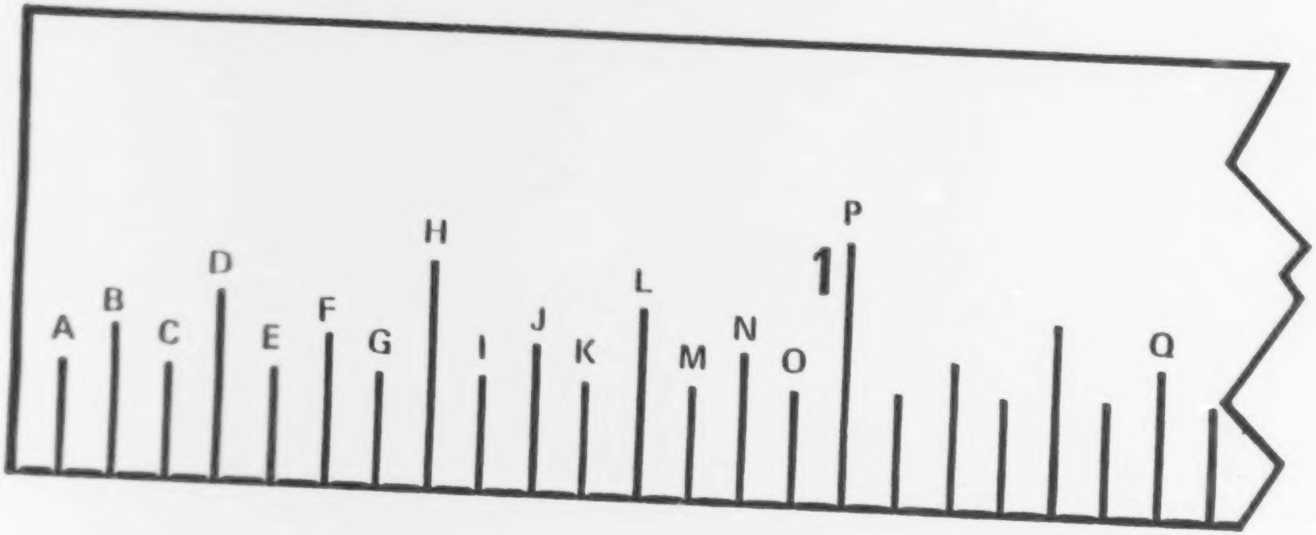
# Eighths Rule



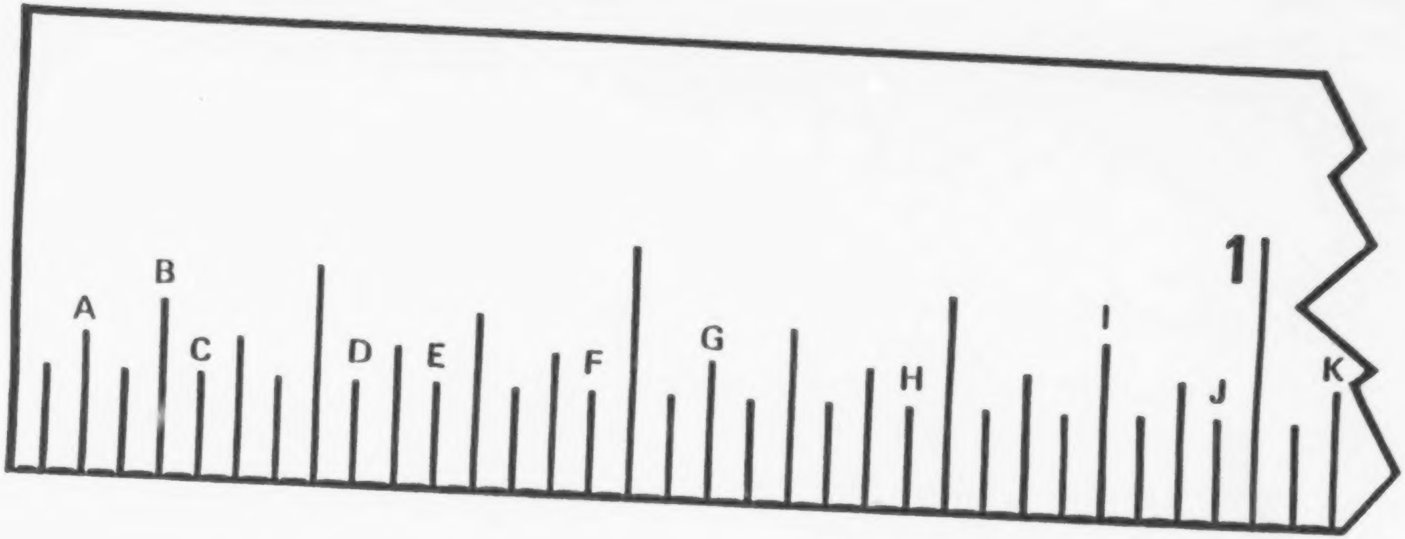
A = 1/8, B = 2/8 = 1/4, C = 3/8, D = 4/8 = 1/2,
E = 5/8, F = 6/8 = 3/4, G = 7/8, H = 8/8 = 1,
I = 13/8 = 1 5/8

# Sixteenths Rule



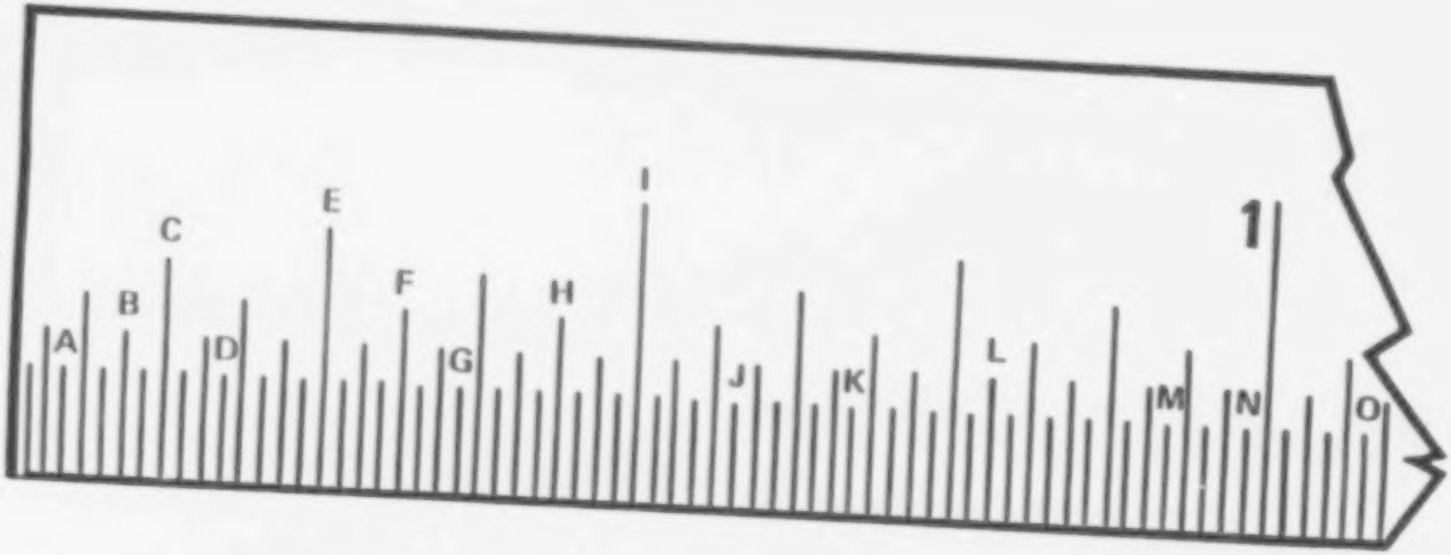
A = 1/16, B = 2/16 = 1/8, C = 3/16, D = 4/16 = 1/4,
E = 5/16, F = 6/16 = 3/8, G = 7/16, H = 8/16 = 1/2,
I = 9/16, J = 10/16 = 5/8, K = 11/16, L = 12/16 = 3/4,
M = 13/16, N = 14/16 = 7/8, O = 15/16, P = 16/16 = 1,
Q = 22/16 = 1 6/16 = 1 3/8

# Thirty-Seconds Rule



A = 2/32 = 1/16, B = 4/32 = 1/8, C = 5/32, D = 9/32,
E = 11/32, F = 15/32, G = 18/32 = 9/16, H = 23/32,
I = 28/32 = 14/16 = 7/8, J = 31/32,
K = 34/32 = 1 2/32 = 1 1/16

# Sixty-Fourths Rule



A = 3/64, B = 6/64 = 3/32, C = 8/64 = 1/8, D = 11/64,
E = 16/64 = 8/32 = 1/4, F = 20/64 = 10/32 = 5/16,
G = 23/64, H = 28/64 = 14/32 = 7/16, I = 32/64 = 1/2,
J = 37/64, K = 43/64, L = 50/64 = 25/32, M = 59/64,
N = 63/64, O = 69/64 = 1 5/64

## RESOURCE 3: GEOMETRIC PATTERNS

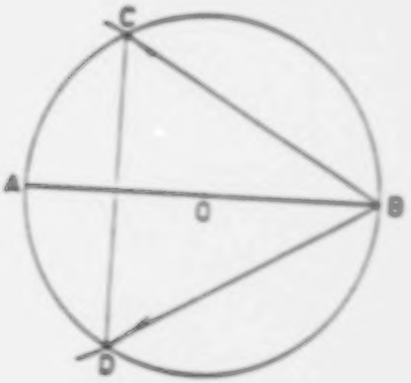
The constructions of inscribed equilateral triangles, regular hexagons, squares and regular octagons can be used to produce a variety of geometric designs.

### A. TRIANGLES AND HEXAGONS

Follow the instructions for inscribing an equilateral triangle and a regular hexagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

1. Inscribing an Equilateral Triangle

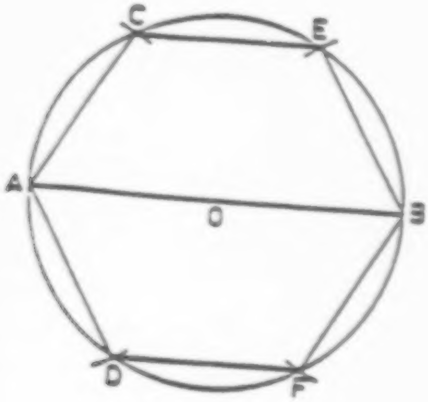
   a) With a compass draw a circle with centre 0.
   b) In this circle draw a diameter AB.
   c) With centre A and radius A0, draw an arc intersecting the circle at C and D.
   d) Connect B, C, and D to form an equilateral triangle.



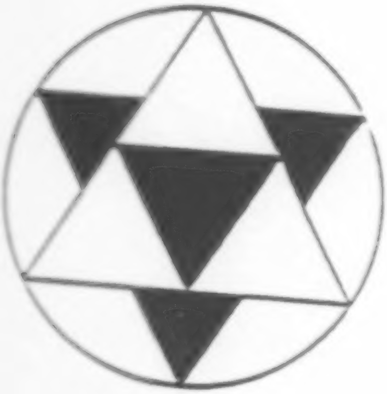
2. Inscribing a Regular Hexagon

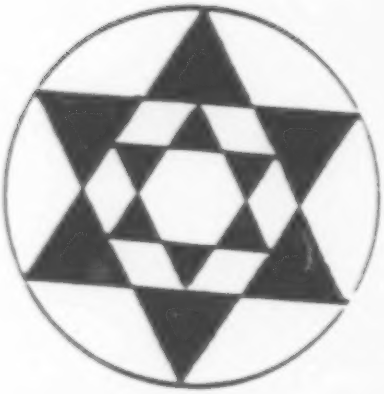
   a) With a compass, draw a circle with centre 0.
   b) In this circle draw a diameter AB.
   c) With centre A and radius A0, draw arcs intersecting the circle at C and D. Similarly, with centre at B and radius B0, draw arcs intersecting the circle at E and F.
   d) Connect the six points on the circle to form a regular hexagon.

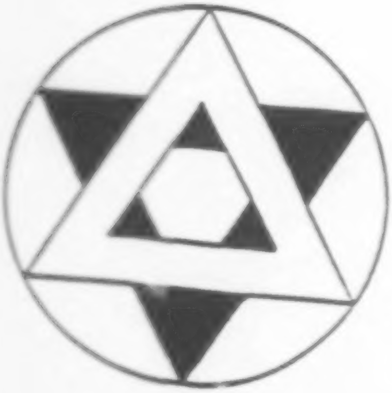
(see diagram on next page)

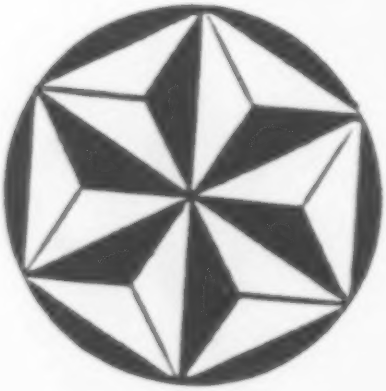


3. Examples of Designs







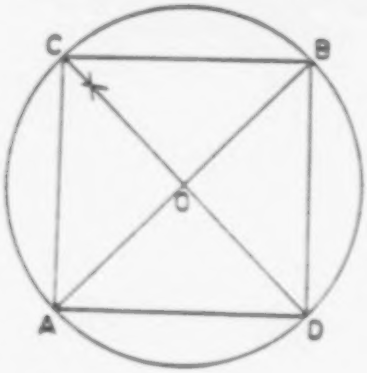


## B. SQUARES AND OCTAGONS

Follow the instructions for inscribing a square and a regular octagon in a circle. By joining various points and colouring or shading parts of the figures, create geometric designs similar to those illustrated in the examples.

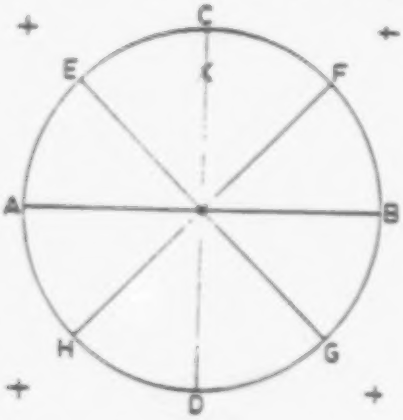
### 1. Inscribing a Square

a) Draw a circle with centre 0 and draw a diameter AB.
b) Construct another diameter CD which is the perpendicular bisector of AB.
c) Connect the four points on the circle to form a square.



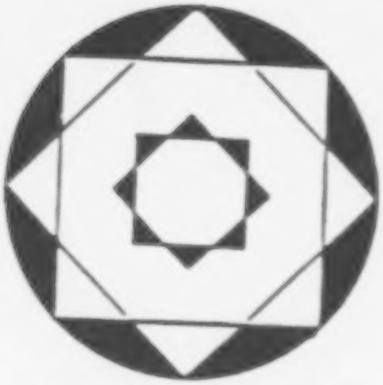
### 2. Inscribing a Regular Octagon

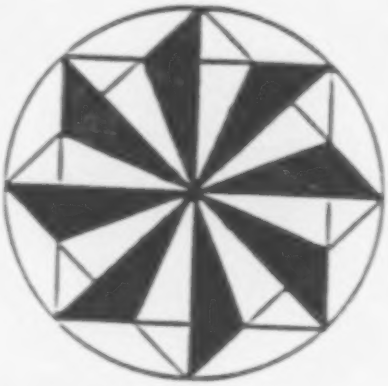
a) Draw a circle with centre 0 and locate points A, B, C and D as in the construction of a square above.
b) Bisect ∠AOC, ∠COB, ∠BOD, ∠DOA to obtain points E, F, G and H on the circle respectively.
c) Connect A, E, C, F, B, G, D and H in order to form a regular octagon.

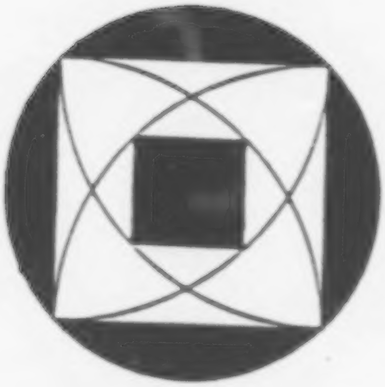


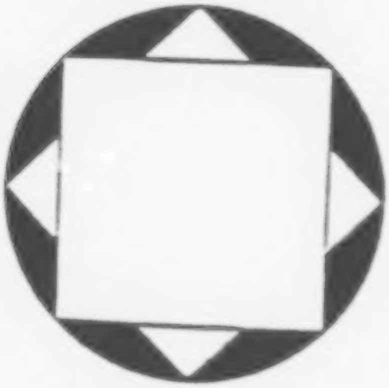
## RESOURCE 3: GEOMETRIC PATTERNS (continued)

3. Examples of Designs





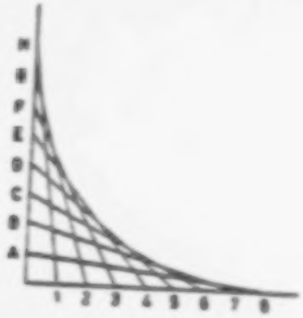




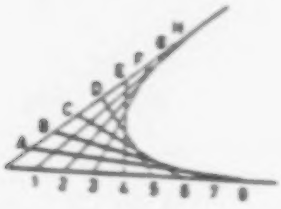
## RESOURCE 4: LINE DESIGN

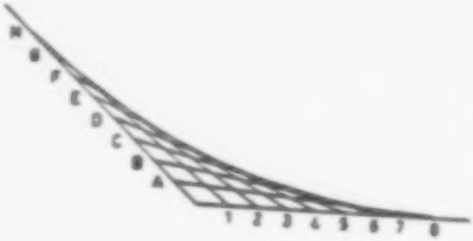
The construction of line segments can produce interesting geometric patterns and designs.

A. Draw a right angle and mark off from the vertex eight segments of equal length along each arm of the angle. (Use either a compass or a ruler to obtain equal lengths.) Label the end points of the segments as in the following diagram and connect A to 8, B to 7, . . . . , and H to 1.

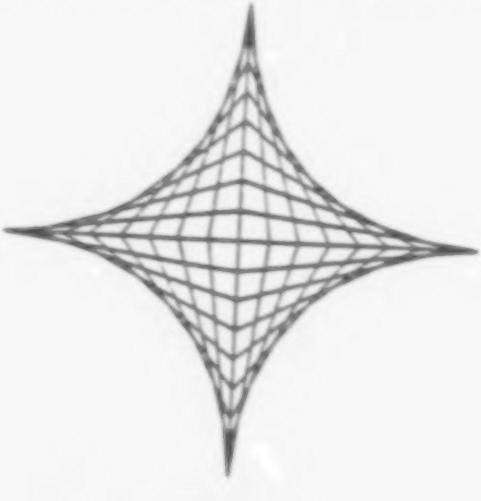


B. Try the procedure outlined above with an acute angle and an obtuse angle.



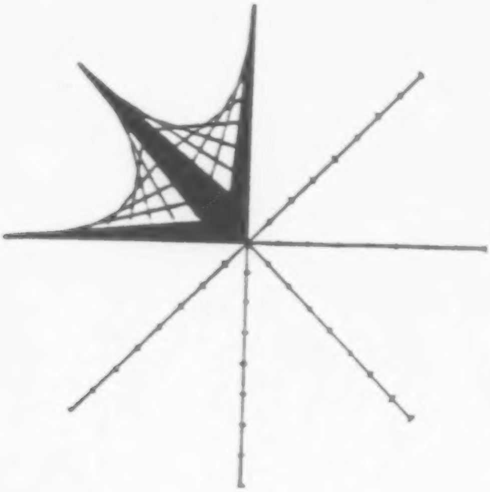


C. Draw a line segment. Construct a second line segment with the same length as the first in such a way that each segment is a perpendicular bisector of the other. Use the four angles formed to create this design.

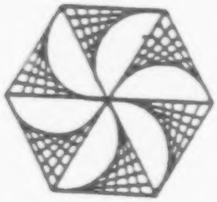


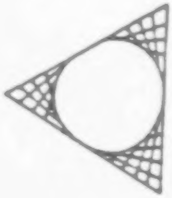
## RESOURCE 4: LINE DESIGN (continued)

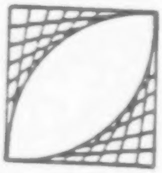
D. More involved designs can be obtained by bisecting angles and using the angles formed for additional designs. The following example makes use of angle bisectors as well as some shading.

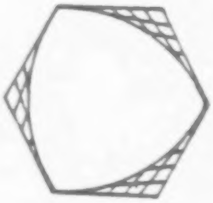


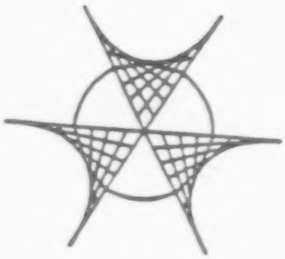
E. Construct additional line designs using the procedures you have learned. The illustrations below may give you some ideas.

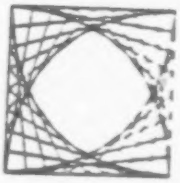


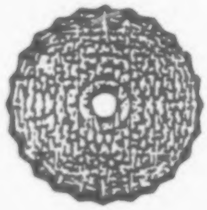


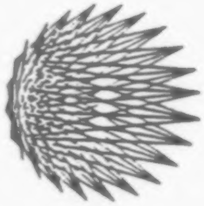










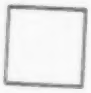


# RESOURCE 5: ATTRIBUTE DOMINOES

**MATERIALS:**

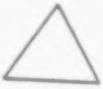
Construct a set of 60 attribute shapes using:

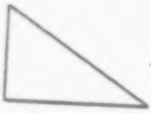
- three different colours of paper (e.g., blue, green, red)
- five large geometric shapes


– large square

– large circle

– large rectangle


– large equilateral triangle


– large isosceles-right triangle

- five small geometric shapes

– small square

– small circle

– small rectangle

– small equilateral triangle

– small isosceles right triangle

- striped and non-striped shapes in each size and colour.

The set of 60 attribute shapes will consist of 20 blue shapes, 20 green shapes and 20 red shapes. The chart below illustrates the attributes that will be common to each set of coloured shapes.

20 blue shapes

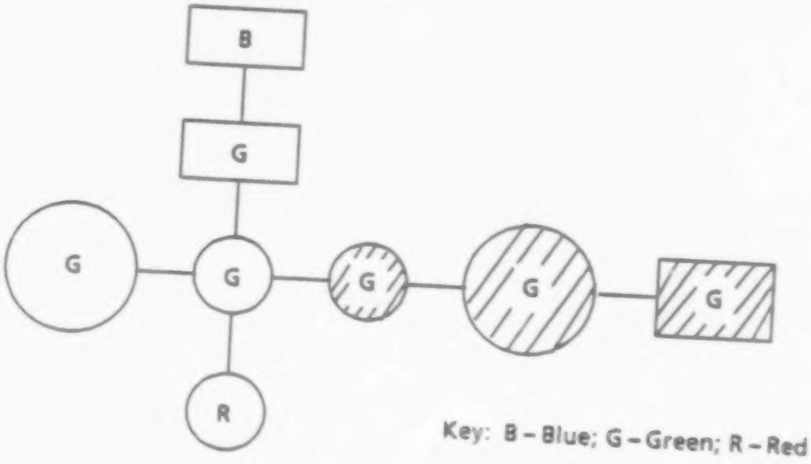
- 10 large
  - 5 striped
    - square
    - circle
    - rectangle
    - equilateral triangle
    - isosceles triangle
  - 5 non-striped
    - square
    - circle
    - rectangle
    - equilateral triangle
    - isosceles triangle
- 10 small
  - 5 striped
    - square
    - circle
    - rectangle
    - equilateral triangle
    - isosceles triangle
  - 5 non-striped
    - square
    - circle
    - rectangle
    - equilateral triangle
    - isosceles triangle

## RESOURCE 5: ATTRIBUTE DOMINOES (continued)

### DIRECTIONS FOR PLAY:

The game can be played with two to four players, each of whom receives ten attribute shapes. The first player places one game piece in the centre of the table. The other players take turns placing a piece in one of four directions, building from the previous piece. In order to play an attribute shape, it must differ from the piece it is placed beside by only one attribute. If a player cannot play, he must draw a new attribute shape from a container holding the extra pieces. If the player cannot play this shape, the play passes to the next person. The first player to play all of their attribute shapes is declared the winner.

A sample game pattern is illustrated below:



Key: B – Blue; G – Green; R – Red

NOTE: Attribute shapes can be varied according to the geometric figures that are being studied. Students in Grade 9 may wish to use the parallelogram, hexagon and octagon in conjunction with two other geometric shapes.

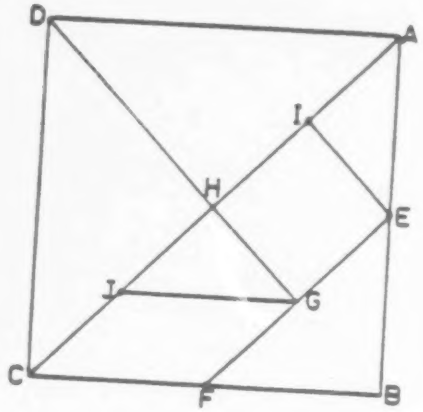
# RESOURCE 6: THE TANGRAM

This exercise involves making a variety of geometric figures by rearranging pieces of a puzzle.

## MAKING THE PUZZLE

If tangram pieces are not available, a set may be constructed by following these directions:

1. On cardboard draw a square ABCD with each side at least 10 cm in length.
2. Locate midpoints E and F of sides $\overline{AB}$ and $\overline{BC}$ respectively.
3. Draw $\overline{EF}$ and $\overline{AC}$.
4. Draw the portion of $\overline{DB}$ from D to $\overline{EF}$ (shown in the diagram as $\overline{DG}$ ). Let H be the point of intersection of $\overline{DG}$ and $\overline{AC}$.
5. Locate I, the midpoint of $\overline{AH}$, and J, the midpoint of $\overline{CH}$. Draw $\overline{EI}$ and $\overline{GJ}$ .
6. Cut out the seven pieces.



## SOLVING THE PUZZLE

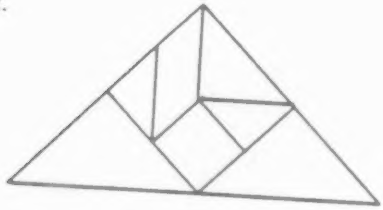
Using the seven tangram pieces, arrange them in such a way as to form:

1. a square
2. a triangle
3. a rectangle which is not a square
4. a parallelogram which is not a rectangle
5. a trapezoid
6. a convex polygon (a polygon in which each of the diagonals fall within its interior; 13 such polygons can be formed with the tangram pieces).

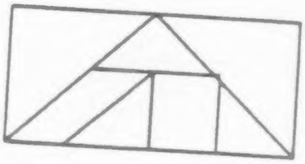
## RESOURCE 6: THE TANGRAM (continued)

### SOME POSSIBLE SOLUTIONS

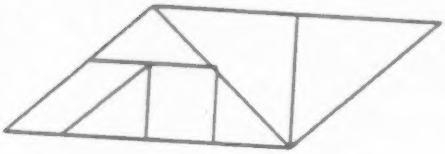
2.



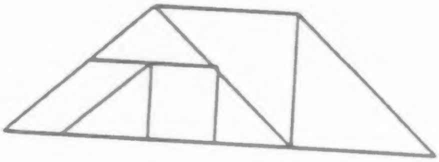
3.



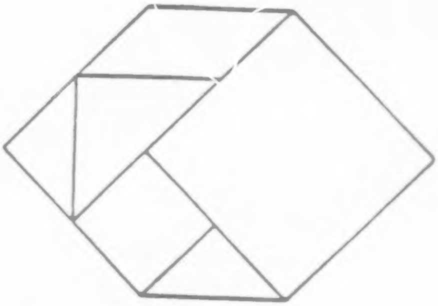
4.



5.



6. One convex polygon can be made in this way. See if you can make the other 12 convex polygons. Record your results.



### FINDING THE AREA

Give the small square, EGHI, an area of one square unit. Compute the area of each of the other six tangram pieces using the square as the unit of measure.

## RESOURCE 7: TESSELLATIONS

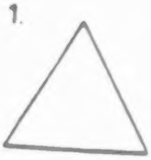
A. A tessellation is a pattern that can completely "fill" a surface or region without overlapping or leaving spaces. The word "tessellation" comes from an ancient Latin word which means "to cover with tiles". Some tessellations which we encounter in our everyday life are tiled floors and wallpapers. Name some other examples of tessellations.

B. Using a piece of cardboard, construct tiles identical to those shown in Part C. You will require several tiles of each shape. Your tiles will include the following shapes:

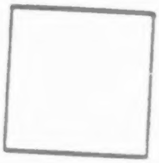
- equilateral triangle
- isosceles right triangle
- parallelogram
- rectangle
- square (three different sizes)
- rhombus
- trapezoid
- regular hexagon.

C. Using only the tiles that have the shape of an equilateral triangle or an isosceles right triangle, cover each of the following shapes. Give the name of the polygon you have covered.

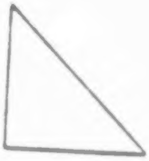
1.



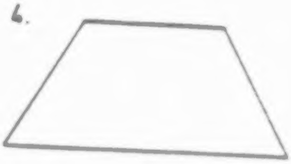
2.



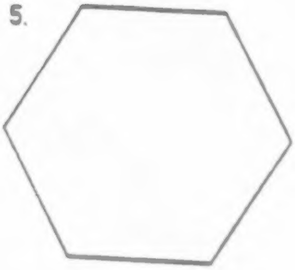
3.



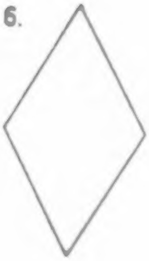
4.



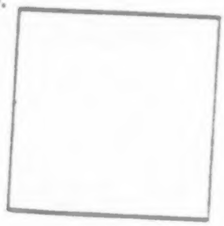
5.



6.

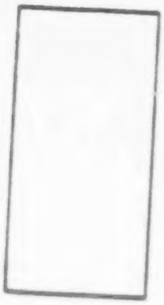


7.

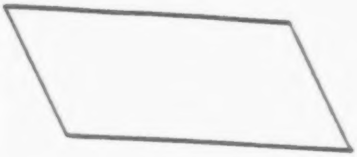


## RESOURCE 7: TESSELLATIONS (continued)

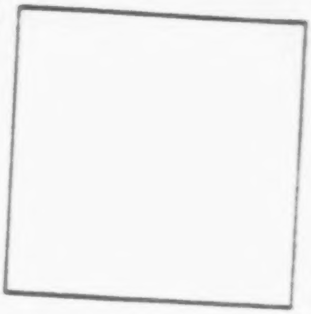
8.



9.



10.



D. Make a sketch to illustrate your use of equilateral triangles to form the hexagon in C5.

Make another sketch to show how a large equilateral triangle can be made from several small ones.

E. Which of the following shapes from Part C can produce tessellations?

| | Yes | No |
|---|---|---|
| square | ____ | ____ |
| parallelogram | ____ | ____ |
| rhombus | ____ | ____ |
| trapezoid | ____ | ____ |

Sketch the tessellations produced by each of these shapes.

In each of these tessellations, can you find a larger version of the basic shape used to make the tessellation? If you can, outline it on your sketch. Which shapes can produce tessellations but cannot be put together to make larger versions of themselves?

## RESOURCE 8: ACCURACY IN ESTIMATION

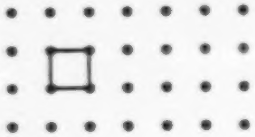
Select a variety of objects in the classroom, to be measured. Estimate the measure of each object. Then, using a variety of measuring tools, measure the objects. Use the formula provided below to express the accuracy of each estimate as a percent.

$$\frac{\text{estimated measure}}{\text{actual measure}} \times 100.$$

| OBJECT | ESTIMATE | ACTUAL | ACCURACY |
|---|---|---|---|
| 1. | | | |
| 2. | | | |
| 3. | | | |
| 4. | | | |
| 5. | | | |
| 6. | | | |
| 7. | | | |
| 8. | | | |
| 9. | | | |
| 10. | | | |

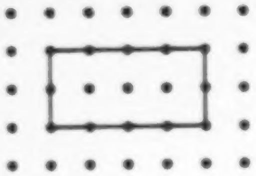
## RESOURCE 9: UNDERSTANDING AREA

You will require the use of a geoboard or dot paper in doing these exercises. The unit square illustrated below will be used in measuring the area of a variety of geometric figures.



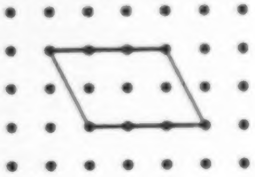
### A. RECTANGLE

Form a rectangle on the geoboard. Determine its area by counting the unit squares. Compare the area of the rectangle to its perimeter.

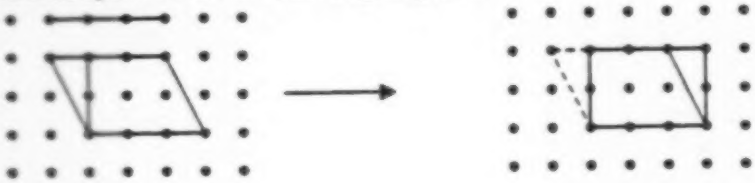


### B. PARALLELOGRAM

Form a parallelogram and find its area.

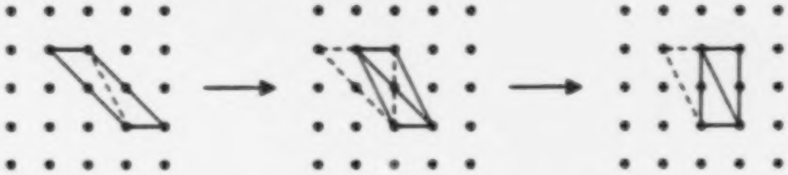


One technique for finding the area is to identify a triangular region within the parallelogram and perform a slide upon it as in the following diagram.



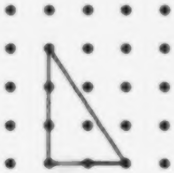
The area of the rectangle can be found by counting. Therefore, the area of the parallelogram is also determined.

In certain cases, this technique has to be applied more than once.

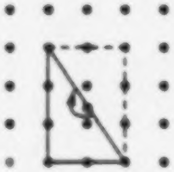


C. RIGHT TRIANGLE

Form a right triangle and determine its area.



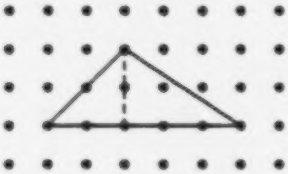
If the triangle is given a half-turn about the midpoint of the side opposite the right angle, a rectangle is identified. The area of the rectangle can be determined by counting. The area of the triangle is one-half the area of the rectangle.



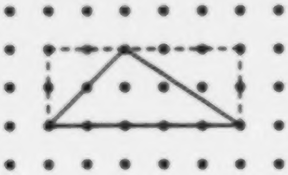
D. SCALENE TRIANGLE

Form a scalene triangle and find its area. Some hints for finding the area are:

1. "Break up" the triangle into right triangles.

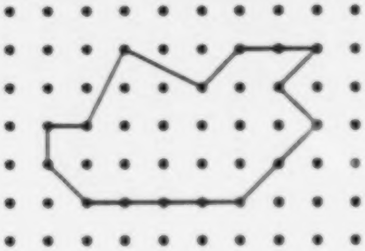


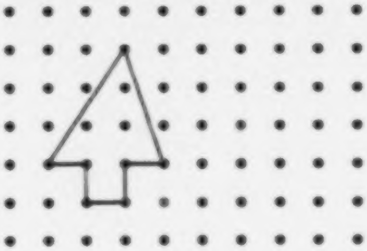
2. Subtract from the area of a larger rectangle.



E. OTHER POLYGONS

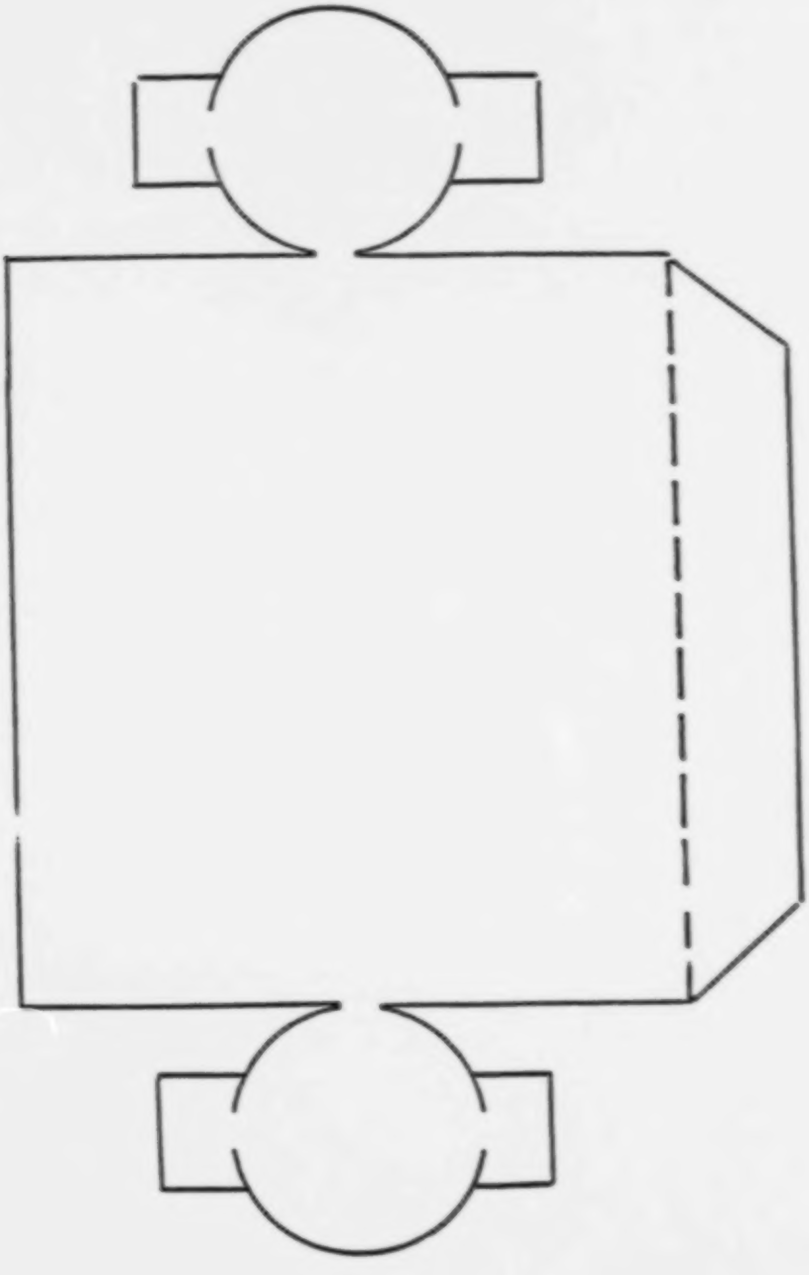
Form other regions similar to the following and devise methods of finding their areas.



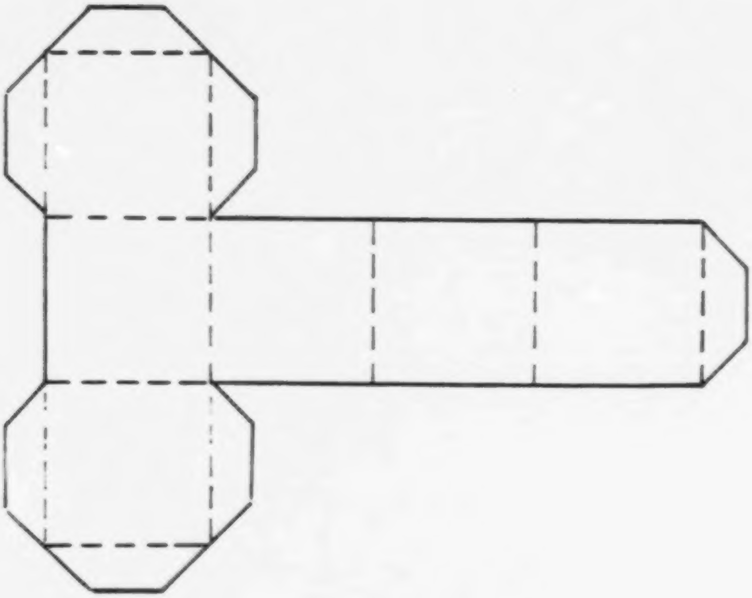


## RESOURCE 10: THREE-DIMENSIONAL PATTERNS

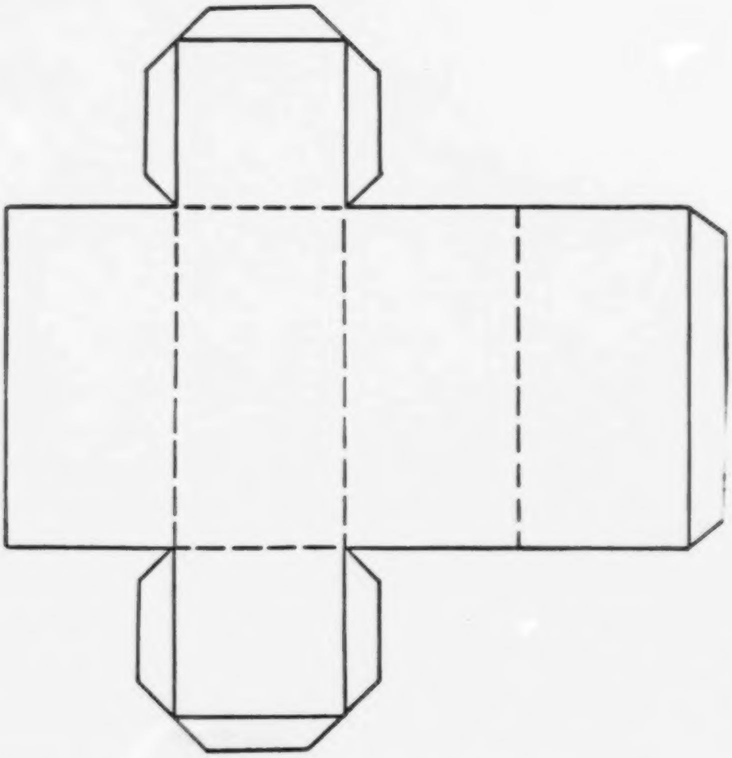
CYLINDER



**CUBE**



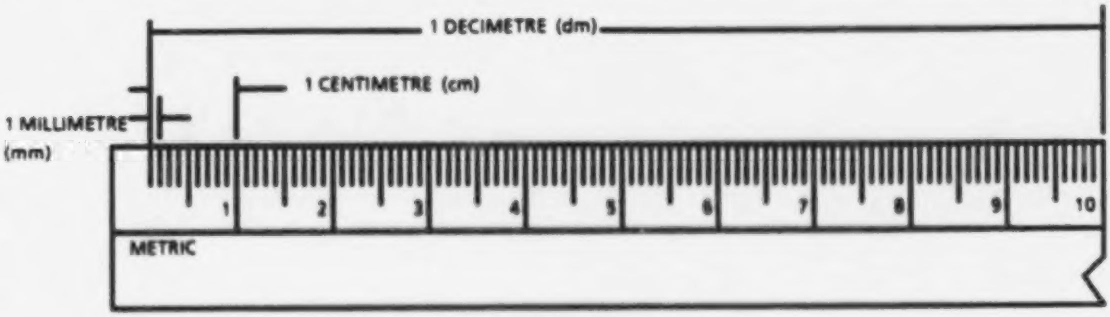
**RECTANGULAR SOLID**



# RESOURCE 11: METERS, SCALES AND GAUGES

## USING A METRIC RULER

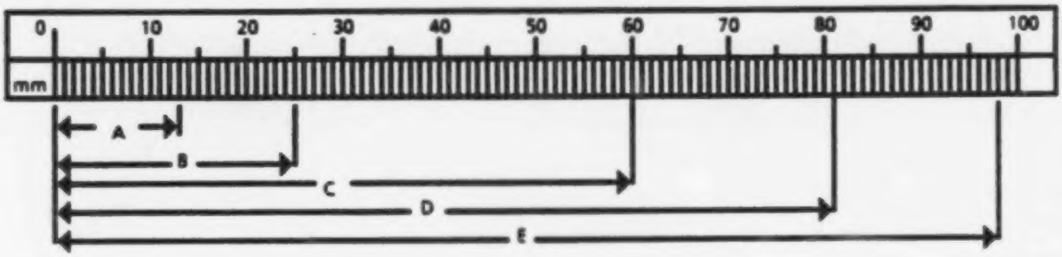
Study the scale on the metric ruler illustrated below.


1 DECIMETRE (dm)
1 CENTIMETRE (cm)
1 MILLIMETRE (mm)
1
2
3
4
5
6
7
8
9
10
METRIC

METRIC SCALE

Exercises:

1. Using the two scales that follow, determine the length of each line to the nearest millimetre. Record your answer first in millimetres, then in centimetres.


0
10
20
30
40
50
60
70
80
90
100
mm
A
B
C
D
E

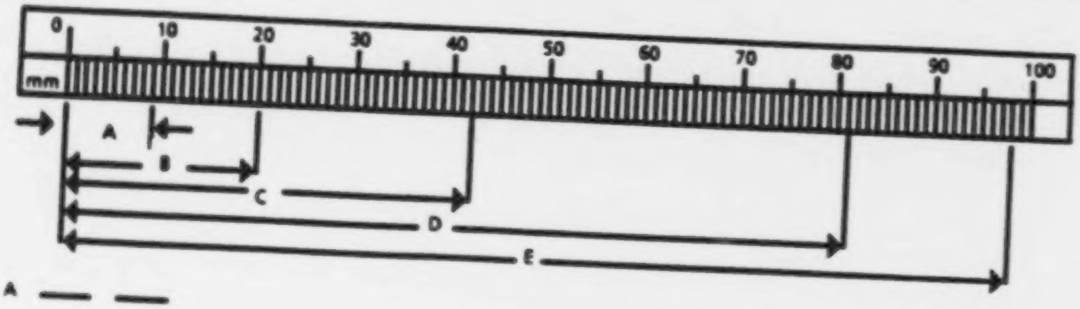
A ____ ____

B ____ ____

C ____ ____

D ____ ____

E ____ ____



A ____ ____

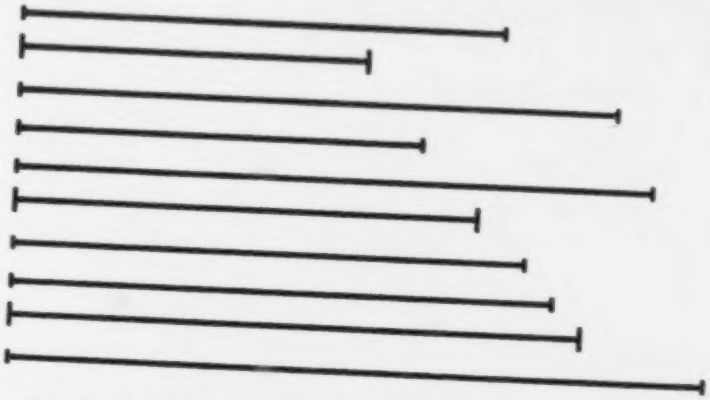
B ____ ____

C ____ ____

D ____ ____

E ____ ____

2. Use a metric scale to measure the length of each line to the nearest millimetre. Record your answer first in millimetres, then in centimetres.



____ ____

____ ____

____ ____

____ ____

____ ____

____ ____

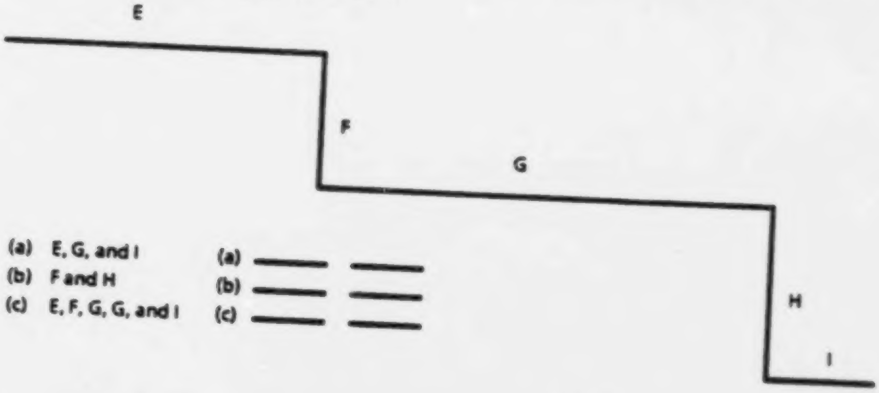
____ ____

____ ____

____ ____

____ ____

3. Use a metric scale to measure the total length of the following diagram. Record your answer first in millimetres, then in centimetres.



(a) E, G, and I

(b) F and H

(c) E, F, G, G, and I

(a) ____ ____

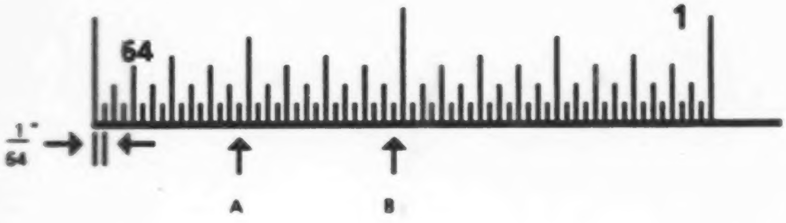
(b) ____ ____

(c) ____ ____

## USING AN INCH RULER

The diagrams below will help you read the different divisions on a ruler. Notice the number 64 printed on the ruler. This number means that the inch is divided into 64 parts, each part equal to 1/64th of an inch. Also notice that the divisions have lines of different length.

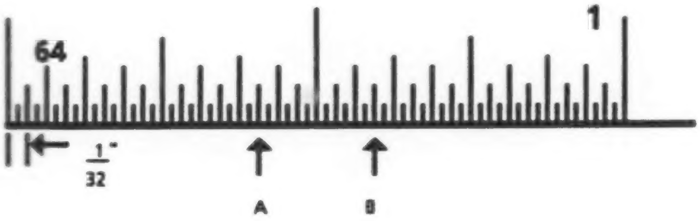
(a) The shortest line represents 64ths of an inch.



Point A represents: counting from the end, we count 15 divisions. Therefore, point A represents 15/64th's of an inch.

Point B represents: ______________________________.

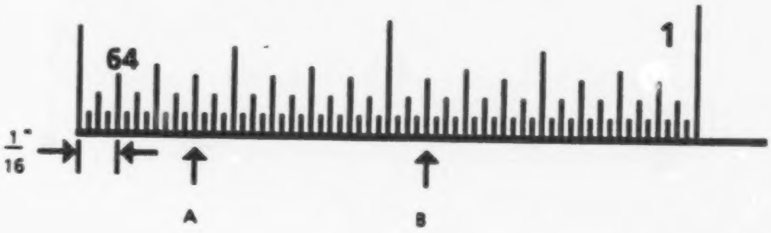
(b) The next longest line represents two parts of an inch (i.e., 1/64 + 1/64 = 2/64 = 1/32). Therefore, each of these lines represent 1/32nd of an inch.



Point A represents: counting every line of this length as well as any longer line, we count 13. Therefore, point A represents 13/32nd's of an inch.

Point B represents: ______________________________.

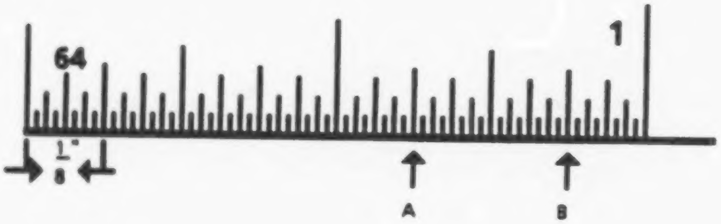
(c) The next longest line represents 4 parts of an inch (i.e., 1/64 + 1/64 + 1/64 + 1/64 = 4/64 or 1/16). Therefore, each of these lines represent 1/16th of an inch.



Point A represents: counting every line of this length or longer, we count 3. Therefore, point A represents 3/16th's of an inch.

Point B represents: ______________________.

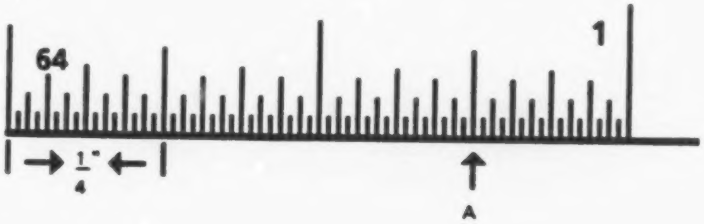
(d) The next longest line represents 8 parts of an inch (i.e., 8/64 or 1/8). Therefore, each of these lines represents 1/8th of an inch.



Point A represents: counting every line of this length or longer, we count 5. Therefore, point A represents 5/8th's of an inch.

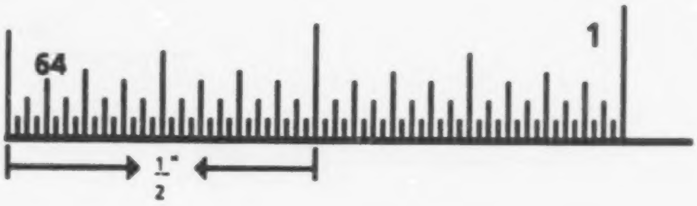
Point B represents: ______________________.

(e) The next longest line represents 16 parts of an inch (i.e., 16/64 or 1/4). Therefore, each of these lines represent 1/4 of an inch.

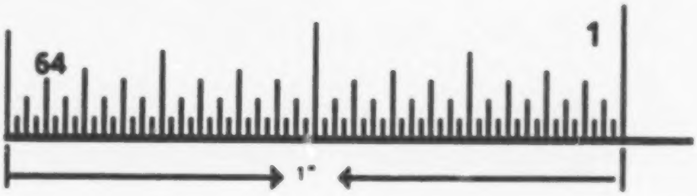


Point A represents: ______________________.

(f) The second longest line represents 32 parts of the inch (i.e., 32/64 or 1/2). Therefore, each of these lines represent 1/2 of an inch.



(g) The longest line represents one whole inch.



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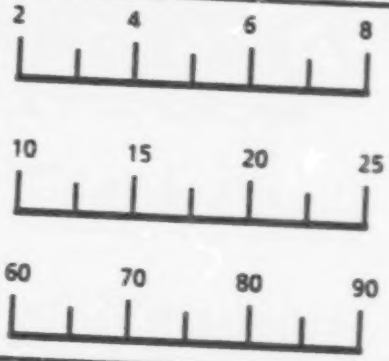
## READING OTHER SCALES AND GAUGES

Before you read any scale, you must examine it carefully to see how it is calibrated.

- Look at the numbers written on the scale. They may not be sequenced in the usual order (e.g., 1, 2, 3, 4 and so on).

  Study the examples:



Numbers go up by 2's.

Numbers go up by 5's.

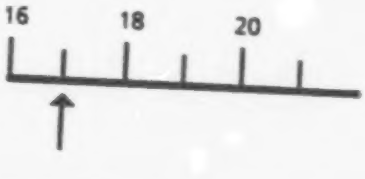
Numbers go up by 10's.

Other scales may have numbers that go up by 20's, 50's, 100's, or 1000's.

- Look at the marks between the numbers. The numbers represented by these marks depend upon:

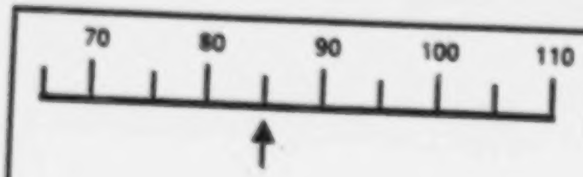
  - the number of spaces between two consecutive numbers
  - the difference between two consecutive numbers on the scale.

Study the examples:

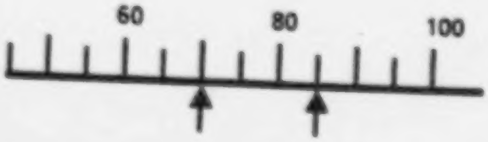


Numbers go up by 2's.
The arrow points to 17.

## RESOURCE 11: METERS, SCALES AND GAUGES (continued)



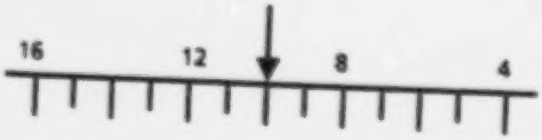
Numbers go up by 10's.
The arrow points to 85.



Numbers go up by 20's.
The first arrow points to 70.
The second arrow points to 85.

### READING SCALES THAT GO BACKWARD

Sometimes a scale appears to go backward. These scales must be read from right to left, as in the example below.



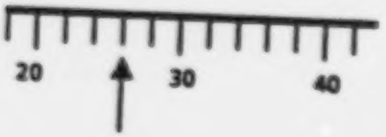
The reading is 10.

"Backward" scales are often found on meters that are turned by gears (e.g., a water meter or an electricity meter in a house).

### READING MORE DIFFICULT SCALES

An easy way to figure out the value of each space on a scale is to divide the difference between two numbers on the scale by the number of spaces between them.

Example 1:



The difference between 20 and 30 is 10.

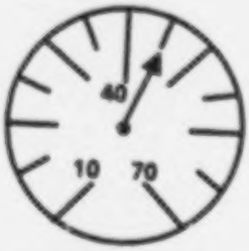
There are 5 spaces between 20 and 30.

Therefore, each space represents $\frac{10}{5}$ or 2 units.

Each space represents 2. The arrow indicates 26.

## RESOURCE 11: METERS, SCALES AND GAUGES (continued)

Example 2:



The difference between 40 and 70 is 30.

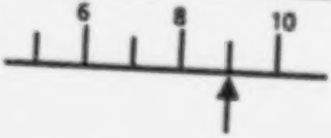
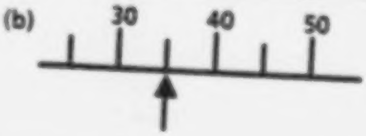
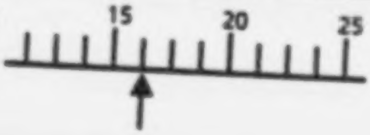
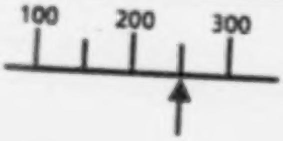
There are 6 spaces between 40 and 70.

Therefore, each space represents $\frac{30}{6}$ or 5 units.

Each space represents 5. The arrow indicates 45.

**EXERCISES**

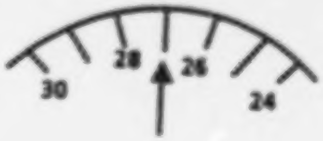
1. Look carefully at each scale. Then answer the questions.

| | |
|---|---|
| (a)  | a. The numbers written on the scale go up by ____.<br>b. The arrow points to the mark that means ___. |
| (b)  | a. The numbers written on the scale go up by ____.<br>b. The arrow points to the mark that means ___. |
| (c)  | a. The numbers written on the scale go up by ____.<br>b. The arrow points to the mark that means ___. |
| (d)  | a. The numbers written on the scale go up by ____.<br>b. The arrow points to the mark that means ___. |

## RESOURCE 11: METERS, SCALES AND GAUGES (continued)

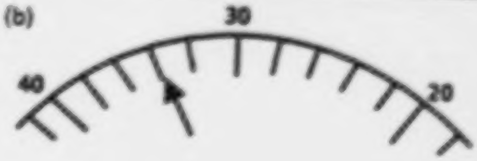
2. Read each of the following scales.

(a)



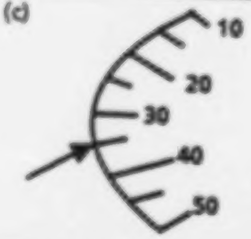
________

(b)



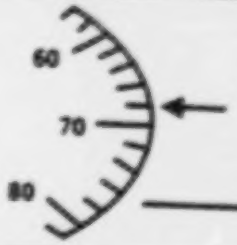
________

(c)


10
20
30
40
50

________

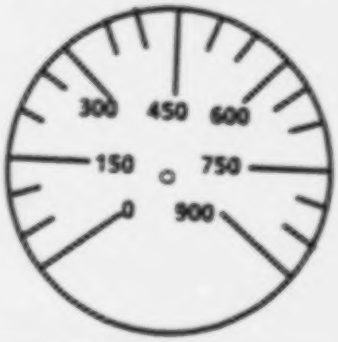
(d)



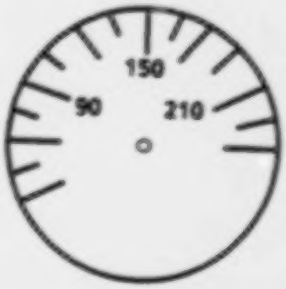
________

3. Give the value of each space on the scales illustrated below.

(a) ________

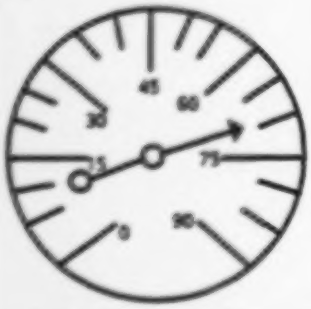

300
450
600
150
750
0
900

(b) ________

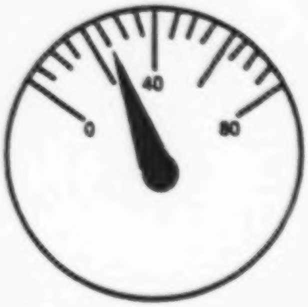


4. Give the reading indicated on each of the following scales.

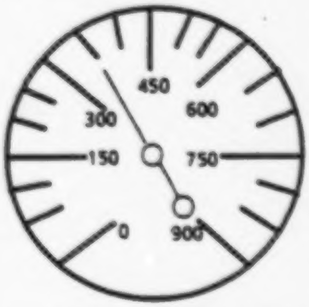
(a) ________


45
60
30
15
75
0
90

(b) ________

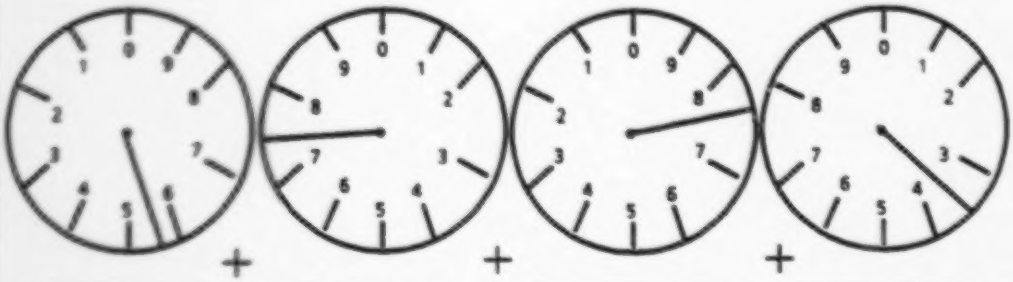


(c) ________


450
600
300
150
750
0
900

5. Give the readings indicated on each of the following electric meters.

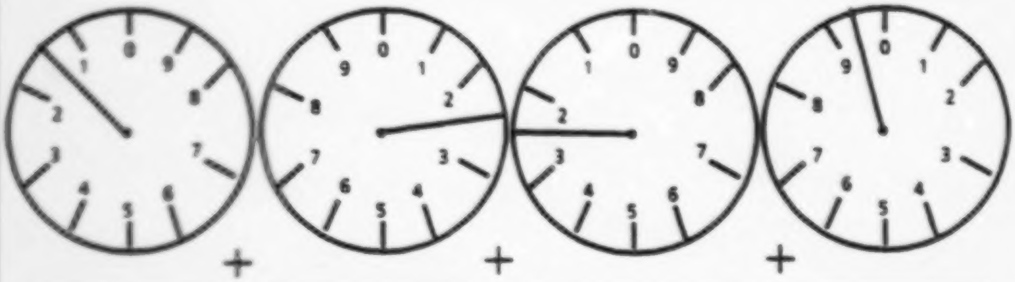
(a)


0 9 8 7 6 5 4 3 2 1
0 1 2 3 4 5 6 7 8 9
0 9 8 7 6 5 4 3 2 1
0 1 2 3 4 5 6 7 8 9

________ + ________ + ________ + ________

= ____ kwh.

(b)


0 9 8 7 6 5 4 3 2 1
0 1 2 3 4 5 6 7 8 9
0 9 8 7 6 5 4 3 2 1
0 1 2 3 4 5 6 7 8 9

________ + ________ + ________ + ________

= ____ kwh.

# ASSESSMENT/EVALUATION

The mathematics program emphasizes an understanding of concepts/skills, problem solving and application. Assessment of understanding and problem solving must go beyond determining the percent of correct responses on a test based on mathematical skills. It should also provide information about how students approach doing mathematics, the level of understanding students have of concepts, and their ability to apply mathematics in new situations. The role of assessment is to provide feedback and evidence of progress toward all instructional goals. A singular assessment technique cannot provide this evidence.

In a report of the National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, the following statement is made about assessment and evaluation within the mathematics program:

> "A common form of assessment is testing for the purpose of assigning grades. But assessment should be conceived of as a much broader and basic task than just testing and grading. Its basic purpose is to determine **what** and **how** students think about mathematics. Assessment should involve the biography of students' learning as well as the continual impact of the instructional program. Such assessment should provide the basis for improving the quality of instruction. Indeed, assessment has no raison d'être unless it is clear how assessment can and will improve instruction"[1]

It is important to recognize that through the process of evaluation we evaluate students' **performance** and not students themselves. This understanding helps one avoid permanently classifying a student as a good student or a poor student. Performance can and does change, and teachers should be aware of these changes.

Suggestions and ideas for assessing and evaluating students' performance have been included in this section of the manual, and focus attention on the use of:

- Observation
- Interviews
- Inventories/Checklists
- Anecdotal Records
- Written Assignments
  - Paper-and-Pencil Tests
  - Diagnostic Writing Assignments
- Self and Peer Evaluation.

The evaluation strategies suggested in this manual are not intended to be discrete, and should be used in conjunction with other strategies. For example, a checklist may be used to document desirable problem-solving behaviour in the classroom, and also to guide discussion and evaluate performance in an interview. Furthermore, teachers are encouraged to extend the application of these evaluation strategies to situations beyond those that are discussed.

Additional suggestions for assessment and evaluation within the mathematics program are provided in:

- *Curriculum and Evaluation Standards for School Mathematics*
- *How to Evaluate Progress in Problem Solving.*
- *Making the Grade, Evaluating Student Progress*

These resources, available through the Learning Resources Distributing Centre, are annotated in Appendix A.

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[1] Reproduced with permission from *Curriculum and Evaluation Standards for School Mathematics: Working Draft*. Copyright 1987 by the National Council of Teachers of Mathematics.

# OBSERVATION

Teachers observe all the time. When observations are documented, their effectiveness as an evaluation strategy increases immensely. Documented observations often provide the raw data required for analysis and diagnosis, and provide the basis on which to make remediation or enrichment decisions.

Mathematics lessons usually have a component in which students work individually or in small groups on assignments and projects. At this time, teachers can observe students at work, look for specific behaviours or outcomes, ask questions and make suggestions. Elements of the learning process that might be monitored through observation include:

- understanding of concepts/skills
- method of attacking problems
- work habits
- level of independence with work
- interpersonal skills and social growth.

Documentation of behaviours that are observed may occur in the form of anecdotal records or checklists. File anecdotal records and checklists in a student folder where samples of daily work, project reports and other artifacts are also placed.

**CLARIFICATION/EXAMPLE**

**Anecdotal Record**

| Name | Date | Situation | Behaviour | Comment |
|---|---|---|---|---|
| John | 1/17 | group lesson, developing meaning of fractions with graph paper | quick to help neighbouring students | understands basic fraction concepts |
| | 1/25 | computation game | unable to use mental arithmetic when multiplying by powers of 10 | redevelop and practise strategies for multiplying by powers of 10 |

A sample checklist that might be used in observing student behaviours has been provided as Resource 1: Observational Checklist of Student Behaviours.

Sometimes audiotapes or videotapes can be used to provide records that can be analyzed in more detail at a later time. Photographs are also useful in providing a record of project work. In addition to providing the tangible kinds of things that parents like to see at conference time, these records of student performance enable the teacher to measure progress with more objectivity than simply through memory of what was done.

## INTERVIEWS

A planned interview with a student or group of students is an effective technique for assessing knowledge, understanding, thinking style, attitude or personal interests. An interview removes the restriction of writing, and enables the teacher to delve more deeply into how a student goes about finding an answer or solving a problem. Although written responses on an assignment may indicate areas of concern, more information is often required before appropriate remediation can be provided. Remediation strategies that are based solely upon the analysis of written responses may in fact be inappropriate at times. Holding interviews with students can reveal both unsuspected weaknesses and surprising strengths in their mathematical thinking.

Some guidelines for conducting interviews include:

- Establish an atmosphere of acceptance. The student must feel comfortable enough to verbalize his or her ideas freely. By accepting the student's responses without judgement, but with encouragement to elaborate further, you are communicating not only respect for the student's thinking but also a curiosity to learn more. Each response, whether right or wrong, has the potential of providing information about the student's level of understanding.
- Ask probing questions. During the interview, ask questions and introduce materials that will cause the student to extend and apply concepts/skills to new areas. Rephrase questions using vocabulary familiar to the student, so as to clarify both your intent and the student's thinking. Although some of the questions you ask should be planned, others should be invented spontaneously in order to test your hunches about the student's thinking. Questions asked may take some of the following forms:
  - How did you . . .?
  - Why did you . . .?
  - How do you know that . . .?
  - Have you . . .?
  - How did you decide whether . . .?
  - Can you describe . . .?
  - Are you sure that . . .?
  - How do you feel about . . .?
- Pace the interview appropriately. By allowing an adequate pause following each question before repeating or rephrasing it, you are giving the student time to interpret the question and construct the response. On some tasks, the student may need more than a thirty-second pause, whereas on others, three seconds will suffice. Also, by allowing an adequate pause following the student's response, you are indirectly encouraging the child to elaborate on his or her initial response.
- Be prepared to coax and encourage the student to make a response. A frequent response given by the student may be "I don't know" or "I forgot". Sometimes an extended pause can coax out productive thinking. At other times, the teacher can encourage a response to questions about which the student is unsure by saying:
  - "I know it's not easy to think about. Just give me your best idea."
  - "Pretend that you did know. How do you think it might be done?"

Interviews should have a definite purpose and both teacher and student should be aware of the purpose. Interviews must be planned in advance. In preparing for an interview, the teacher should consider:

- What questions will I ask?
- What basic understandings will I initially assess?
- How can I incorporate the use of manipulative materials?
- How can I vary the task and questions to obtain different perspectives on the student's ability?

Maximum benefits can be gained from an interview by reflecting on your interaction after listening to an audiotape playback. Considerable value can also be gained from sharing your tape with a colleague. By discovering how students interpret and view a problem, the teacher will be better able to make effective on-the-spot decisions in the classroom.

Several guides for interviewing have been included at the end of this section of the manual:

- Resource 2: Sample Protocol for Student Interviews
- Resource 3: Interview Guide for Problem Solving
- Resource 4: Interview Guide for Project Work.

## INVENTORIES/CHECKLISTS

Inventories and checklists are documentation strategies that are used conjunctively with other evaluation strategies. They can be easily designed and customized to meet many different needs and situations. Generally a matrix is created, with indicators of desirable behaviours/outcomes on one side, and ratings/skill levels along another side. As teachers note a particular behaviour, they need only check the appropriate column that evaluates or rates the behaviour.

Checklists lend themselves very well to documenting such elements of the program as:

- understanding of a concept when using a manipulative
- mastered knowledge, skills or process
- work habits
- problem-solving strategies
- social skills.

**CLARIFICATION/EXAMPLE**

**Checklist/Inventory of Social Skills**

| Behaviour to be Observed | Frequently | Sometimes | Never |
|---|---|---|---|
| Is sensitive to the needs and problems of others. | | | |
| Willingly shares ideas/materials. | | | |
| Accepts suggestions and help. | | | |
| Adheres to group plans/decisions. | | | |
| Works cooperatively with others. | | | |
| Respects the property of others. | | | |
| Appears to like group work. | | | |

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A variety of inventories/checklists useful in monitoring the development of problem-solving skills are provided in the "Problem Solving" section of this manual:

- Attitude Inventory Items
- Observational Checklist of Problem-Solving Attitudes and Behaviours
- Observational Rating Scale of Problem-Solving Attitudes and Behaviours
- Checklist of Problem-Solving Strategies
- Problem-Solving Strategy Inventory.

Similar checklists can be developed to monitor and evaluate other components of the mathematics program.
e.g., – use of estimation strategies
– facility with mental arithmetic.

## ANECDOTAL RECORDS

Anecdotal records refer to the spontaneous documentation of notable behaviour, effort and achievement. These records provide specific and dated information that can form the basis for conclusions and assessments. Anecdotal records often prove invaluable in clarifying assessments and add credibility to observations and recommendations being offered in student, parent and/or teacher meetings.

Anecdotal records may include observations on:

- attitude/work habits
- social skills
- effort and homework
- changes in performance
- specific strengths/deficiencies
- communication skills.

**CLARIFICATION/EXAMPLE**

**Anecdotal Record Card**

Student: Sue Jones

Date: 04/10

Comments:

- knows how and when to look for a pattern in problem solving
- keeps trying even when she has trouble finding a solution
- shows interest in helping others
- often enters numbers into the calculator in the wrong order when dividing

Anecdotal records may be kept in a daily or weekly diary, in student files, in the marks record book, or in a common file of short dated notes.

## WRITTEN ASSIGNMENTS

### PAPER-AND-PENCIL TESTS

Traditional paper-and-pencil tests are probably the most widely used method of evaluating student performance. While effective in assessing factual and procedural knowledge, these tests often elicit feelings of inadequacy and self-doubt for students who have experienced previous difficulty or failure. These negative feelings affect test performance, and may cause a cycle of repeated failure to continue.

Students may have learned the information presented in class, but are unable to demonstrate the knowledge because of poor reading skills, visual perception problems, inadequate reasoning and comprehension, fine-motor difficulties or other related deficiencies. Special needs of the student can be met through minor alterations in the construction of teacher-made tests. Constructing tests according to special needs can mean the difference between success and failure for some students.

The suggestions that follow will assist teachers to construct their tests according to the needs of individual students.

#### TEST DIRECTIONS

- Keep directions simple and avoid unnecessary words.
- Define words that are unfamiliar or abstract.
- Give an example of how the student is to respond.

**CLARIFICATION/EXAMPLE**

Directions

Add the fractions.
Give the answer in lowest terms.

Example:

$$\begin{array}{rl} & \frac{1}{3} = \frac{2}{6} \\ + & \\ & \frac{1}{2} = \frac{3}{6} \\ \hline & \qquad \frac{5}{6} \end{array}$$

Some teachers feel that providing a model invalidates a test designed to measure knowledge of mathematical process. However, some students may never be able to remember a formula or complex set of processes without visual prompts. For them, failure is almost a certainty without modifications.

- Avoid oral directions as the only means of communication. Read directions orally as well as clearly writing them on the test.

## TEST ITEMS

- Provide manipulative objects that make the problems more concrete.
- When using computation problems, avoid mixing different problem formats in the same section.

**CLARIFICATION/EXAMPLE**

A student with organizational or visual tracking problems may have difficulty in "changing gears" from problem A to problem B. It might be better to put these questions in two different sections of the test.

| Problem A | Problem B |
|---|---|
| $\begin{array}{r} 468 \\ +\ \ 31 \\ 896 \\ \hline \end{array}$ | $670+40+861=$ |

- Provide visual prompts for computational problems.

**CLARIFICATION/EXAMPLE**

$$\begin{array}{r} \square\ \ 73 \\ +\ \ 69 \\ \hline \end{array} \qquad \text{OR} \qquad \begin{array}{r} \text{R} \\ 61\overline{)263} \\ -\ \underline{\qquad} \end{array}$$

- Provide formulas and explain the meaning of special symbols (e.g., $A=1\times w$, $<$ means "less than").
- Provide a set of written steps for applying difficult algorithms.

**CLARIFICATION/EXAMPLE**

Long Division

1. Divide
2. Multiply
3. Subtract
4. Check
5. Bring down

- Incorporate the use of the calculator into computational problems that are not intended to assess understanding of the paper-and-pencil process.

- Design word problems that:
  - are relevant to the student's personal experience
  - consist of simple sentences and familiar words.

- Underline or circle key words in word problems (e.g., less, more).

- Ask students to circle the correct response in multiple choice items. This reduces the possibility of copying errors when transferring letters to blanks. Arrange the answer and distractors vertically on the page.

### CLARIFICATION/EXAMPLE

You have a board 48 centimetres long. If you cut off a 6-centimetre piece, how much is left?

a. 38 centimetres
[✓ b. 42 centimetres]
c. 48 centimetres

- Keep all matching items brief, and have only one correct answer for each item. Use no more than ten items in the matching lists. If you have more than ten items, group them by concepts in clusters of ten.

## TEST DESIGN

- Construct the test in logical sequential order, from simple to complex problems.

- Use test items that reflect the content taught and techniques used to teach.

- Prepare a study guide for the test that matches the design of the actual test.

- Design the test to reflect the student's knowledge, rather than ability to follow complicated directions, to use difficult vocabulary, or to work under time constraints.

- Adjust the readability level of the test to meet student needs.

- Prepare the test in short sections that can be administered individually if necessary.

- Use graph paper for paper-and-pencil computational problems. The squares may help the student to keep figures aligned.

## DIAGNOSTIC WRITING ASSIGNMENTS

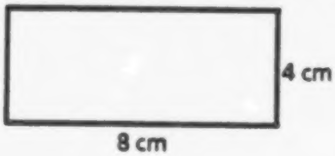
Communication, both oral and written, plays a vital role in the teaching and learning of mathematics. Students should not only be involved in group problem-solving activities where they must share ideas with one another, but should also have the opportunity to express their ideas in writing.

Writing in mathematics requires students to reflect on their reasoning and thus enhances cognitive development. Diagnostic writing assignments require students to respond to specific mathematical questions in an expressive writing style. Written responses often force students to examine their own understanding of concepts, and communicates to teachers how much students really know about a concept. Written responses also provide insight into how students think. Information revealed through written responses will enable teachers to diagnose students' strengths and weaknesses, and plan lessons that promote mathematical thinking and understanding. Appropriate remedial and enrichment activities can be determined on the basis of understanding demonstrated for a given concept.

### CLARIFICATION/EXAMPLE

**Comparing Perimeter and Area**

What is wrong with each of the following statements about the rectangle?



- The area is 32 cm.
- The perimeter is 12 cm.
- The area is 12 $cm^2$.
- If the length is increased by 2 cm, the area will be increased by 2 $cm^2$.
- If the perimeter is doubled, the area will be doubled.
- The perimeter of any rectangle is always smaller than the area.

Diagnostic writing assignments (i.e., writing to describe thinking and reasoning) should be kept relatively short. As this type of writing may be new to your class, encourage students to spend time going beyond the "right answer". Written assignments should not be graded for a mark but, rather, assessed for understanding. Students might be awarded bonus points based on their effort and presentation.

### CLARIFICATION/EXAMPLE

0 points – no effort
1 point – mediocre attempt with little or no understanding
2 points – good attempt, but with some lack of understanding
3 points – high level of effort and understanding

Diagnostic writing assignments can be kept in a student diary or logbook, and might be assigned on a regular basis (once or twice a week), or in lieu of a regular quiz.

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## SELF AND PEER EVALUATION

Self and peer evaluation involves having students rate their own work or the work of others, using clearly defined criteria and guidelines supplied by the teacher. This process encourages students to examine their work in some depth, and facilitates the development of analytical and critical thinking skills. As students often perceive their classmates as more accepting of their thoughts and ideas than teachers, peer evaluation may reduce fear of failure and help students to develop confidence about expressing their ideas.

## GUIDELINES FOR IMPLEMENTING SELF AND PEER EVALUATION

- Do not assume that students can evaluate their own or others' work with little or no preparation. Provide clearly defined criteria and guidelines for students to use in making judgments about a piece of work (e.g., a checklist of what to look for, a list of simple questions that students should ask themselves, a sample solution).

- Provide students with concrete examples of student work that differs in quality, and explain how particular criteria and guidelines were used to determine each mark. Teachers may wish to coach the class in marking a sample assignment, using established guidelines.

- Organize for peer evaluation in partner situations or in small groups (e.g., 3-5 students). Provide for an appropriate selection of students in each group by assigning membership rather than letting students form their own groups. Group membership may be changed from one peer evaluation session to another.

- Throughout the process of self and peer evaluation, encourage students to focus attention on:
  - positive features of the work being evaluated
  - aspects of the work that are not clear, incomplete and/or incorrect
  - specific suggestions for improvement.

**CLARIFICATION/EXAMPLE**

**Guidelines for Self Evaluation in Group Problem Solving**

1. I contributed ideas without waiting to be asked. One idea I contributed was ______________________.
2. I supported my suggestions/ideas with specific details and examples.
3. I can recall other group members' ideas. One important idea was ______________________.
4. I asked other group members questions about their ideas.
5. I showed respect for other members' ideas and opinions, even if I disagreed.
6. I learned something as a result of listening to another member's ideas. I learned that ______________________.
7. I think I helped someone else to learn as a result of an idea I contributed. The idea/suggestion was ______________________.
8. I have a clearer understanding of the problem and related issues as a result of my participation in the group.

## RESOURCE I: OBSERVATIONAL CHECKLIST OF STUDENT BEHAVIOURS

### LEVEL OF INDEPENDENCE WITH WORK:

| | ALWAYS | OFTEN | SOMETIMES | NEVER |
|---|---|---|---|---|
| • Settles down to work upon entering class. | | | | |
| • Spends time on task; has satisfactory attention span. | | | | |
| • Takes responsibility for making up work after absences. | | | | |
| • Takes responsibility for supplies and equipment. | | | | |
| • Follows directions; completes tasks with minimal assistance. | | | | |
| • Asks for and accepts help when needed. | | | | |
| • Accepts a challenge; works productively on tasks of increasing difficulty. | | | | |
| • Displays self-confidence and pride in work. | | | | |

### UNDERSTANDING OF CONCEPTS AND SKILLS:

| | | | | |
|---|---|---|---|---|
| • Uses the necessary vocabulary and concepts. | | | | |
| • Uses appropriate operations, strategies and principles. | | | | |
| • Asks questions, volunteers answers. | | | | |
| • Answers questions that involve thought (e.g., What do you think?). | | | | |
| • Demonstrates understanding through ability to generalize and apply. | | | | |
| • Displays curiosity about concepts, relationships and applications. | | | | |
| • Works independently on projects and research. | | | | |

## RESOURCE I: OBSERVATIONAL CHECKLIST OF STUDENT BEHAVIOURS (continued)

### ABILITY TO INVESTIGATE AND SOLVE PROBLEMS:

| | ALWAYS | OFTEN | SOMETIMES | NEVER |
|---|---|---|---|---|
| • Understands and defines problems. | | | | |
| • Develops a systematic plan of attack. | | | | |
| • Gathers information, using a variety of sources. | | | | |
| • Carries out plans and procedures, seeking help when necessary. | | | | |
| • Uses appropriate strategies and processes. | | | | |
| • Considers alternatives before reaching a solution/decision. | | | | |
| • Evaluates solutions to the problem and decisions made. | | | | |
| • Considers other ideas/opinions/solutions. | | | | |

### INTERPERSONAL SKILLS AND SOCIAL GROWTH :

| | | | | |
|---|---|---|---|---|
| • Resists aggressive and impulsive behaviours. | | | | |
| • Volunteers to work in group situation. | | | | |
| • Cooperates and contributes to group goals. | | | | |
| • Listens to peers; considers the opinions of others. | | | | |
| • Participates in oral discussions. | | | | |
| • Helps others willingly. | | | | |

# RESOURCE 2: SAMPLE PROTOCOL FOR STUDENT INTERVIEWS[1]

Sample interview questions that might be used in assessing the student's understanding of operations, basic facts and algorithmic procedures are provided. It is not suggested that every question be used during each interview. Their purpose is simply to illustrate how carefully sequenced and well-phrased questions can provide worthwhile information about the student's understanding of process and skill.

## UNDERSTANDING OF OPERATIONS

The following questions and tasks relate to the operation of division. Similar questions and tasks could be developed for other operations. These questions will indicate:

- how students interpret the operation
- whether the action can be identified and simulated
- whether the operation can be related to personal experience.

**INTERVIEW QUESTIONS:**

1. Read "56 ÷ 7" for me.

2. Do you know other ways of reading this? Students may frequently read statements like 56 ÷ 7 or equivalent subtraction statements (e.g., 56 − 7) in either direction.

3. Do you know other names for " ÷ "? (Point to the symbol, or use terminology from the student's response to the previous task.)

4. Use counters (or draw a sketch) to show 12 ÷ 3.
   - Is the student able to simulate the action for 12 ÷ 3 with counters?
   - Is the student's preferred division interpretation one of measurement or partition, and will the student be consistent throughout the interview?

5. Make up a word problem for 12 ÷ 3.

## KNOWLEDGE OF BASIC FACTS

The following examples deal with multiplication facts. Similar strategies could be developed for basic facts related to other operations. Through appropriate interview questions, the teacher can assess the student's ability to:

- recall simple basic facts
- use known facts to derive answers for other facts
- apply the properties of "zero" and "one"
- use strategies for checking answers or finding an answer for a fact that is not known.

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[1] Reprinted with permission from *The Arithmetic Teacher*. Copyright 1988 by the National Council of Teachers of Mathematics.

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## RESOURCE 2: SAMPLE PROTOCOL FOR STUDENT INTERVIEWS (continued)

**INTERVIEW QUESTIONS:**

1. Give the answer for 4×0.
2. What is your rule for finding the answer when a number is multiplied by zero? Use your rule for 25×0.
3. Give the answer for 6×1.
4. What is your rule for finding the answer when a number is multiplied by one? Use your rule for 37×1.
5. If the answer for 7×9 is 63, how could you use this answer to find the answer for 8×9?
6. Pretend you forgot the answer for 7×8. How would you find or calculate this answer?
7. As I show you a flash card, tell me, without calculating the answer, whether you think the fact is easy or hard. (After the cards have been sorted as "easy" and "hard", answers are solicited and checked by turning the appropriate cards over, first for the facts classified as easy and then for those identified as hard.)

## UNDERSTANDING OF ALGORITHMIC PROCEDURES

Subtraction is used to illustrate a possible protocol for an algorithmic procedure during an interview. These procedures can be adapted for other operations. Questions posed during the interview should relate to:

- the order of "attack" used by the student
- the reasons for "moves" made during the calculation
- the meaning of the digits that are being manipulated.

**INTERVIEW QUESTIONS:**

1. Show me how you would find the answer for 57 – 34 = ____. Talk to me as you are doing it.

   Explanations provided by the student should provide answers for the following questions:
   - Why did you record one number below the other?
   - Why did you begin "here"?
   - You said "5 – 3". What do these digits really mean?
2. Explain to me what you are thinking as you find the answer for 71 – 48 = ____.
3. Show me how you would explain how to find the answer for 4003 – 897 = ____ to a student in a lower grade (or to a younger brother or sister).
4. Use these base ten blocks to show how you would find the answer for 605 – 67 = ____.

# RESOURCE 3: INTERVIEW GUIDE FOR PROBLEM SOLVING[1]

1. Establish rapport to help the student feel comfortable.

2. Ask the student to "talk about what he or she is doing or thinking" while solving the problem. Explain that this will enable you to help the student to become a better problem solver.

3. Give this problem to the student:

   At an amusement park, Mike and his 5 friends decided to take enough roller coaster rides so that each person would take a ride with every other person exactly once. How many rides were taken if only 2 students went on each ride?

4. As the student attempts to understand the problem question and conditions, observe the student and ask questions such as the following, if appropriate:

   a. What did you do first when given the problem? Next?
   b. What question is asked in the problem? What are the important facts and conditions in the problem? Do you need any information not given in the problem?
   c. Is there anything you don't understand about the problem?

5. As the student works on a solution to the problem, remind him or her again to talk about it, and ask questions such as the following, if appropriate:

   a. What strategy are you using? Do you think it will lead to a solution? Have you thought about using other strategies? Which ones?
   b. Where are you having difficulty? What are your ideas about where to go from here?

6. As the student finds an answer to the problem, observe the ways, if any, in which he or she checks the answer and its reasonableness as a solution. Ask questions such as:

   a. Are you sure this is the correct answer to the problem? Why?
   b. Do you think it is important to check your answer? Why?

7. After the student has solved the problem, ask questions such as:

   a. Can you describe the solution to the problem and how you found it?
   b. Is this problem like any other problem you've solved? How?
   c. Do you think this problem could be solved in another way? What are your ideas?
   d. How did you feel while you were solving this problem? How do you feel now that you have found a solution?

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[1] Reprinted with permission from *How to Evaluate Progress in Problem Solving*. Copyright 1987 by the National Council of Teachers of Mathematics.

## RESOURCE 4: INTERVIEW GUIDE FOR PROJECT WORK[1]

### THE TASK ENVIRONMENT

1. How did you select this investigation/project? ____________________

____________________

2. Are you interested in it?
___ very ___ somewhat ___ not at all

3. What did the teacher do when giving out the assignment? ____________________

____________________

Examples:
- give verbal guidelines
- give written guidelines
- select the topic
- provide a strategy
- increase your interest

4. Who do you expect will examine the results of your investigation/project? ____________________

____________________

### PREVIOUS KNOWLEDGE

5. Have you undertaken an investigation/project like this before?
___ yes ___ no

6. What did you know about this topic before you started? ____________________

____________________

7. Which of the following mathematical ideas/processes did you use in carrying out your investigation/project?

___ geometric shapes
___ order of operations
___ fractions
___ rounding numbers
___ mental arithmetic
___ units of measure
___ banking skills
___ scale drawing

___ large numbers
___ decimals
___ estimation
___ calculator usage
___ graphs and tables
___ budgeting
___ comparative shopping
___ patterns and designs

## RESOURCE 4: INTERVIEW GUIDE FOR PROJECT WORK (continued)

### PLANNING

8. How did you plan your investigation/project? ______________________

______________________________________

9. Did you make an outline of the steps you would take?
____ yes ____ no

What kind of thinking did you do first? ______________________

______________________________________

10. Did you do any research? ____ yes ____ no

What sources of information did you use? ______________________

______________________________________

### REVIEWING AND APPLYING

11. What strategies did you find most helpful in completing your investigation/project?

Examples:

- ____ reading and research
- ____ following directions received
- ____ explanations from the teacher
- ____ studying diagrams/models
- ____ receiving help from parents
- ____ studying with a partner
- ____ explaining problems to classmates
- ____ having group discussions about solutions
- ____ other

12. Explain three ways that you might make use of the results of your investigation in everyday life.

______________________________________

______________________________________

______________________________________

### EVALUATING

13. What have you learned in completing this investigation/project? ______________________

______________________________________

14. What grade did you think you would get? ______________________

Why? ______________________________________

15. What was the teacher's evaluation? ______________________

How was this evaluation different from your anticipated grade? ______________________

______________________________________

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[1] For permission to reprint copyrighted material grateful acknowledgement is made to the following: Division for Learning Disabilities, *Learning Disabilities Focus* for the material adapted from "Alternatives in the Assessment of the Learning Disabled Adolescent: A Learning Strategies Approach" by Judith Wiener, Spring 1986.

# APPENDIX A: ANNOTATED LIST OF LEARNING RESOURCES

## STUDENT RESOURCES

Alberta Consumer and Corporate Affairs. *2000 A. D., A Guide to Financial Awareness*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1986. A booklet that provides an introduction to the subject of investing money. The booklet describes a variety of financial alternatives, along with associated risks and rewards. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *Consumer Complaints: A Self-Help Handbook*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1985. Provides information that will help students resolve consumer complaints about products that don't perform as they should, inadequate services, and businesses that fail to live up to the terms of their sales contracts. Information is presented in magazine format. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *"Consumer Talk" Information Sheets*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs. Consists of a series of eight consumer information sheets presented in tabloid-newspaper format. Provides information useful in developing awareness of consumer rights/responsibilities and personal strategies for dealing with consumer problems. Information sheets in the series include *Buying a Used Car*. *Credit*, *Shop Around*, *Shopping at Home*, *Returns*, *Tenants: Moving In*, *Tenants: Living There* and *Tenants: Moving Out*. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *Consumer Tips*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs. Consists of a series of consumer information sheets dealing with cars, general shopping matters, housing/real estate, insurance, money management/credit, securities/investments, and consumer services. Information sheets in the series that are particularly relevant include *How to Use Credit Responsibly* and *Protect Your Credit Rating*. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *Keeping Your Balance: Information for Financial Consumers*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs. Consists of a series of consumer information sheets that provide strategies for dealing with consumer problems and making responsible financial decisions. Information sheets in the series that are particularly relevant include *Putting Your Spending Plan on Paper* and *Take Care of Your Savings*. Class sets are available upon request.

Alberta Consumer and Corporate Affairs. *Moving Out*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1986. Provides information that is useful in developing consumer skills. Topics relate to housing, credit, transportation, the marketplace, lifestyle and careers. Information is presented in magazine format. Class sets are available upon request. An *Instructor's Guide* is also available. The guide provides programming ideas, discussion questions, case studies, community resources and activity suggestions.

Alberta Consumer and Corporate Affairs. *Taking Charge of Your Money*. Edmonton, Alberta: Alberta Consumer and Corporate Affairs, 1986. Provides strategies for making the most of your money through personal planning and budgeting. Topics relate to setting financial goals, estimating costs, planning spending, keeping records and using credit. Information is presented in magazine format. Class sets are available upon request.

Canadian Bankers' Association. *Credit Wise*. Toronto, Ontario: Canadian Bankers' Association, 1983. A booklet providing strategies that will assist individuals to analyze their financial situation and make appropriate credit choices. The publication is useful in developing a general understanding of the use of credit in society. Class sets are available upon request.

Canadian Bankers' Association. *Helping You Bank*. Toronto, Ontario: Canadian Bankers' Association, 1989. A booklet that provides suggestions and information about how to choose a bank account that suits your needs, how to open a bank account, and how to use a bank account. The booklet explains common banking terms, is well illustrated and very readable. Class sets are available upon request.

Canadian Imperial Bank of Commerce. *CIBC's Greatest Hits*. Toronto, Ontario: Canadian Imperial Bank of Commerce, 1989. An interactive youth presentation that encourages student participation through discussion with a representative of the local Canadian Imperial Bank of Commerce. The program involves use of print and video resources, and can be designed to address one or more relevant banking products and/or services. Contact the local Canadian Imperial Bank of Commerce for further information.

Cutting, Robert, et al. *Math You Need, Book 2*. Scarborough, Ontario: Nelson Canada, A Division of Thomson Canada Limited, 1982. A workbook of developmental and practice activities that involve the application of mathematical skills in everyday situations. Topics relate to mathematics in sports, automotive mathematics, keeping a place of your own, spending and earning money, savings and interest, loans and credits, and buying insurance. The workbook also provides a bank of exercises on basic computation and measurement skills that students need to be able to use in coping with everyday situations.

Dladay, Les. *Decisions, Making Personal Economic Choices*. St. Paul, Minnesota: EMC Publishing Corporation, 1985. A workbook that provides an extensive set of student activity worksheets on finding employment, computing income, using bank accounts, preparing budgets and comparison shopping. Activities focus attention on the use of critical and creative thinking skills when making personal economic choices. Some topics involve references to American standards/practices and do not reflect the Canadian situation.

Elrick, Thomas. *Forms in Your Life: A Student Workbook and Guide to Everyday Forms*. Toronto, Ontario: Globe/Modern Curriculum Press, 1981. A workbook that provides guidelines for using a variety of forms encountered in everyday and work-related situations. Specimen forms included in the publication are those currently used by Canadian companies, institutions and federal and provincial government agencies.

Lennox, Art. *Banking, Budgeting and Employment*. Phoenix, New York: Frank E. Richards Publishing Company Incorporated, 1984. A workbook that is designed to supplement instruction in common banking, budgeting and employment terminology and procedure. The publication includes crossword and maze puzzles, vocabulary lists, exercises and story problems. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

Lunney, Jeri, et al. *MathSense, Book 2*. Scarborough, Ontario: Nelson Canada, A Division of Thomson Canada Limited, 1991. A thematic workbook that provides for the application of math skills to real-life situations at home and at work. The workbook is very readable, contains numerous charts/graphs, and integrates calculator skills and problem-solving strategies throughout. Topics relate to obtaining a job, getting paid, repairing/renovating, using your money wisely, investing/borrowing, comparison shopping, travelling, and fitness/sports.

Lyng, Merwin, et al. *Applied Mathematics*. Markham, Ontario: Houghton Mifflin Canada Limited, 1989. A student textbook that supports the development of mathematical skills frequently used in the world of work. Activities focus attention on industrial measurements, algebra in the workplace, applied geometry, occupational simulations, and trigonometry and statistics at work. A *Teacher's Manual and Answer Key* that provides teaching strategies, warm-up activities, unit tests, teacher aids and a complete answer key is also available from the publisher.

Revenue Canada Taxation. *Teaching Taxes Student's Workbook*. Ottawa, Ontario: Revenue Canada Taxation, published annually. Designed for use in junior and senior high schools, this workbook is intended for students who are just becoming familiar with the Canadian income tax system. Students are introduced to the General Tax Guide, Tax Tables, as well as numerous other forms and guides through a variety of questions, exercises and projects. The *Teacher's Update*, also published annually, provides background information, teaching suggestions and answers to the student's exercises. The *Taxopedia* (1989), a permanent reference book for students studying the Canadian tax system, outlines the history of taxation in Canada and describes the Canadian tax system. These publications are available, free of charge, from district taxation offices.

Wool, John. *Useful Arithmetic, Volume 1*. Phoenix, New York: Richards Publishing Company, 1987. A workbook designed to assist students to apply mathematical skills in practical situations. Activities focus attention on buying goods and services, comparing prices, the work week, pay deductions and the cost of different transportation alternatives. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

Wool, John. *Useful Arithmetic, Volume 2*. Phoenix, New York: Richards Publishing Company, 1988. A workbook designed to assist students to apply mathematical skills in practical situations. Activities focus attention on money problems, mathematics in the marketplace, earning money and banking services. Material within the workbook has been simplified and is very readable. Some activities involve references to American standards/practices and do not reflect the Canadian situation.

## TEACHER RESOURCES

Alberta Education. *Problem Solving Challenge for Mathematics*. Edmonton, Alberta: Alberta Education, 1985. Produced as a junior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, a bank of problems, methods of evaluating problem solving in performance and a 36-item problem-solving biography.

BEST COPY AVAILABLE

Alberta Education. *Problem Solving in Mathematics: Focus for the Future*. Edmonton, Alberta: Alberta Education, 1987. Produced as a senior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, project and computer related problems, a bank of mathematics problems, methods of evaluating problem solving performance and a 123-item problem-solving bibliography.

Board of Education for the City of Etobicoke. *Making the Grade: Evaluating Student Progress*. Scarborough, Ontario: Prentice-Hall Canada Inc., 1987. Provides a variety of suggestions for modifying and differentiating among evaluation techniques used with students of differing ability levels.

Charles, R., et al. *How to Evaluate Progress in Problem Solving*. Reston, Virginia: The National Council of Teachers of Mathematics, 1987. Discusses goals of problem-solving evaluation, evaluation techniques, organizing and managing an evaluation program, and using evaluation results.

Easterday, K., et al. *Activities for Junior High School and Middle School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1981. Provides meaningful activities in a wide range of mathematics topics (e.g., a manipulative approach to integer addition/subtraction; paper-folding activities for exploring two-dimensional relationships; games that provide practice in mental arithmetic/fractions/percent/decimals/proportions/measurement/geometry; probability activities using non-standard dice/marbles/the alphabet; a card game to practise fraction-decimal-percent equivalencies; alternative approaches for developing equation-solving skills).

Hope, J., et al. *Mental Math in Junior High*. Palo Alto, California: Dale Seymour Publications, 1988. Provides teaching notes and blackline masters for a series of 50 lessons in mental calculation. Topics covered include working with compatible numbers, working from the front-end, dealing with trailing zeros, as well as more advanced lessons that develop methods of calculating mentally with fractions, decimals and percents. The publisher grants permission to individual teachers who have purchased this book to reproduce the blackline masters as needed for use with their own students.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1989. Provides a set of standards for mathematics curricula in North American schools (K-12) and for evaluating the quality of both the curriculum and student achievement. The document represents the consensus of NCTM's members about the fundamental content that should be included in the school mathematics curriculum and about key issues concerning the organization and implementation of student and evaluation programs. Establishes a broad framework to guide reform in school mathematics in the next decade.

## TECHNOLOGY AND MEDIA

### COMPUTER SOFTWARE

*Managing Lifestyles: Survival Math Skills*. Pleasantville, New York: Sunburst Communications Incorporated, 1984. (Apple II, TRS-80.) This package provides three simulations that are modelled after practical situations that students may encounter in everyday life. ("Comparison Shopping" involves shopping for a week's supply of groceries and planning a menu. "Budget for Success" requires students to plan and manage living expenses for six simulated months. "Calorie Counter" incorporates students' personal data into an activity involving counting calories to help maintain a desired weight.) Simulations within the package are designed to enhance students' ability to perform basic operations with whole numbers/decimals/fractions/percent, to develop and use formulas, and to determine unit prices and interest rates. The package includes one disk, backup, teacher's guide with reproducible worksheets, masters for overhead transparencies and student activities.

*Math Strategies: Problem Solving*. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1983. (Apple II, TRS-80.) This program is designed to provide instruction and practice in solving multiple-step problems using appropriate strategies. Topics include simplifying a problem, breaking a problem into parts, identifying needed additional information, and making a model of the problem. The package includes two disks, a teacher's guide and 20 student texts.

*Money Manager*. Washington, D.C.: Computer Age Education Incorporated, 1984. (Apple II.) This program is a simulation designed to help students develop and practise skills in financial management. Students begin with a set amount of money in cash and in their chequing and savings account. During the simulated eight week period, the user earns money, pays bills, makes purchases, manages cash, controls bank accounts, and makes other financial decisions. The package includes one disk, a teacher's guide and a user's guide.

*SRA Computer Drill and Instruction, Mathematics, Level D*. Willowdale, Ontario: Science Research Associates (Canada) Limited, 1981-83. (Apple II, IBM PC.) This program provides drill exercises in computation, numbers and numeration, fractions, decimals, ratio and percent, measurement, pre-algebra and applications. The program does not provide answers or a method of solving problems. May be used by students in unmanaged or teacher-managed form. Multi-disk package includes a teacher's guide.

*Spreadsheets for Students*. Mississauga, Ontario: Copp Clark Pitman Limited, 1985. (Apple II, Commodore 64.) This spreadsheet-style computer program requires students to apply mathematical skills to practical situations involving shopping and sales tax, simple budgeting, savings accounts and loan variables. Activities within the program encourage students to estimate, approximate and round, formulate strategies and predict outcomes. The software is simplified and does not present full-size spreadsheet programs with advanced features. The package includes two disks, a set of reproducible blackline masters, and teacher's notes.

### VIDEO PROGRAMS

*A History of Income Tax in Canada*. Revenue Canada Taxation. A 13-minute video that outlines the history of income tax in Canada. Can be borrowed, free of charge, from the public affairs officer at any district taxation office. Teachers may make copies if they wish.

*Mathways*. Kinetic, 1980. Program consists of a series of four 15-minute videos on various topics in mathematics. Designed to develop/review concepts and skills for students in Grades 7 to 12. Method of presentation is animation with voice-over. Available through ACCESS NETWORK and/or Regional Resource Libraries.

> *The Decimal Point*: Introduces or reviews fundamental decimal point concepts. Explains the meanings of various positions of the decimal point within a numeral.
> *The Percent*: Introduces the percent symbol. Uses a "sales pitch" to convince the viewer that the percent symbol is the hardiest symbol ever devised.
> *Areas*: Assumes that prior to viewing this program students understand the general concept of area and how to find areas of triangles and quadrilaterals. Introduces the symbol π and its use in computing area of a circle and surface area of a cylinder.
> *Volumes*: Uses layers of cubes to fill a rectangular prism to illustrate volume. Covers the computation of the volume of prisms, cylinders and cones.

*Math Wise*. Bloomington, Indiana: Agency for Instructional Television, 1981. This part-documentary and part-drama series consists of twelve 15-minute videos. The program is designed to motivate students to use math skills in making better decisions and solving problems in everyday situations. The videos are not mathematics "lessons"; rather, the actors use mathematics in a variety of practical situations. A commentator introduces the mathematics topic of each video and closes by reviewing the mathematically significant points in each of the "stories". The program provides a teacher's guide that suggests ideas for discussion and a student worksheet for each of the videos. Available through ACCESS NETWORK and/or Regional Resource Libraries.

Measuring 01: *Measuring Instruments*
Measuring 02: *Formulas*
Comparing 01: *Numerical Comparisons*
Comparing 02: *Percent*
Comparing 03: *Finding a Common Unit*
Comparing 04: *Proportion*
Locating/Interpreting 01: *Organizing Information*
Locating/Interpreting 02: *Graphs*
Locating/Interpreting 03: *Maps, Charts, and Tables*
Predicting 01: *Probability*
Predicting 02: *Choosing a Sample*
Predicting 03: *Averages*

*Revenue for Canada*. Revenue Canada Taxation. A 10-minute video that explains the basis of income tax, the self-assessment system and the role of the tax department. The video is suitable for students who have no familiarity with the world of taxation. Can be borrowed, free of charge, from the public affairs officer at any district taxation office. Teachers may make copies if they wish.

*Street Cents*. Alberta Consumer and Corporate Affairs/Canadian Broadcasting Corporation, 1990. This program consists of twelve 30-minute videos designed to develop prudent judgement and consumer skills among Canadian teens. The program features food reoorts, product surveys, views from the marketing and advertising worlds, and suggests ways for teens to save and make the most of their money. Available through both ACCESS NETWORK and regional offices of Consumer and Corporate Affairs.

Program 1: *Advertising*
Program 2: *Food*
Program 3: *Jobs*
Program 4: *Music*
Program 5: *Fashion*
Program 6: *Future*
Program 7: *Millionaire*
Program 8: *The Environment*
Program 9: *Cheap Stuff*
Program 10: *Image*
Program 11: *Holidays*
Program 12: *Wrap-Up*

# APPENDIX B: ALBERTA CONSUMER AND CORPORATE AFFAIRS OFFICES

Calgary

310 Centre 70
7015 Macleod Trail South
Box 5880, Postal Station "A"
Calgary, Alberta
T2H 2M9
Phone: 297-5700

Edmonton

3rd Floor Capilano Centre
9945 - 50 Street
Edmonton, Alberta
T6A 0L4
Phone: 427-5782

Fort McMurray

Fort McMurray Provincial Building
5th Floor West Tower
9915 Franklin Avenue
Fort McMurray, Alberta
T9H 2K4
Phone: 743-7231

Grande Prairie

Grande Prairie Provincial Building
10320 - 99 Street
Box 7
Grande Prairie, Alberta
T8V 6J4
Phone: 538-5400

Lethbridge

300 Professional Building
740 - 4 Avenue South
Bag Service 3014
Lethbridge, Alberta
T1J 4C7
Phone: 381-5360

Medicine Hat

303 Provincial Building
770 - 6 Street S.W.
Medicine Hat, Alberta
T1A 4J6
Phone: 529-3535

Peace River

Peace River Provincial Building
9621 - 96 Avenue
Bag 900, Box 9
Peace River, Alberta
T0H 2X0
Phone: 624-6214

Red Deer

2nd Floor Provincial Building
4920 - 51 Street
Red Deer, Alberta
T4N 6K8
Phone: 340-5241

# APPENDIX C: REGIONAL OFFICES OF EDUCATION

Grande Prairie Regional Office

12th Floor, 214 Place
9909 - 102 Street
Grande Prairie, Alberta
T8V 2V4
Phone: 538-5130
FAX: 538-5135

Edmonton Regional Office

7th Floor, Westcor Building
12323 Stony Plain Road
Edmonton, Alberta
T5N 3Y5
Phone: 427-2952
FAX: 422-9682

Red Deer Regional Office

3rd Floor West, Provincial Building
4920 - 51 Street
Red Deer, Alberta
T4N 6K8
Phone: 340-5262
FAX: 340-5305

Calgary Regional Office

Room 1200, Rocky Mountain Plaza
615 Macleod Trail, S.E.
Calgary, Alberta
T2G 4T8
Phone: 297-6353
FAX: 297-3842

Lethbridge Regional Office

Provincial Building
200 - 5th Avenue, South
Lethbridge, Alberta
T1J 4C7
Phone: 381-5243
FAX: 381-5734

# COPYRIGHT ACKNOWLEDGEMENTS

Alberta Education wishes to thank the following authors, publishers, and agents for granting permission to include copyrighted materials:

Alberta Consumer and Corporate Affairs for the excerpts from *Moving Out*, 1988.

Alberta Consumer and Corporate Affairs for the excerpts from *Taking Charge of Your Money*, 1988.

Alberta Consumer and Corporate Affairs for the excerpts from *TIPS*.

Alberta Consumer and Corporate Affairs for the excerpts from *2000 A.D. A Guide to Financial Awareness*, 1989.

Alberta Education for the excerpts from *Junior High Mathematics Teacher Resource Manual*, 1988.

Alberta Education for the excerpts from *Let Problem Solving be the Focus of the 1980's*, 1983.

Alberta Education for the excerpts from *Problem Solving Challenge for Mathematics*, 1985.

Alberta Education for the excerpts from *Problem Solving in Mathematics: Focus for the Future*, 1987.

Boston, Allyn and Bacon for excerpts from *Strategies for Teaching Students with Learning and Behaviour Problems*, 1988, by Dr. C. Bos and S. Baughn.

Canadian Bankers' Association for excerpts from *Credit Wise*, 1983.

Division for Learning Disabilities, *Learning Disabilities Focus*, for material adapted from "Alternatives in the Assessment of the Learning Disabled Adolescent: A Learning Strategies Approach", by Judith Wiener, 1986.

Kitchener-Waterloo Record Ltd. for material adapted from *That Figures*, 1981, by Marilyn Hintz and Jerry Ziegler.

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National Council of Teachers of Mathematics for the excerpts from *The Arithmetic Teacher*, 1985 and 1988.

National Council of Teachers of Mathematics for the excerpts from *Curriculum and Evaluation Standards for School Mathematics, Working Draft*, 1987.

National Council of Teachers of Mathematics for the excerpts from *How to Evaluate Progress in Problem Solving*, 1987, by Randall Charles et al.

National Council of Teachers of Mathematics for the excerpts from *The Mathematics Teacher*, 1989.